Heisenberg-Limited Spin Squeezing via Bosonic Parametric Driving

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Spin-spin interactions generated by a detuned cavity are a standard mechanism for generating highly entangled spin squeezed states. We show here how introducing a weak detuned parametric (two-photon) drive on the cavity provides a powerful means for controlling the form of the induced interactions. Without a drive, the induced interactions cannot generate Heisenberg-limited spin squeezing, but a weak optimized drive gives rise to an ideal two-axis twist interaction and Heisenberg-limited squeezing. Parametric driving is also advantageous in regimes limited by dissipation, and enables an alternate adiabatic scheme which can prepare optimally squeezed, Dicke-like states. Our scheme is compatible with a number of platforms, including solid-state systems where spin ensembles are coupled to superconducting quantum circuits or mechanical modes.

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Introduction.—The field of quantum sensing focuses on enhancing measurements by exploiting entanglement. Among the most studied approaches are those based on spin squeezing [1], where one uses an entangled state of N spin-$1/2$ particles to reduce the imprecision of a Ramsey-type phase measurement. While there are many approaches for generating spin squeezing (see, e.g., Refs. [2–8]), new methods are still of interest if they can transcend limitations of standard approaches. The most widely studied deterministic method is based on exploiting an all-to-all Ising interaction, the so-called one-axis twist (OAT) Hamiltonian [2]; this has been implemented in several groundbreaking experiments [9–13] in atomic and trapped ion systems, and also shows potential in solid state implementations (as has been analyzed theoretically [5,14–17]). While conceptually simple, this method cannot achieve fundamental $1/N$ Heisenberg scaling of the squeezing. In contrast, the so-called two-axis twist (TAT) Hamiltonian is known to achieve Heisenberg scaling [2], but is typically difficult to implement physically.

In this work, we show how adding a detuned parametric drive (PD) to the standard setup of spins coupled to a cavity (Fig. 1) can be used to exactly implement the TAT interaction, and thus achieve Heisenberg-limited spin squeezing. Our scheme is compatible with standard spin-echo techniques, thus giving it robustness against the effects of inhomogeneous broadening and low-frequency noise; it also outperforms standard OAT in the presence of realistic dissipation. For stronger PD strengths, one can alternatively implement an adiabatic protocol that produces Dicke-like states which achieve the maximum possible level of spin squeezing (outperforming TAT by a factor of 2) [18,19]. Our approach could in principle be implemented in a host of systems, including solid state spins coupled to driven mechanical modes [5,20] (see Ref. [21] along with Refs. [22–26]), driven superconducting cavities [27], or trapped ions [13] (also see Ref. [21] and Refs. [28–30]).

Note that our protocols differ significantly from previous ideas using a PD for spin squeezing. Reference [31] considered how a PD could enhance OAT in a trapped ion setup; we consider a different basic spin-boson coupling, and demonstrate methods that go beyond OAT. Reference [32] considered how a PD in consort with strong cavity frequency modulation could realize dissipative spin squeezing [6]. Our approaches in contrast require no frequency modulation, and are based on induced coherent interactions. Furthermore, our induced two-axis twist (ITAT) protocol leads to more modest cooperativity requirements.

Model.—We consider N two-level systems (splitting frequency ω_s) coupled via a standard Tavis-Cummings interaction (strength g) to a bosonic mode subject to a parametric (i.e., two-photon) drive at frequency $2\omega_p$:

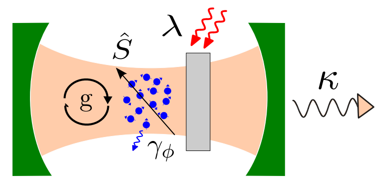

FIG. 1. A collective spin $\hat{S}$ comprised of N spin-$1/2$ particles is coupled to a parametrically driven cavity, with drive amplitude λ (the grey rectangle represents a nonlinear crystal, that could let one implement such a drive in an optical cavity). The cavity has a decay rate κ, and the single-spin dephasing rate is γ_ϕ.

$$\hat{H}_{\rm lab} = \omega_c \hat{c}^\dagger \hat{c} + \omega_s \hat{S}_z + \left(g\hat{c}\hat{S}_+ + \frac{\lambda}{2} e^{i2\omega_p t}\hat{c}^2 + \text{H.c.}\right), \quad (1)$$

where we introduce collective spin operators $\hat{S}_\pm = \hat{S}_x \pm i\hat{S}_y$ and $\hat{S}_k = \frac{1}{2}\sum_j \hat{\sigma}^k_{(j)}$ for $k \in \{x, y, z\}$, with $\hat{\sigma}^k_{(j)}$ denoting a standard Pauli operator acting on the jth spin.

