Fractional Mutual Statistics on Integer Quantum Hall Edges

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Fractional charge and statistics are hallmarks of low-dimensional interacting systems such as fractional quantum Hall (QH) systems. Integer QH systems are regarded as noninteracting, yet they can have fractional charge excitations when they couple to another interacting system or time-dependent voltages. Here, we notice Abelian fractional mutual statistics between such a fractional excitation and an electron, and propose a setup for detection of the statistics in which a fractional excitation is generated at a source and injected to a Mach-Zehnder interferometer (MZI) in the integer QH regime. In a parameter regime, the dominant interference process involves braiding, via double exchange, between an electron excited at an MZI beam splitter and the fractional excitation. The braiding results in the interference phase shift by the phase angle of the mutual statistics. This proposal for directly observing the fractional mutual statistics is within experimental reach.

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Fractional charge excitations emerge in various interacting systems, including fractional quantum Hall (QH) systems [1,2] and Luttinger liquids [3–6]. The fractional charges have been observed [7–12].

The excitations obey fractional statistics and are called anyons [13–15]. Upon winding of an anyon around another or their double exchange, their state gains a fractional phase angle in cases of Abelian anyons or evolves into another state (sometimes orthogonal to the initial state) in non-Abelian anyons. In fractional QH cases, it has been proposed [16] that the statistics can be identified in an interferometer where an anyon propagating along edge channels winds around localized anyons in the QH bulk, and the number of the bulk anyons is controlled. This strategy is not applicable to detecting the fractional statistics in one-dimensional systems, such as Luttinger liquids [3], having no bulk anyon to braid of.

Interestingly, fractional charge excitations can also be generated in noninteracting integer QH edge channels, by coupling the channels to interacting systems such as a metallic island (as shown in this Letter), a fractional QH system [17], an interedge-interaction region [18–22], or to voltage pulse [23,24] (see Fig. 1). They are expected to propagate along the integer QH edge without decay. A question is whether the fractional charges behave as anyons, although they are not of topological order.

In this Letter, we notice that the fractional charges generated from the sources in Fig. 1 obey fractional statistics on integer QH edges, and propose how to detect fractional mutual statistics between a fractional charge and an electron, using a Mach-Zehnder interferometer (MZI) [25,26] in the integer QH regime, which has length difference ΔL between the two MZI arms. We consider the regime $w \ll \Delta L \lesssim L_\beta$ of the spatial width w of the fractional charge, thermal length $L_\beta = \hbar v \beta/\pi$, thermal energy $1/\beta$, and electron velocity v on the arms. The dominant process of this regime consists of (i) injection of a fractional charge from a source of Fig. 1 to the MZI, and (ii) excitation of an electron at one (the other) MZI beam splitter in one (the other) subprocess. The interference of the two subprocesses involves braiding, via double exchange, between the fractional charge and the electron. The resulting phase shift of the interference is determined

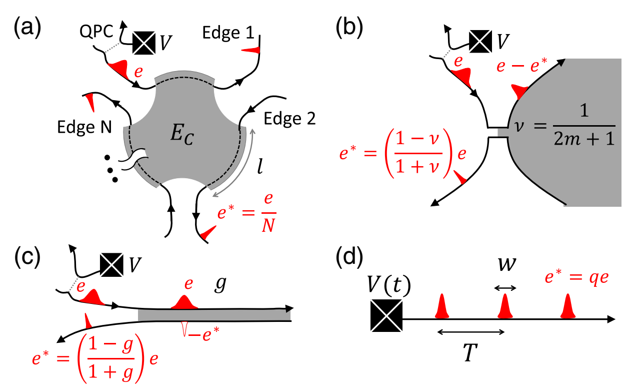

FIG. 1. Sources of fractional charges on integer QH edges (solid lines). (a)–(c) An electron wave packet (thick red peaks) carrying charge e is generated on an edge at a quantum point contact (QPC) by electron tunneling (dotted line) from another edge biased by static voltage V. The electron is fractionalized into charges e^* or $e - e^*$ (thinner peaks) while scattered by an interacting region (shade) of (a) a metallic island coupled to N integer QH edges, (b) fractional QH filling factor ν, or (c) interedge interaction g. (d) Fractional charges of period T and spatial width w, generated by an Ohmic contact (cross) biased by periodic voltage pulses.

by the mutual statistics and is identified by comparing the interference with a reference signal from the same setup.

