Boosting Entanglement Generation in Down-Conversion with Incoherent Illumination

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Entangled photons produced by spontaneous parametric down-conversion have been of paramount importance for our current understanding of quantum mechanics and advances in quantum information. In this process, the quantum correlations of the down-converted photons are governed by the optical properties of the pump beam illuminating the nonlinear crystal. Extensively, the pump beam has been modeled by either coherent beams or by the well-known Gaussian–Schell model, which leads to the natural conclusion that a high degree of optical coherence is required for the generation of highly entangled states. Here, we show that when a novel class of partially coherent Gaussian pump beams is considered, a distinct type of quantum state can be generated for which the amount of entanglement increases inversely with the degree of coherence of the pump beam. This leads to highly incoherent yet highly entangled multiphoton states, which should have interesting consequences for photonic quantum information science.

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Introduction.—Quantum correlated photons generated in the process of spontaneous parametric down-conversion (SPDC) have played a key role in the development of quantum information science over the last decades. Originating from the conservation of momentum, there is robust entanglement generation between the transverse spatial variables (position and momentum) of the down-converted photons [1], which has attracted considerable attention as it can be used to define high-dimensional quantum systems [2–9] as well as engineered in a number of ways [10–14]. This is due to the fact that the down-converted photons can inherit properties of the pump laser beam [10,15], which provides interesting relations between the classical optical properties of the pump field and the nonclassical characteristics of the multiphoton state. Thus, SPDC provides rich and flexible quantum state engineering that is crucial for many fundamental studies and applied research.

Several authors have already studied the spatial entanglement of SPDC in a more generalized framework than usual, where partially coherent pump beam illumination is considered [16–21]. In particular, Refs. [18–20] addressed the position and wave vector quantum correlations of down-converted photons produced by a pump beam with partial transverse spatial coherence, described by the well-known Gaussian–Schell model (GSM) [22]. They showed that, in this case, highly entangled states can only be observed when the pump beam has a large degree of spatial coherence.

However, the GSM beam is not the only example of a partially coherent beam. In 1993, Simon and Mukunda introduced the "twisted Gaussian Schell Model" (TGSM), which is a more general partially coherent Gaussian beam with rotational symmetry around the propagation axis [23,24]. This model predicted the existence of novel correlations between the transverse variables of Gaussian beams, a property they dubbed the "twist phase." However, the twist phase is not a phase in the usual sense and in fact vanishes in the coherent limit. These beams have been experimentally realized [25,26], and recent studies have shown their improved resilience against turbulence-induced degradation effects when compared to traditional GSM beams [27,28] with applications in imaging [29]. They also carry orbital angular momentum and therefore can find applications in biophysics and metrology [30,31].

Motivated by these results, we present here several advantages of adopting TGSM beams for the process of SPDC. In particular, we show that highly mixed, highly entangled states can be produced by exploiting the twist phase in TGSM beams. Counterintuitively, in this case the entanglement actually increases with the incoherence of the pump beam. This effect is a consequence of the infinite dimension of the quantum states defined in terms of the position and momentum of down-converted photons, a fact that allows for separable and highly entangled states to be arbitrarily close together in state space [32–34]. In addition, we are able to connect the SPDC entanglement with the twist phase of the TGSM pump beam, a novel optical trait in its own right, with useful applications in optics. This should open the way for producing highly entangled photons in a highly mixed state, which have many potential applications in quantum information science.

TGSM beams.—The spatial degree of freedom of a paraxial and monochromatic optical field with wave number k can be described by the near-field (position) and far-field variables (wave vector). For a paraxial field propagating in the z direction, let us define the position in a transverse plane as $\boldsymbol{r} = (r_x, r_y) = (x, y)$, and the transverse wave vector as $\boldsymbol{q} = (q_x, q_y)$. Defining a vector $\xi = (x, q_x, y, q_y)$, the second moments of these variables can be written in a 4×4 covariance matrix (CM) V. Optical fields with a Gaussian transverse profile are completely and uniquely described by the CM V up to a translation. The same can be said for Gaussian states in quantum mechanics [35].