The parametric drive will give us a powerful means for controlling the form of the cavity mediated spin-spin interactions. We next move to a rotating frame (for both spins and cavity) in which the Hamiltonian is time independent:

$$\hat{H}_{\rm rot} = \Delta_c \hat{c}^\dagger \hat{c} + \Delta_s \hat{S}_z + \left(g\hat{c}^\dagger \hat{S}_- + \frac{1}{2}\lambda \hat{c}^2 + \text{H.c.}\right). \quad (2)$$

Here $\Delta_{c/s} \equiv \omega_{c/s} - \omega_p$ are the respective detunings of the cavity and spins from the parametric drive. Note that parametric driving has been realized in several platforms compatible with spin squeezing. Methods include dielectrophoretic modulation of a diamond mechanical resonator [33], flux-pumping a superconducting microwave resonator that could be coupled to solid state spins (e.g., theoretical analysis in Ref. [34]), modulating the trapping frequency of trapped ions [29], or utilizing the nonlinearity of atomic transitions in cavity QED platforms [35,36]. In Ref. [21] we provide more details on potential trapped ion implementations, as well as a comprehensive discussion of an implementation based on a diamond optomechanical crystal with nitrogen-vacancy (NV) center spins.

Without loss of generality, we take the parametric drive amplitude λ to be real and positive, and consider the regime $|\Delta_c| \geq \lambda$, ensuring a stable system. We can then diagonalize the cavity Hamiltonian in terms of a Bogoliubov mode $\hat{\beta} \equiv \cosh r\hat{c} + \sinh r\hat{c}^\dagger$, where the parameter r satisfies $\tanh 2r = \lambda/\Delta_c$. Defining $E_\beta \equiv \sqrt{\Delta_c^2 - \lambda^2}$, this yields

$$\hat{H}_{\rm sq} = E_\beta \hat{\beta}^\dagger \hat{\beta} + \Delta_s \hat{S}_z + g(\hat{\beta}^\dagger \hat{\Sigma} + \text{H.c.}). \quad (3)$$

where the spin Bogoliubov mode is defined as $\hat{\Sigma} \equiv \cosh r\hat{S}_- - \sinh r\hat{S}_+$.

We next consider the case where $\sqrt{N}g \ll E_\beta$, and where the parametric drive is almost resonant with the spins, such that $\Delta_s \sim g^2/E_\beta \ll E_\beta$. In this case, we can eliminate the cavity-spin interaction to leading order using a standard Schreiffer-Wolff transformation [5,37] (see Ref. [21] for details); this is analogous to standard derivations of a cavity-mediated OAT [5,38]. Retaining terms to order g^2, we obtain an effective interacting spin Hamiltonian:

$$\hat{H}_{\rm eff} \simeq E_\beta \hat{\beta}^\dagger \hat{\beta} + \Delta_s \hat{S}_z - \chi \hat{\Sigma}^\dagger \hat{\Sigma} - 2\chi \hat{S}_z \hat{\beta}^\dagger \hat{\beta}. \quad (4)$$

with $\chi \equiv g^2/E_\beta$. Superficially, this is identical to the Hamiltonian for a cavity-mediated OAT, except the spin lowering operator has been replaced by $\hat{\Sigma}$, the spin Bogoliubov operator. As we now show, this has dramatic consequences.