Fractional mutual statistics.—Excitations of fractional charge $e^* = qe$ on an integer QH edge (labeled by α) behave as "particles" as they form wave packets moving along the chiral edge without deformation [18–23], although they are not energy eigenstates. To see their statistics, we identify their creation operator $\eta^\dagger_{\alpha,q}(x)$ on position x in the bosonization [27], where the Hamiltonian $\mathcal{H}_{\text{edge},\alpha} = (\hbar v/4\pi)\int dx[\partial_x\phi_\alpha(x)]^2$ and electron operator $\psi_\alpha(x) \propto e^{i\phi_\alpha(x)}$ of the edge are represented by a bosonic field ϕ_α. Since the excitation carries the charge qe, $\eta^\dagger_\alpha(x)$ satisfies $[\rho_\alpha(x), \eta^\dagger_\alpha(x')] = q\delta(x-x')\eta^\dagger_\alpha(x')$, where $\rho_\alpha(x) = \partial_x\phi_\alpha/(2\pi)$ is the electron density operator. Combining it with the Kac-Moody algebra $[\phi_\alpha(x), \phi_\beta(x')] = \pi i\delta_{\alpha\beta}\text{sgn}(x-x')$, we identify $\eta_{\alpha,q}(x) \propto e^{iq\phi_\alpha(x)}$. We will show that the fractional charges generated from the sources in Fig. 1 are, indeed, described by $\eta_{\alpha,q}$.

The identification, combined with the Kac-Moody algebra, allows us to find that position exchange of fractional charges qe and $q'e$ obeys the fractional statistics

$$\eta_{\alpha,q}(x)\eta_{\alpha,q'}(x') = \eta_{\alpha,q'}(x')\eta_{\alpha,q}(x)e^{-i\pi qq'\text{sgn}(x-x')}, \quad (1)$$

with statistical angle $\pi qq'$. For example, a fractional charge qe and an electron ($q'=1$) satisfy the mutual statistics $\eta_{\alpha,q}(x)\psi_\alpha(x') = \psi_\alpha(x')\eta_{\alpha,q}(x)e^{-i\pi q\text{sgn}(x-x')}$. with statistical angle πq.

The angle πq is either quantized [Figs. 1(a) and 1(b)] or continuously tuned [Figs. 1(c) and 1(d)], determined by geometry, interaction strength, or voltage pulse. This is in stark contrast to the fractional QH cases that the mutual statistics between a fractional particle and an electron is trivial with statistics angle π [15]. These may be due to the fact that the fractional excitations $\eta_{\alpha,q}$ are not energy eigenstates.

Fractional charge generation.—Below, we show how to generate fractional charges using a metallic island in Fig. 1(a) and that they are described by $\eta_{\alpha,q}$. The island is useful [28–33] for simulating Luttinger liquids and multichannel Kondo effects.

The island couples with N integer QH edges. The coupling region in each edge has the same length l for simplicity. The island has the interaction $E_C(n_{\text{tot}} - n_g)^2$ of excess charge $e(n_{\text{tot}} - n_g)$. E_C is the charging energy, en_{tot} is the total charge in the N coupling regions, and en_g is tuned by gate voltages. Combining the boson modes $\phi_{\alpha=1,2,\ldots,N}$ of the edges, we introduce the charge mode $\tilde{\phi}_c(x) = \sum_{\alpha=1}^N \phi_\alpha(x)/\sqrt{N}$ and neutral modes $\tilde{\phi}_{n,j=1,\ldots,N-1}$ orthonormal to each other. Then, the Hamiltonian $\mathcal{H}_{\text{island}} = \mathcal{H}_c + \mathcal{H}_n$ is decoupled into the charge part $\mathcal{H}_c = (\hbar v/4\pi)\int_x(\partial_x\tilde{\phi}_c)^2 + E_C(n_{\text{tot}} - n_g)^2$ and neutral part $\mathcal{H}_n = (\hbar v/4\pi)\sum_{j=1}^{N-1}\int_x(\partial_x\tilde{\phi}_{n,j})^2$. While the neutral part is noninteracting, the charge mode feels the charging energy. Since $n_{\text{tot}} = \sqrt{N}[\tilde{\phi}_c(l) - \tilde{\phi}_c(0)]/(2\pi)$, where $x = 0$ ($x = l$) is the starting (ending) coordinate of the coupling regions, the ratio of the charging energy to kinetic energy of the charge mode is $\sim NE_C$; the charging energy effectively increases with N.