A TGSM beam is therefore uniquely characterized by its CM [23,24]:

$$T = \begin{pmatrix} \sigma^2 & -\frac{k\sigma^2}{R} & 0 & ku\sigma^2 \\ -\frac{k\sigma^2}{R} & \tau^2 & -ku\sigma^2 & 0 \\ 0 & -ku\sigma^2 & \sigma^2 & -\frac{k\sigma^2}{R} \\ ku\sigma^2 & 0 & -\frac{k\sigma^2}{R} & \tau^2 \end{pmatrix}. \qquad (1)$$

Here σ is the beam waist, $\tau^2 = (1/\delta^2) + (1/4\sigma^2) + k^2[(\sigma^2/R^2) + u^2\sigma^2)]$ is the variance of the wave vector distribution, δ is the transverse coherence length, and R is the radius of curvature. The parameter u is the so-called twist phase, here with dimension given by length^{-1}. These are Gaussian beams in which the position coordinates x and y are coupled to the wave vector coordinates q_y and q_x, respectively. This results in a nonzero angular momentum $\langle L_z \rangle = 2\hbar k u \sigma^2$, giving origin to the nomenclature "twist." As with particular cases of a TGSM beam, the well-known (rotationally symmetric) GSM [22,36] is obtained by setting $u = 0$, and a spatially coherent Gaussian beam is recovered by setting $\delta = \infty$. The positivity constraints on the CM (1) lead to [23,24]: $|u| \leq 1/k\delta^2$. Thus, the twist phase tends to zero for a perfectly coherent beam ($\delta \to \infty$). We note that the twist phase appears in the variance of the momentum coordinates τ^2 through the term $k^2u^2\sigma^2$, which causes increased beam divergence as a function of u [23–25]. TGSM beams have been produced and studied experimentally, and they can be constructed as a convex (incoherent) combination of coherent Gaussian beams with different transverse phases [25,26], as illustrated in Fig. 1.

Partially coherent SPDC.—Consider now that a TGSM is used to pump a nonlinear down-conversion crystal, as shown in Fig. 1. Let us assume that the down-converted photons (1,2) are degenerate, so that $k_1 = k_2 = k/2$, where k is the wave number of the pump beam. Let us define the global coordinates:

$$\boldsymbol{q}_\pm = \boldsymbol{q}_1 \pm \boldsymbol{q}_2; \qquad \boldsymbol{r}_\pm = \frac{1}{2}(\boldsymbol{r}_1 \pm \boldsymbol{r}_2). \qquad (2)$$

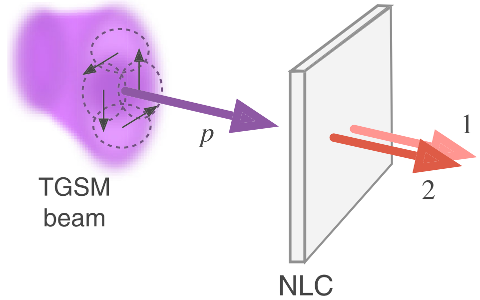

FIG. 1. A partially coherent TGSM beam incident on a nonlinear crystal producing photon pair. The small black arrows illustrate transverse momentum.

Under appropriate conditions [37], it is well known that the two-photon state is nearly separable in these global sum and difference coordinates [15,18,43–45]. For example, for a partially coherent pump beam, the two-photon Wigner function is $\mathcal{W}(\boldsymbol{\xi}_+, \boldsymbol{\xi}_-) = W_+(\boldsymbol{\xi}_+)W_-(\boldsymbol{\xi}_-)$, where $\boldsymbol{\xi}_\pm = (x_\pm, q_{\pm x}, y_\pm, q_{\pm y})$ are the global phase space coordinates [37]. Here W_+ is the Wigner function describing the spatial properties of the pump laser, and W_- is the Wigner function of the so-called phase matching function $\mathcal{S}$ [44,45]. Though $\mathcal{S}$ is not a Gaussian function, it can be approximated using a double-Gaussian representation [45] in which both $\mathcal{S}$ and its Fourier transform $\tilde{\mathcal{S}}$ are each approximated by Gaussian functions giving the same variance. In most situations, this is enough to describe the salient features of the two-photon state [3,43,45].

The separable form of the two-photon state with respect to the global coordinates allows for several immediate conclusions. First, the purity of the two-photon state is given by $\mu_{12} = \mu_+\mu_-$, where $\mu_\pm$ is the purity associated with $W_\pm$ [37]. The second implication is that the 8×8 two-photon CM is

$$G = \begin{pmatrix} V_+ & 0 \\ 0 & V_- \end{pmatrix}, \qquad (3)$$

where $V_\pm$ is the 4×4 CM describing second moments of $\boldsymbol{\xi}_\pm$. Thus, to study properties of the two-photon state when the pump is a TGSM beam, we simply set V_+ equal to the CM of Eq. (1). In this way, the purity $\mu_+ = [4\sqrt{\mathrm{Det}(V+)}]^{-1} = \beta^2$, where $\beta^2 = (1 + 4\sigma^2/\delta^2)^{-1}$ is the dimensionless normalized coherence parameter of the pump beam [25] ($0 \leq \beta^2 \leq 1$). For a coherent beam, we have $\beta^2 = 1$ ($\delta^2 \gg \sigma^2$), while for a completely incoherent beam, $\beta^2 = 0$ ($\delta^2 \ll \sigma^2$). Thus, the purity of the two-photon state $\mu_{12} = \mu_-\beta^2$ is proportional to the square of the coherence of the TGSM pump beam.