Induced Two-axis twist.—We first ignore the last dispersive coupling term in Eq. (4). In this case the spins and cavity are decoupled, and the spin-only terms in Eq. (4) describe an unusual kind of cavity-mediated spin-spin interaction. Expanding these terms out, and defining $\tilde{\chi} = \chi \cosh 2r$, $\tilde{\Delta} = \Delta_s - \chi$, we have

$$\hat{H}_s = \tilde{\Delta}\hat{S}_z - \tilde{\chi}[(\hat{S}^2_{\rm tot} - \hat{S}^2_z) - \tanh(2r)(\hat{S}^2_x - \hat{S}^2_y)], \quad (5)$$

with $\hat{S}^2_{\rm tot} = \hat{S}^2_x + \hat{S}^2_y + \hat{S}^2_z$. Without a parametric drive (i.e., $r = 0$) we have a standard cavity-induced OAT Hamiltonian [5,38]. For nonzero r, the new interaction terms have the form of the TAT Hamiltonian introduced in Ref. [2]; these terms on their own are capable of generating spin squeezing with Heisenberg-limited scaling, something that is impossible with an OAT Hamiltonian.

At first glance, it seems like our scheme can never realize a pure TAT interaction, both because the OAT-like terms will always dominate (as $\tanh 2r \leq 1$), and because of the spurious linear-in-$\hat{S}_z$ term. This pessimism is unfounded. First, the unwanted linear term can be eliminated by simply tuning the spin detuning to $\Delta_s = \chi$; this could be done, e.g., by just slightly shifting the parametric drive frequency. Second, if we also tune the parametric drive amplitude so that $\lambda = \Delta_c/3$, we have $\tanh 2r \to \tanh 2r_0 = 1/3$, and the resulting Hamiltonian can be written

$$\begin{aligned}\hat{H}_s &\to -\tilde{\chi}[(\hat{S}^2_{\rm tot} - \hat{S}^2_z) - \frac{1}{3}(\hat{S}^2_x - \hat{S}^2_y)] \\ &= -\frac{2}{3}\tilde{\chi}[\hat{S}^2_{\rm tot} - \hat{S}^2_z + \hat{S}^2_y]. \end{aligned} \quad (6)$$

Since $\hat{S}_{\rm tot}$ is a constant of motion for $\hat{H}_s$, the effective dynamics of $\hat{H}_s$ are equivalent to the desired two-axis twist Hamiltonian.

Equation (6) is a central result of our work, and represents a new mechanism for implementing the TAT Hamiltonian. Previous proposals for realizing a TAT either require carefully tailored bang-bang control of the spin ensemble [39,40], multiple drive lasers, atomic levels and cavity transitions [41–43], or very weak higher-order interaction processes [44]. In contrast, our scheme utilizes a standard Tavis-Cummings coupling, and does not require an elaborate pulsed driving of the spin system. It also requires only a modest-amplitude parametric drive (far from any regime of instability). Note one could alternatively tune $\lambda = -\Delta_c/3$; in this case an equivalent TAT Hamiltonian in the z–x plane is generated. By tuning the parametric drive amplitude, one can also realize other kinds of spin-spin interactions, including an OAT Hamiltonian along the y axis, and a "twist-and-turn" Hamiltonian [45,46] (see Ref. [21]).

As is standard, we quantify the amount of useful spin squeezing using the Ramsey spin squeezing parameter [47]:

$$\xi_R^2 \equiv N\langle\Delta\hat{S}_\perp^2\rangle/\langle\vec{\hat{S}}\rangle^2, \tag{7}$$

where $\langle\Delta\hat{S}_\perp^2\rangle$ is the minimum variance in a direction perpendicular to the direction of the mean of the collective spin.

We now return to the issue of the dispersive interaction in Eq. (4). In the absence of dissipation, $\hat{\beta}^\dagger\hat{\beta}$ is a conserved quantity. Further, assuming the cavity starts in a vacuum state, the $\hat{\beta}$ mode starts in a squeezed state characterized by r_0, and thus has a small but nonzero population. Hence, the small mean value $\langle\hat{\beta}^\dagger\hat{\beta}\rangle = \sinh^2 r_0 \simeq 0.03$ can be easily canceled by slightly shifting the spin-drive detuning to $\Delta_s = \chi(1 + 2\sinh^2 r_0)$. The remaining static fluctuations of the Bogoliubov-mode number operator have a dephasing effect, which is also insignificant due to the smallness of the required parametric drive amplitude (i.e., $\langle(\hat{\beta}^\dagger\hat{\beta})^2\rangle - \langle(\hat{\beta}^\dagger\hat{\beta})\rangle^2 \simeq 0.06$). They have a negligible effect on the optimal squeezing (numerical simulations show that at $N = 5000$, the change in ξ_R^2 is much smaller than 1 dB), and furthermore, their effects can be *completely* canceled using a dynamical decoupling protocol (which corresponds to applying fast π pulses about, e.g., the x axis). This highlights another key advantage of our scheme: like the standard cavity-based OAT [5], it is fully compatible with widely used spin-echo techniques for suppressing the effects of inhomogeneous broadening and low-frequency dephasing. This is of particular importance in potential solid-state implementations.