For $NE_C \gg \hbar v/l$, we find [34] charge teleportation with fractionalization. In Fig. 1(a), an electron is injected from an edge (different from the edges $\alpha = 1, 2, \ldots, N$) biased by voltage V to the edge $\alpha = 1$ via tunneling through the quantum point contact (QPC). When the electron enters the coupling region ($x = 0$), a fractional charge $qe = e/N$ with width $w = \hbar v/(\text{eV})$ immediately appears at the end ($x = l$) of the coupling region of each edge, independently of l,

$$\psi_1^\dagger(x=0,t) \to \prod_{\alpha=1}^N \eta^\dagger_{\alpha,(1/N)}(x=l,t). \quad (2)$$

The teleportation of the electron into the N fractional charges happens due to the edge chirality and the large charging energy NE_C that prohibits charge modulation inside the coupling regions. It can be seen [34] from the equation of motion of the charge mode, $\tilde{\phi}_c(x,t) = \tilde{\phi}_c^{(0)}(x-l,t) + 2\pi n_g/\sqrt{N}$ for $x > l$. $\tilde{\phi}_c^{(0)}$ denotes the charge mode of the $E_C = 0$ case. The teleportation is accompanied [not shown in Eq. (2)] by neutral excitations moving inside the coupling regions. Since the neutral excitations decay out before moving out of the coupling regions [35], the setup of Fig. 1(a) generates fractional charges $\eta^\dagger_{\alpha,1/N}$, obeying Eq. (1), on each edge. The teleportation of the $N = 1$ case (without the fractionalization) has been proposed [36] and experimentally supported [31].

Similarly, the setups in Figs. 1(b) and 1(c) generate fractional charges, described [34] by $\eta_{\alpha,q}$ in a stochastic way utilizing a QPC and an interaction region. The fraction q is governed by the filling factor or interedge interaction.

By contrast, the setup in Fig. 1(d) generates fractional charges on demand [23]. Here, a voltage pulse (per period) $V(t)$ of Lorentzian shape with temporal width w/v satisfying $e\int V(t)dt = 2\pi q\hbar$ generates a charge qe with width w. When $q < 1$, it generates a fractional charge. Although the fractional charge is composed of many electron-hole pair excitations [23], it is described by $\eta_{\alpha,q}$ and satisfies Eq. (1). This can be seen from the time evolution operator $\mathcal{T}e^{-(i/\hbar)\int_{t'}\mathcal{H}_V(t')}$ in the interaction picture under the Hamiltonian $\mathcal{H}_V(t) = eV(t)\int_{-\infty}^0 \rho_\alpha(x'-vt)dx'$ due to the voltage pulse applied at, say, $x < 0$ on edge α, where $\mathcal{T}$ is the time ordering. Since the voltage pulse is well localized (like a delta function), the evolution operator reduces to $\eta^\dagger_{\alpha,q}(x)$, $\mathcal{T}e^{-(i/\hbar)\int_{t'}\mathcal{H}_V(t')} \to \eta^\dagger_{\alpha,q}(x)$, on length scales $\gg w$. This generation is beneficial as one can tune q and the number of charges (per period).