For the double-Gaussian representation of the phase matching function, we use the diagonal CM $V_- = \mathrm{diag}(\sigma_-^2, \Delta_-^2, \sigma_-^2, \Delta_-^2)$, since it is approximately separable in the x and y spatial directions [13]. Following Schneeloch and Howell [45], the variances are $\sigma_-^2 = 9L/10k$ and $\Delta_-^2 = 3k/2L$, where L is the length of the crystal in the longitudinal direction. We note that these are the variances calculated from non-Gaussian $\mathcal{S}$ and $\tilde{\mathcal{S}}$, which are not Fourier-transform limited. Thus, the purity $[4\sqrt{\mathrm{Det}(V-)}]^{-1} = (4\sigma_-^2\Delta_-^2)^{-1} = 5/27$ is less than unity, which would be obtained from the actual Wigner function.

To write the two-photon CM G in coordinates describing the individual photons, we define $\boldsymbol{\xi}_{12} = (\boldsymbol{\xi}_1, \boldsymbol{\xi}_2)$, with $\boldsymbol{\xi}_j = (x_j, q_{j_x}, y_j, q_{j_y})$ $(j = 1, 2)$. Then, the CM can be obtained by $V_{12} = \boldsymbol{R}G\boldsymbol{R}^T$, where $\boldsymbol{R}$ is the matrix representing the inverse of the coordinate transformations (2). It is straightforward to calculate

$$V_{12} = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}, \tag{4}$$

where 4×4 matrices of type A (B) refer to photon 1 (2), and the C matrices refer to correlations between the photons. We have $A = B$, with

$$A = \begin{pmatrix} \sigma^2 + \sigma_-^2 & -\frac{k\sigma^2}{2R} & 0 & \frac{ku\sigma^2}{2} \\ -\frac{k\sigma^2}{2R} & \frac{1}{4}(\tau^2 + \Delta_-^2) & -\frac{ku\sigma^2}{2} & 0 \\ 0 & -\frac{ku\sigma^2}{2} & \sigma^2 + \sigma_-^2 & -\frac{k\sigma^2}{2R} \\ \frac{ku\sigma^2}{2} & 0 & -\frac{k\sigma^2}{2R} & \frac{1}{4}(\tau^2 + \Delta_-^2) \end{pmatrix}. \tag{5}$$

Thus, there are cross-correlations between the near-field (position) and far-field (momentum) coordinates of each down-converted photon that are proportional to the twist phase u. This results in orbital angular momentum that is one-half that of the pump beam: $\langle L_z \rangle = \hbar k u \sigma^2$, implying that it is conserved from the pump to the down-converted photons.

The matrices C describe correlations between photons 1 and 2. We have

$$C = \begin{pmatrix} \sigma^2 - \sigma_-^2 & -\frac{k\sigma^2}{2R} & 0 & \frac{ku\sigma^2}{2} \\ -\frac{k\sigma^2}{2R} & \frac{1}{4}(\tau^2 - \Delta_-^2) & -\frac{ku\sigma^2}{2} & 0 \\ 0 & -\frac{ku\sigma^2}{2} & \sigma^2 - \sigma_-^2 & -\frac{k\sigma^2}{2R} \\ \frac{ku\sigma^2}{2} & 0 & -\frac{k\sigma^2}{2R} & \frac{1}{4}(\tau^2 - \Delta_-^2) \end{pmatrix}. \tag{6}$$

The wave vector correlations diverge more rapidly due to the presence of the twist phase in the τ^2 term. We see that $\langle x_1 q_{y2} \rangle - \langle x_2 q_{y1} \rangle = ku\sigma^2$. This is not an optical angular momentum *per se* but rather a coupling between perpendicular components of the position of one photon and the wave vector of the other [14].