For our induced TAT Hamiltonian, we start with an initial product state where all spins are polarized along the x direction. As shown in Fig. 2, if we apply a dynamical decoupling protocol to cancel the effects of the dispersive coupling, the induced TAT Hamiltonian (in the absence of dissipation) generates spin squeezing at an optimal time, that scales as $\xi_R^2 \sim 4/N$. This represents Heisenberg-limited scaling, something that is impossible with a standard OAT protocol (i.e., our setup with zero parametric drive), where $\xi_R^2 \sim 1/N^{2/3}$ at best. Figure 2 also shows that our protocol is robust against variations in the parametric drive amplitude; even when λ is far away from its optimal value of $\Delta_c/3$ the performance is superior to a OAT.

Impact of dissipation.—It is also crucial to understand the ITAT scheme in the presence of dissipation. As discussed, standard spin-echo pulses are compatible with our scheme, and hence can be used to suppress the impact of inhomogeneous broadening and low-frequency dephasing noise. For the remaining dissipative processes, we assume each spin is dephased by a Markovian bath (rate γ_ϕ) and that the cavity has an energy damping rate κ due to coupling to a zero-temperature environment. The dissipative dynamics of our system is then described by

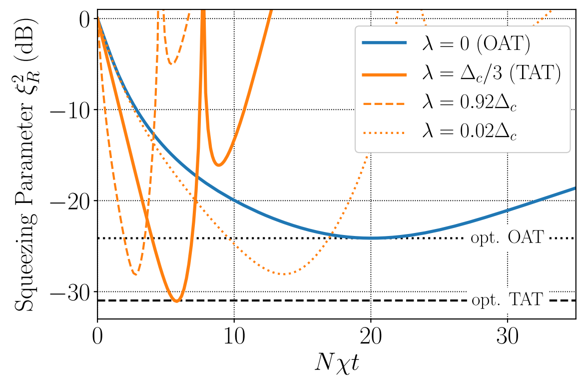

FIG. 2. Dissipation-free evolution of spin squeezing under the spin-spin interaction in Eq. (5) for different parametric drive amplitudes ($\tilde{\Delta} = 0$, $N = 5000$, spins initially polarized along x). The solid blue (orange) curve corresponds to the OAT (TAT) evolution with the parametric drive amplitude $\lambda = 0$ ($\lambda = \Delta_c/3$). The dotted (dashed) orange curve shows results for nonoptimal amplitude of $\lambda = 0.02\Delta_c$ ($\lambda = 0.92\Delta_c$). Even nonideal choices of λ lead to performance that surpasses a OAT. Horizontal lines indicate the optimal squeezing for a OAT and TAT. We assume that a dynamical decoupling protocol is being applied to cancel the effects of the dispersive interaction. In the absence of such a protocol, the optimal squeezing for $\lambda = \Delta_c/3$ is degraded by less than 1 dB.

$$\dot{\hat{\rho}} = -i[\hat{H}_s, \hat{\rho}] + \Gamma\mathcal{D}[\hat{z}(r)]\hat{\rho} + \frac{\gamma_\phi}{2}\sum_{k=1}^{N}\mathcal{D}[\hat{\sigma}_{(k)}^z]\hat{\rho}, \tag{8}$$

where $\mathcal{D}[z]\hat{\rho} = \hat{z}\hat{\rho}\hat{z}^\dagger - \{\hat{z}^\dagger\hat{z}, \hat{\rho}\}/2$ is the standard Linblad dissipative superoperator. $\Gamma = \kappa\chi/E_\beta$ is the rate associated with cavity-induced spin dissipation; the jump operator describing this process is

$$\hat{z}[r] = e^{-2r}\hat{S}_x - ie^{2r}\hat{S}_y. \tag{9}$$

For large parametric drives (e.g., as used in the scheme of Ref. [31]), the drive causes strong amplification of the cavity-induced dissipation, potentially nullifying any advantage. In contrast, our scheme only requires a small parametric drive (i.e., $e^{2r_0} = \sqrt{2}$), leading to minimal amplification of dissipation.