Detecting the mutual statistics.—The statistics in Eq. (1) can be detected by injecting fractional charges to an MZI. In Fig. 2, we illustrate the case of dilute injection, which is

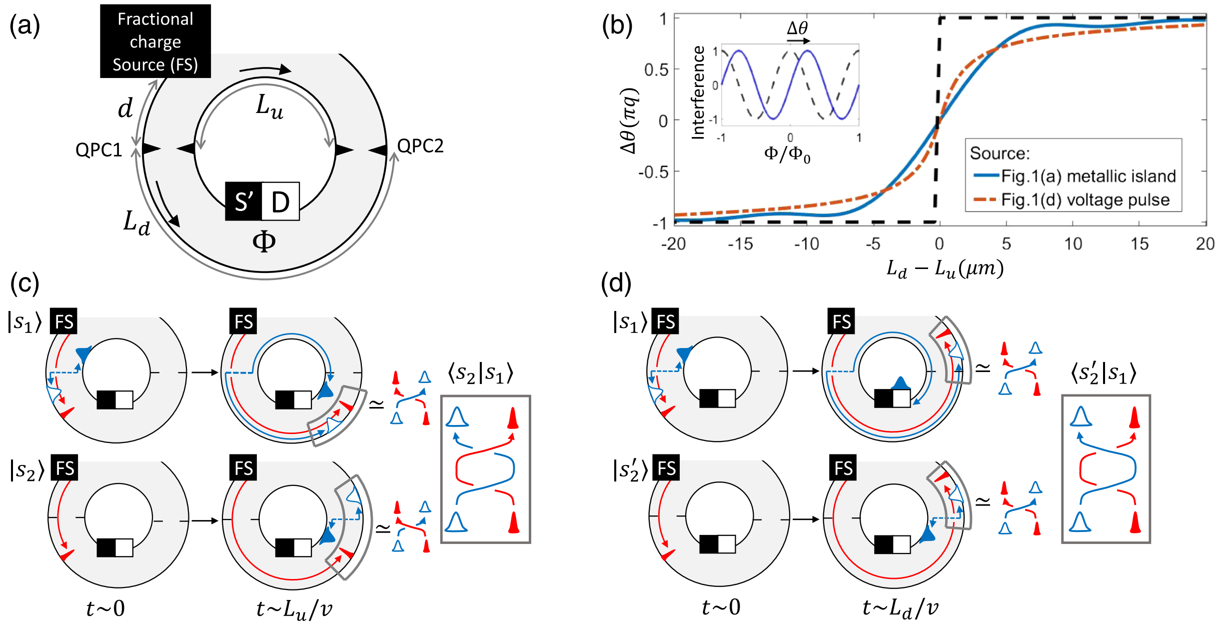

FIG. 2. MZI for detecting the fractional mutual statistics. (a) The MZI has two (upper and lower) integer QH edge channels (the boundaries of the shaded region, with chirality indicated by thick arrows) and two QPCs (its two beam splitters). The length L_u (L_d) of the upper (lower) MZI arm is chosen as $L_d > L_u$. The source (one of Fig. 1) generates fractional charges qe on the lower edge, so the charges are injected to the MZI. Dilute injection leading to only one or no fractional charge inside the arms at any time is considered. Interference signals are detected at reservoir D. (b) Phase shift $\Delta\theta$ of the Aharonov-Bohm interference signal versus $L_d - L_u$. It approaches the statistics angle $\pm\pi q$ at large $\Delta L = |L_d - L_u|$, and the black dashed line as $w \to 0$. The signals are interference conductance (blue solid) computed with a metallic island of $N = 3$ ($q = 1/3$), $V = 40\ \mu$V ($w = 1.6\ \mu$m), $E_C \to \infty$ [37], and interference current (orange dash-dotted) with voltage pulses of $q = 1/3$, temporal width $w/v = 3$ ps, period $T = 250$ ps. Temperature 20 mK and $v = 10^5$ m/s are chosen. Inset: The phase shift is obtained by comparing the signal (solid) with a reference (dashed). The reference interference is obtained with turning off the fractional charge source and applying a small voltage to reservoir S′. (c)–(d) Interference processes. In a subprocess $|s_1\rangle$, an electron (wide blue packet) and a hole (empty) are pair excited (dashed line) at QPC1 at time $t = 0$, after a fractional charge (thin red) passes QPC1. In $|s_2\rangle$ (respectively, $|s_2'\rangle$), a pair excitation happens at QPC2 at $t \sim L_u/v$ (respectively, $t \sim L_d/v$) before (respectively, after) the fractional charge passes QPC2. The interference $\langle s_2|s_1\rangle$ between $|s_1\rangle$ and $|s_2\rangle$ involves a double exchange between the electron and fractional charge, while the interference $\langle s_2'|s_1\rangle$ does not (see the boxes).