Twist phase and entanglement.—In typical SPDC experiments, entanglement can be identified by observing correlations in the near-field (position) variables and the far-field (wave vector) variables, leading to violation of one of two inequalities [46]:

$$\langle \Delta r_\pm \rangle \langle \Delta q_{r\mp} \rangle \geq \frac{1}{2}, \tag{7}$$

where $r_\pm = x_\pm, y_\pm$ and $q_{r\pm} = q_{x\pm}, q_{y\pm}$. Violation of the inequalities (7) occurs when either the two-photon state is anticorrelated in the near field and correlated in the far field ("$+-$"), or correlated in the near field and anticorrelated in the far field ("$-+$"). The latter is typically the case in SPDC, and a number of experiments have used these or similar inequalities to identify the entanglement of the down-converted photons [1,7,9,19,20,47].

While the above criteria are sufficient for many experiments, they fail to capture entanglement that arises from correlations between different spatial d.o.f. A more complete analysis is achieved by calculating the four symplectic eigenvalues $\{\lambda_i\}$ of the CM of the partially transposed state [35]. This allows us to investigate the so-called distillable entanglement [48]. Partial transposition corresponds to changing the sign of the wave vector coordinates of one of the photons [49]. To simplify the analysis, we perform a local scaling of the coordinates of each photon using the transformation $\xi'_{12} = \boldsymbol{S}\xi_{12}$, with $\boldsymbol{S} = \oplus_{k=1}^4 S_k$ and $S_k = \mathrm{diag}(1/\sqrt{2}, \sqrt{2})$. In this case, observing $\lambda_i < 1/2$ implies that the state has a negative partial transpose, which indicates entanglement [35,49]. Furthermore, the smallest symplectic eigenvalue can be used to quantify Gaussian entanglement using the negativity or other quantifiers [35]. The symplectic eigenvalues of Eq. (4) are twofold degenerate and given by [37]

$$\lambda_\pm = \frac{1}{\sqrt{2}} \left| \sqrt{a_+ \pm \sqrt{4k^2\Delta_-^2\sigma_-^2\sigma^4\left(u^2 + \frac{1}{R^2}\right) + a_-^2}} \right|, \tag{8}$$

where $a_\pm = \tau^2\sigma_-^2 \pm \Delta_-^2\sigma^2$.

We can now analyze the entanglement produced in the SPDC process as a function of the spatial coherence of the TGSM pump beam and its twist phase. Figure 2(a) shows a plot of the smallest symplectic eigenvalue (red surface) as a function of the absolute value of the twist phase $|u|$ and the spatial coherence parameter β of the TGSM pump beam. Here, $|u|$ is scaled by $1/k\delta^2$, so that it varies from 0 to 1. Two regions of entanglement are clearly visible. One corresponds to larger coherence parameter β. This is the usual case, where entanglement increases as a function of the pump beam coherence [18–20]. However, we also observe a second region where entanglement grows inversely with the coherence and is present even though

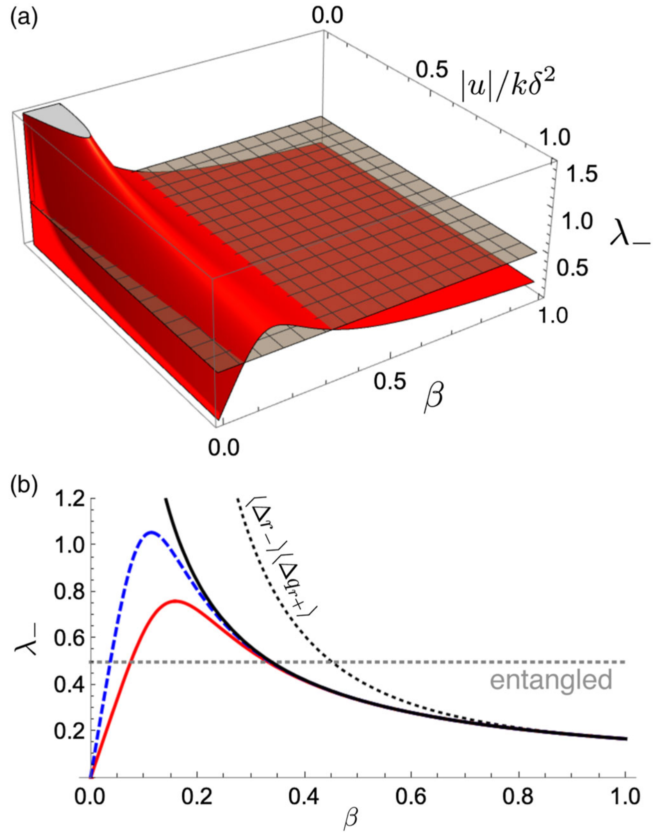

FIG. 2. Smallest symplectic eigenvalue of partially transposed state. (a) Evaluation of smallest symplectic eigenvalue λ_- (red surface) as a function of the normalized pump beam coherence β and normalized twist phase. Entanglement is confirmed when $\lambda_- < 1/2$ (gray horizontal plane). The SPDC parameters are $R = \infty$, $\lambda_p = 400$ nm, $\sigma_p = 50$ μm, and $L = 1$ cm. (b) Profile plots of λ_- for normalized twist phase $|u|/k\delta^2$ equal to zero (black solid line), 1 (red solid line), and $1/2$ (blue dashed line). The dotted black curve is the near-field and far-field entanglement criteria (7).