Shown in Fig. 3 are results from numerical simulations of the full master equation [48,49], depicting optimal spin squeezing versus N (with E_β optimized for each N). We pick parameters such that $\kappa \gg \gamma_\phi$ and the collective cooperativity $\mathcal{C} \equiv Ng^2/(\kappa\gamma_\phi)$ is 5 for $N = 1$, and always evolve starting with spins fully polarized in the x direction. Even with dissipation, a parametric drive corresponding to $r = r_0$ (i.e., ITAT) appreciably improves performance for all values of N over the undriven ($r = 0$, OAT) case. Our results are also consistent with an approximate $\xi_R^2 \sim 1/\sqrt{\mathcal{C}}$ scaling, as would be expected from a standard linearized treatment of our system (see Ref. [21]). We also consider optimizing the value of r (i.e., parametric drive strength) for

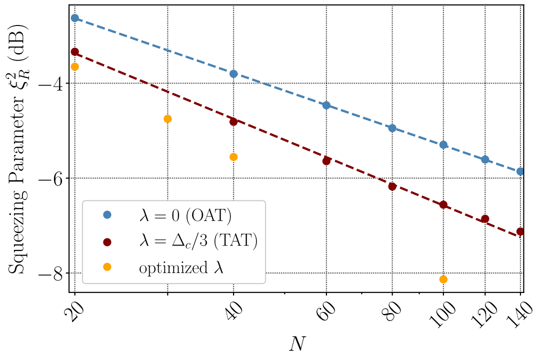

FIG. 3. Optimized squeezing parameter ξ_R^2 in the presence of dissipation versus number of spins. The dots represent the effective master equation [Eq. (8)] simulation data for OAT (blue) corresponding to $\lambda = 0$, ITAT (maroon) corresponding to $\lambda = \Delta_c/3$, and finally a case where the drive strength λ is itself (approximately) optimized (orange), with values $\lambda = \alpha\Delta_c$, where $\alpha = 0.70, 0.74, 0.76, 086$ for $N = 20, 30, 40, 100$, respectively. In all cases we take $\kappa = 10g$, $\gamma_\phi = 0.02g$ and optimize over E_β and protocol time. The dashed lines depict corresponding numerical fits to $a\mathcal{C}^{-b}$, with $3.2\mathcal{C}^{-0.4}$ (blue), $3.8\mathcal{C}^{-0.5}$ (maroon). (Also, see Ref. [21] for a comparison with an ideal, no-noise OAT and TAT evolution).

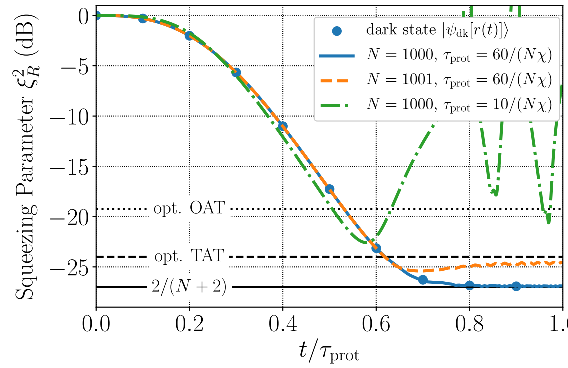

FIG. 4. Squeezing parameter ξ_R^2 versus time for the adiabatic scheme, for different protocol times: $\tau_{\rm prot} = 60/(N\chi)$ (blue, solid line), $\tau_{\rm prot} = 10/(N\chi)$ (green, dash-dotted line). In both cases $N = 1000$. The blue dots correspond to the performance one would obtain from an ideal dark state $|\psi_{\rm dk}[r(t)]\rangle$. The orange dashed curve depicts the evolution with $\tau_{\rm prot} = 60/(N\chi)$, but with an odd number of spins; $N = 1001$.

each N. We find that these optimized r values (orange points in Fig. 3), are always larger than the value r_0 that would yield the TAT Hamiltonian. At a heuristic level, increasing r increases the initial rate at which squeezing is produced, something that is likely advantageous in the presence of dissipation.