achieved by tuning the QPC or voltage pulse in Fig. 1. The MZI is in the regime of $w \ll \Delta L \lesssim L_\beta$, and its two QPCs are in the electron-tunneling regime. The conventional interference of the MZI occurs [34] as interference of processes with and without splitting of an injected fractional charge qe at QPC1 into e in the upper arm and $-(1-q)e$ in the lower arm. However, this interference is negligible since the width w of the fractional charge is much shorter than the arm length difference ΔL. Instead, there is a new interference process in which an electron braids, via double exchange, with the fractional charge, with the help of electron tunneling at the QPCs; the splitting of the fractional charge does not happen in the new process. This interference is visible when the thermal length is not too short ($\Delta L \lesssim L_\beta$).

To illustrate the new process, first, we discuss the corresponding process in thermal equilibrium with no fractional charge injection. We consider an electron on the lower edge of the MZI, located at QPC1 (whose location is $x = 0$ on both edges) at time $t = 0$. In a subprocess $|s_1\rangle_0$, this electron jumps to the upper edge at QPC1 via tunneling at $t = 0$. The resulting state is $|s_1\rangle_0 = \psi_u^\dagger(0,0)|\rangle_u \psi_d(0,0)|\rangle_d$, where $\psi_{u(d)}^\dagger(x,t)$ creates an electron on the Fermi sea $|\rangle_{u(d)}$ of the upper (lower) edge. The electron arrives at QPC2 at $t = L_u/v$. In another subprocess $|s_2\rangle_0$ (respectively, $|s_2'\rangle_0$), which interferes with $|s_1\rangle_0$ with the largest overlap, the electron moves along the lower edge and electron tunneling from the lower to upper edge happens at QPC2 at $t = L_u/v$ (respectively, $t = L_d/v$); $|s_2\rangle_0 = \psi_u^\dagger(L_u, L_u/v)|\rangle_u \psi_d(L_d, L_u/v)|\rangle_d$ and $|s_2'\rangle_0 = \psi_u^\dagger(L_u, L_d/v)|\rangle_u \psi_d(L_d, L_d/v)|\rangle_d$. The interference between $|s_1\rangle_0$ and $|s_2\rangle_0$ cancels that between $|s_1\rangle_0$ and $|s_2'\rangle_0$, $\langle s_2'|s_1\rangle_0 + \langle s_2|s_1\rangle_0 = 0$. This is proved [34] with the Fermi statistics and the chirality $\psi(x,t) = \psi(x - vt)$. The full cancellation is natural in equilibrium.

The full cancellation does not happen when a fractional charge qe is injected to the MZI. The fractional charge is generated at $x = -d$ on the lower edge at $t = -d/v + t_0$, much earlier than the other events (d is assumed large). Then, the subprocesses are modified

$$|s_1\rangle = \psi_u^\dagger(0,0)|\rangle_u \psi_d(0,0)\eta_{d,q}^\dagger\left(-d,-\frac{d}{v}+t_0\right)|\rangle_d,$$
$$|s_2\rangle = \psi_u^\dagger\left(L_u,\frac{L_u}{v}\right)|\rangle_u \psi_d\left(L_d,\frac{L_u}{v}\right)\eta_{d,q}^\dagger\left(-d,-\frac{d}{v}+t_0\right)|\rangle_d,$$
$$|s_2'\rangle = \psi_u^\dagger\left(L_u,\frac{L_d}{v}\right)|\rangle_u \psi_d\left(L_d,\frac{L_d}{v}\right)\eta_{d,q}^\dagger\left(-d,-\frac{d}{v}+t_0\right)|\rangle_d, \tag{3}$$