β nears zero. We note that this only occurs for larger values of the twist phase.

To better visualize these results, in Fig. 2(b), we plot λ_- as a function of β for $|u|/k\delta^2 = 0, 1/2, 1$. In all cases we can identify entanglement when the pump beam is more coherent. Also shown is the lhs of the inequality, Eq. (7), showing a violation for larger β. This indicates that for larger pump coherence, the entanglement involves the same spatial d.o.f. of the photons and can be detected using the standard approach. Indeed, for a coherent pump beam, with $R = \delta = \infty$ and twist phase $u = 0$, the symplectic eigenvalues $\lambda_\pm$ reduce to the lhs of the entanglement criteria, Eq. (7). Nevertheless, Fig. 2(b) shows that this "standard" entanglement decreases and eventually disappears as the pump beam becomes more incoherent, as observed in Refs. [18–20].

For the nonzero twist phase, our study reveals that a different type of entangled quantum state can be generated for small values of β. Here the two-photon state is highly mixed since the purity $\mu_{12} \propto \beta^2$. Interestingly, the entanglement can be larger when the pump beam is less coherent. In this region, there is no violation of the near-field and far-field criteria, Eq. (7). We numerically tested a wide range of parameters $\sigma_-, \Delta_-, \sigma$ and observed qualitatively similar results. To identify this entanglement, one can measure the elements of the CM by measuring five coincidence images in combinations of detection planes (see [37] for detailed description) using simple lens systems [47] and then determine the symplectic eigenvalues of the partial transposition.

To provide an intuitive explanation for this phenomenon, note that when $u \neq 0$, the correlation matrix C contains extra covariance terms with modulus $\propto k\sigma^2|u|$, which are upperbounded by σ^2/δ^2. These covariances originate from the correlations already present in the pump beam, which transformed into quantum correlations between the down-converted photons. When the twist phase is appreciable, the correlations grow at the same rate that the purity decreases ($\mu_{12} \propto \beta^2 \sim \delta^2/\sigma^2$). On the other hand, when $u = 0$, these covariances are zero, demonstrating that this is an effect that could only be revealed while considering this general class of TGSM beams in the SPDC process. The existence of highly mixed yet highly entangled states is a known phenomenon that appears for infinite dimensional systems (for another example, see Ref. [34]) since for these systems there is no dense region of separable states in the state space [32,33]. Thus, there exist entangled mixed states that lie arbitrarily close to separable mixed states. We note that there is usually some physical (e.g., energy) constraint in the generation of infinite dimensional systems. In our case, the relevant physical parameter is the number of transverse modes supported by the optical systems, providing a lower limit to β. We note that SPDC experiments have been realized with $\beta \lesssim 0.1$ [20], indicating that experimental observation of this phenomenon is feasible with current technology.

Conclusions.—In this Letter, we bridge together the more general theory of partially coherent Gaussian beams, namely the twisted Gaussian–Schell model, and the process of spontaneous parametric down-conversion that has been used extensively over the last decades to produce entangled photons. By doing so, we reveal new phenomena that allow for the generation of a class of multiphoton states with unique entanglement and coherence properties. Even though similar entangled states might be created in quantum optics of entangled qumodes [34,50], here we are able to interconnect the amount of entanglement with the so-called twist phase of the pump beam, an intriguing optical phenomenon first introduced in 1993 [23]. We note that twist phase is a property of incoherent Gaussian beams that vanishes in the coherent limit. Thus, the novel entanglement produced here is directly related to the incoherence of the pump beam. We can envisage a number of potential applications, and we expect that these highly mixed yet highly entangled states should allow for the exploitation of highly entangled photons in quantum adaptations of applications originally designed for incoherent beams, such as imaging and optical communications. For example, a very recent study has shown that partially coherent multiphoton states are more resistant to atmospheric turbulence [51]. Our results provide a way to increase the spatial entanglement in this scenario.

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