Adiabatic preparation of optimally squeezed states.—While the TAT Hamiltonian is able to produce Heisenberg-limited spin squeezing, it is well known that states exist which are squeezed by an *additional* factor of 2 [18,19]. Such states are infinitesimally close to so-called "Dicke states": collective spin eigenstates that have a maximal $\hat{S}^2_{\rm tot}$ and are also annihilated by, e.g., $\hat{S}_z$. As we now show, by making our parametric drive time dependent, our setup can also produce such states.

To see how this works, note that for even N the spin Bogoliubov operator $\hat{\Sigma}[r]$ has a unique state in its kernel, $|\psi_{\rm dk}[r]\rangle$ [3,6,50]. This state exhibits optimal spin squeezing properties, with $\xi_R^2 \to 2/N$ in the large-r limit. Furthermore, it can be naturally produced by driving a spin ensemble with squeezed light [3,50], and can also be stabilized using dissipative protocols involving multilevel atoms and engineered Raman processes [6].

Our setup provides an alternate, fully coherent method for generating such states. For even N, $|\psi_{\rm dk}[r]\rangle$ is the unique zero-energy ground state of the drive-modified spin-spin interaction in Eq. (4); all other higher-lying states are separated by a gap. As discussed in Ref. [21], by making the complex parametric drive amplitude λ, frequency ω_p, and spin Larmor frequency ω_s all time dependent, we obtain a Hamiltonian with the same form as Eq. (4), except with a time-dependent squeezing parameter $r(t)$. This Hamiltonian always has an instantaneous zero-energy eigenstate $|\psi_{\rm dk}[r(t)]\rangle$. Our protocol thus consists of starting with $\lambda(0) = r(0) = 0$, with an initial state having all of the spins polarized along the z axis (i.e., $|\psi_{\rm dk}[r = 0]\rangle = |N/2, -N/2\rangle$). We then slowly ramp up $r(t)$ from 0 to r_f by appropriately varying $\lambda(t)$, $\omega_p(t)$ and $\omega_s(t)$ (see Ref. [21] and Refs. [51,52]). The adiabatic theorem then implies that the system evolves from its initial product form to the highly entangled state $|\psi_{\rm dk}[r_f]\rangle$. One can show analytically that for large r_f, $|\psi_{\rm dk}\rangle$ exhibits spin squeezing with $\xi_R^2 \sim 2/N$ [3,50]. Adiabaticity requires a total evolution time $\tau_{\rm prot}$ that is much longer than the relevant inverse gap, which in our case scales as Ng^2/E_β.

Shown in Fig. 4 are numerical results for the coherent (no-noise) time evolution of the squeezing under this adiabatic protocol for $N = 1000$ and for different total protocol times. We take the time dependence of $r(t)$ to smoothly increase from 0 to 4 during the evolution (see Ref. [21]). Even for faster evolution times where non-adiabatic errors are prevalent, large amounts of spin squeezing are produced, and performance can still surpass that of a standard OAT. In practice, the dark state $|\psi_{\rm dk}[r(t)]\rangle$ only exists for even N. Figure 4 shows that our protocol nonetheless produces considerable squeezing even when N is odd. Finally, in Ref. [21] we also show numerical results of a scenario where both cavity decay and local spin dephasing are included. Because of the requirement for longer evolution times, in that case, we use weaker noise strengths, namely, $\kappa = 0.1g$ and $\gamma_\phi = 0.02g$.

Conclusions.—We have explored how parametrically driving a cavity coupled to a spin ensemble can be used to optimize the generation of highly squeezed spin states. An optimally detuned parametric drive allows a direct realization of the ideal TAT spin squeezing Hamiltonian, enabling Heisenberg-limited scaling. This protocol also significantly improved performance over the undriven system in regimes limited by dissipation. We also described an alternate protocol using a time-dependent parametric drive, which adiabatically produced optimally spin-squeezed states which approach Dicke states.