such that electron tunneling happens at a QPC as in the previous case, while the fractional charge stays on the lower edge, passing QPC1 at time $t_0 \neq 0, (L_u - L_d)/v$ so that it does not overlap with the electron. In the domain of t_0 between 0 and $(L_u - L_d)/v$, the fractional charge passes QPC1 before the electron tunneling in $|s_1\rangle$ and passes QPC2 after the electron tunneling in $|s_2\rangle$ in the setup with $L_d > L_u$ [Fig. 2(c)]. Hence, an exchange between the fractional charge and electron happens in $|s_1\rangle$ in the direction opposite to $|s_2\rangle$. The interference between $|s_1\rangle$ and $|s_2\rangle$ gains, in comparison with the case of no fractional charge injection, a phase factor $e^{-i2\pi q}$ of the mutual statistics angle due to double exchange [see Eq. (1) with $q' = 1$]. In the same t_0 domain of the $L_d < L_u$ case, the events happen in reverse time ordering, and the interference gains $e^{i2\pi q}$. This is shown as

$$\langle s_2|s_1\rangle = \frac{1}{2\pi a}\langle s_2|s_1\rangle_0 e^{\mp i2\pi q}, \tag{4}$$

with $e^{-i2\pi q}$ for $L_d > L_u$, $e^{i2\pi q}$ for $L_d < L_u$, and $2\pi a$ from the short-length cutoff of $\eta_{d,q}$. In the other domain of t_0 [but $t_0 \neq 0, (L_u - L_d)/v$], the double exchange does not happen, $\langle s_2|s_1\rangle = \langle s_2|s_1\rangle_0/(2\pi a)$. By contrast, $|s_2'\rangle$ has an exchange in the same direction with $|s_1\rangle$ for any t_0 [Fig. 2(d)], hence, $\langle s_2'|s_1\rangle = \langle s_2'|s_1\rangle_0/(2\pi a) = -\langle s_2|s_1\rangle_0/(2\pi a)$. Thus, the two interferences cancel only partially, $\langle s_2|s_1\rangle + \langle s_2'|s_1\rangle \propto (e^{\mp i2\pi q} - 1)\langle s_2|s_1\rangle_0$ in the domain of t_0 between 0 and $(L_u - L_d)/v$.

The partial cancellation due to the mutual statistics has direct consequence in interference signals [differential conductance for the sources in Figs. 1(a)–1(c) and current for Fig. 1(d)] measured at detector D. For $w \ll \Delta L$, the processes and their complex conjugate lead [34] to

$$\begin{aligned}\text{Interference signal} &\propto \int dt_0 \mathrm{Re}[\langle s_2|s_1\rangle + \langle s_2'|s_1\rangle]\\ &\propto \frac{\Delta L}{\beta v}\Lambda_\beta \mathrm{Re}[\pm i(e^{\mp i2\pi q} - 1)e^{-2\pi i(\Phi/\Phi_0)}]\\ &\propto \frac{\Delta L}{\beta v}\Lambda_\beta \sin(\pi q)\cos\left(2\pi\frac{\Phi}{\Phi_0} \pm \pi q\right),\end{aligned} \tag{5}$$

with the magnetic flux Φ enclosed by the MZI, the flux quantum $\Phi_0 = h/e$, the thermal dephasing factor $\Lambda_\beta = \mathrm{csch}(\Delta L/L_\beta)$, and the sign $\pm$ determined by $\mathrm{sgn}(L_d - L_u)$. The second line is obtained using Eq. (4) and $\langle s_2|s_1\rangle_0 \propto \pm i(\Lambda_\beta/\beta)e^{-2\pi i(\Phi/\Phi_0)}$. The factor $\Delta L/v$ comes from the t_0 domain of the partial cancellation, showing that the double exchange is more probable at larger $\Delta L/w$. The factor $\sin(\pi q)$ indicates full cancellation between $\langle s_2|s_1\rangle$ and $\langle s_2'|s_1\rangle$ when an electron, instead of a fractional charge, is injected ($q = 1$). Because of the thermal dephasing, the signal is visible at $\Delta L \lesssim L_\beta$. Equation (5) shows that the interference pattern is shifted by πq due to the mutual statistics, which can be read out by comparing the pattern with a reference [Fig. 2(b)].