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[1] J. Ma, X. Wang, C.-P. Sun, and F. Nori, Phys. Rep. **509**, 89 (2011).
[2] M. Kitagawa and M. Ueda, Phys. Rev. A **47**, 5138 (1993).
[3] G. S. Agarwal and R. R. Puri, Phys. Rev. A **49**, 4968 (1994).
[4] M. H. Schleier-Smith, I. D. Leroux, and V. Vuletić, Phys. Rev. A **81**, 021804(R) (2010).
[5] S. D. Bennett, N. Y. Yao, J. Otterbach, P. Zoller, P. Rabl, and M. D. Lukin, Phys. Rev. Lett. **110**, 156402 (2013).
[6] E. G. Dalla Torre, J. Otterbach, E. Demler, V. Vuletic, and M. D. Lukin, Phys. Rev. Lett. **110**, 120402 (2013).
[7] J. Hu, W. Chen, Z. Vendeiro, A. Urvoy, B. Braverman, and V. Vuletić, Phys. Rev. A **96**, 050301(R) (2017).
[8] R. J. Lewis-Swan, M. A. Norcia, J. R. K. Cline, J. K. Thompson, and A. M. Rey, Phys. Rev. Lett. **121**, 070403 (2018).
[9] I. D. Leroux, M. H. Schleier-Smith, and V. Vuletić, Phys. Rev. Lett. **104**, 073602 (2010).
[10] M. F. Riedel, P. Böhi, Y. Li, T. W. Hänsch, A. Sinatra, and P. Treutlein, Nature (London) **464**, 1170 (2010).
[11] C. Gross, T. Zibold, E. Nicklas, J. Estève, and M. K. Oberthaler, Nature (London) **464**, 1165 (2010).
[12] O. Hosten, R. Krishnakumar, N. J. Engelsen, and M. A. Kasevich, Science **352**, 1552 (2016).
[13] J. G. Bohnet, B. C. Sawyer, J. W. Britton, M. L. Wall, A. M. Rey, M. Foss-Feig, and J. J. Bollinger, Science **352**, 1297 (2016).
[14] S. Dooley, E. Yukawa, Y. Matsuzaki, G. C. Knee, W. J. Munro, and K. Nemoto, New J. Phys. **18**, 053011 (2016).
[15] Y.-L. Zhang, C.-L. Zou, X.-B. Zou, L. Jiang, and G.-C. Guo, Phys. Rev. A **92**, 013825 (2015).
[16] K. Xia, Sci. Rep. **7**, 1 (2017).
[17] K. Xia and J. Twamley, Phys. Rev. B **94**, 205118 (2016).
[18] A. André and M. D. Lukin, Phys. Rev. A **65**, 053819 (2002).
[19] L. Pezzè, A. Smerzi, M. K. Oberthaler, R. Schmied, and P. Treutlein, Rev. Mod. Phys. **90**, 035005 (2018).
[20] D. Lee, K. W. Lee, J. V. Cady, P. Ovartchaiyapong, and A. C. B. Jayich, J. Opt. **19**, 033001 (2017).
[21] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.125.203601 for derivations as well as details related to potential physical realizations.
[22] M. J. Burek, J. D. Cohen, S. M. Meenehan, N. El-Sawah, C. Chia, T. Ruelle, S. Meesala, J. Rochman, H. A. Atikian, M. Markham *et al.*, Optica **3**, 1404 (2016).
[23] J. V. Cady, O. Michel, K. W. Lee, R. N. Patel, C. J. Sarabalis, A. H. Safavi-Naeini, and A. C. B. Jayich, Quantum Sci. Technol. **4**, 024009 (2019).
[24] Y.-I. Sohn, M. J. Burek, V. Kara, R. Kearns, and M. Lončar, Appl. Phys. Lett. **107**, 243106 (2015).
[25] D. R. Koenig, E. M. Weig, and J. P. Kotthaus, Nat. Nanotechnol. **3**, 482 (2008).
[26] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Rev. Mod. Phys. **86**, 1391 (2014).
[27] A. Bienfait, P. Campagne-Ibarcq, A. H. Kiilerich, X. Zhou, S. Probst, J. J. Pla, T. Schenkel, D. Vion, D. Esteve, J. J. L. Morton, K. Moelmer, and P. Bertet, Phys. Rev. X **7**, 041011 (2017).
[28] D. Leibfried, R. Blatt, C. Monroe, and D. Wineland, Rev. Mod. Phys. **75**, 281 (2003).
[29] S. C. Burd, R. Srinivas, J. J. Bollinger, A. C. Wilson, D. J. Wineland, D. Leibfried, D. H. Slichter, and D. T. C. Allcock, Science **364**, 1163 (2019).
[30] D. Kienzler, H.-Y. Lo, B. Keitch, L. d. Clercq, F. Leupold, F. Lindenfelser, M. Marinelli, V. Negnevitsky, and J. P. Home, Science **347**, 53 (2015).
[31] W. Ge, B. C. Sawyer, J. W. Britton, K. Jacobs, J. J. Bollinger, and M. Foss-Feig, Phys. Rev. Lett. **122**, 030501 (2019).
[32] W. Qin, Y.-H. Chen, X. Wang, A. Miranowicz, and F. Nori, arXiv:1912.04039v1.
[33] Y.-I. Sohn, M. J. Burek, V. Kara, R. Kearns, and M. Lončar, Appl. Phys. Lett. **107**, 243106 (2015).
[34] T. Yamamoto, K. Inomata, M. Watanabe, K. Matsuba, T. Miyazaki, W. D. Oliver, Y. Nakamura, and J. S. Tsai, Appl. Phys. Lett. **93**, 042510 (2008).
[35] V. Josse, A. Dantan, L. Vernac, A. Bramati, M. Pinard, and E. Giacobino, Phys. Rev. Lett. **91**, 103601 (2003).
[36] A. Ourjoumtsev, A. Kubanek, M. Koch, C. Sames, P. W. H. Pinkse, G. Rempe, and K. Murr, Nature (London) **474**, 623 (2011).
[37] J. R. Schrieffer and P. A. Wolff, Phys. Rev. **149**, 491 (1966).
[38] G. S. Agarwal, R. R. Puri, and R. P. Singh, Phys. Rev. A **56**, 2249 (1997).
[39] P. Cappellaro and M. D. Lukin, Phys. Rev. A **80**, 032311 (2009).
[40] Y. C. Liu, Z. F. Xu, G. R. Jin, and L. You, Phys. Rev. Lett. **107**, 013601 (2011).
[41] J. Borregaard, E. Davis, G. S. Bentsen, M. H. Schleier-Smith, and A. S. Sørensen, New J. Phys. **19**, 093021 (2017).
[42] Y.-C. Zhang, X.-F. Zhou, X. Zhou, G.-C. Guo, and Z.-W. Zhou, Phys. Rev. Lett. **118**, 083604 (2017).
[43] F. Anders, L. Pezzè, A. Smerzi, and C. Klempt, Phys. Rev. A **97**, 043813 (2018).
[44] V. Macri, F. Nori, S. Savasta, and D. Zueco, Phys. Rev. A **101**, 053818 (2020).
[45] H. Strobel, W. Muessel, D. Linnemann, T. Zibold, D. B. Hume, L. Pezzè, A. Smerzi, and M. K. Oberthaler, Science **345**, 424 (2014).
[46] W. Muessel, H. Strobel, D. Linnemann, T. Zibold, B. Juliá-Díaz, and M. K. Oberthaler, Phys. Rev. A **92**, 023603 (2015).
[47] D. J. Wineland, J. J. Bollinger, W. M. Itano, and D. J. Heinzen, Phys. Rev. A **50**, 67 (1994).
[48] J. Johansson, P. Nation, and F. Nori, Comput. Phys. Commun. **184**, 1234 (2013).
[49] N. Shammah, S. Ahmed, N. Lambert, S. De Liberato, and F. Nori, Phys. Rev. A **98**, 063815 (2018).
[50] G. S. Agarwal and R. R. Puri, Phys. Rev. A **41**, 3782 (1990).
[51] J. P. Davis and P. Pechukas, J. Chem. Phys. **64**, 3129 (1976).
[52] N. Wiebe and N. S. Babcock, New J. Phys. **14**, 013024 (2012).