The interference with the double exchange is compared with the conventional process of the MZI. The conventional interference occurs with $t_0 = 0$ or $(L_u - L_d)/v$ such that electron tunneling happens at QPC1 or QPC2 (leaving a fractional hole behind) when a fractional charge arrives at the QPC [34]. This process is negligible when $\Delta L/w \gg 1$. This is confirmed in the calculation [Fig. 2(b)] including all the processes and obtained with experimentally feasible parameters and the Keldysh Green functions. The calculation becomes identical to the result of Eq. (5) at $\Delta L \gg w$.

When one changes the voltage pulse shape (varying the pulse period T or applying a group of pulses) in Fig. 1(d), one can further control the number of injected fractional charges inside the MZI. Then, in the interference $\langle s_2|s_1\rangle$, the electron can braid more than one fractional charge, $n = \lfloor \Delta L/vT \rfloor$ or $n+1$ fractional charges with probability $p_n = \lceil \Delta L/vT \rceil - (\Delta L/vT)$ or $p_{n+1} = 1 - p_n$. Here, $\lfloor x \rfloor$ is the largest integer $\leq x$ and $\lceil x \rceil$ is the smallest integer $\geq x$. Another interference $\langle s_2'|s_1\rangle$ has no braiding. For $w \ll \Delta L \lesssim L_\beta$, the time-averaged interference current is found [34]

$$\overline{I_D^{\mathrm{int}}} \propto |f|\Lambda_\beta \cos\left(2\pi\frac{\Phi}{\Phi_0} - \arg f\right), \tag{6}$$

where $f = \pm i(p_n e^{\mp i2\pi q n} + p_{n+1}e^{\mp i2\pi q(n+1)} - 1)$ corresponds to the factor $\pm i\Delta L(e^{\mp i2\pi q} - 1)$ in Eq. (5). Equation (6) reduces to Eq. (5) for $vT > \Delta L$.

Concluding remark.—We demonstrated that fractional charges on integer QH edges obey the fractional mutual statistics. Our proposal for directly detecting the statistics is within experimental reach: The setups in Fig. 1 are experimentally available. The MZI with long coherence length has been realized many times. The parameters used in Fig. 2 are realistic; in Fig. 2, the phase shift $\pm\pi q$ is obtained for $\Delta L \sim 10\ \mu$m, with which the interference visibility is expected larger than 0.7.

We note that environmental effects can cause the same amount of an additional phase shift in both the interference signal and the reference. Hence, they do not affect the detection of the phase shift by the mutual statistics. Moreover, the dynamical phase of injected fractional charges does not cause a phase shift in Eq. (5), since a fractional charge propagates the same distance in the interfering subprocesses. There is one obstacle to this direction of detecting the mutual statistics, when the MZI is realized with $\nu = 2$ QH edges. There, interedge Coulomb interaction can result in additional unwanted fractionalization [20–22]. One can avoid this by gapping out the inner edge as done recently [38].

Our main process with the double exchange is nontrivial, existing with the help of the fractional statistics. The process has been unnoticed before, maybe because the full cancellation between $\langle s_2|s_1\rangle$ and $\langle s_2'|s_1\rangle$ is restored (causing no response in observables) when fractional charges are replaced by electrons ($q = 1$).

The process relies on the double exchange between two particles on QH edges rather than braiding with anyons in QH bulk. Such a double exchange will also be useful for detecting anyonic statistics in fractional QH interferometries [39,40] or anyon collision setups [41,42].

Our work suggests exploring fractional statistics in noninteracting systems, which will be useful for engineering anyons and for detecting, nonlocally with an MZI, the information of the interaction regions (e.g., the interedge interaction). It will be valuable to generalize our work to the fractional statistics of Luttinger liquids.

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