Silence of Binary Kerr Black Holes

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A nontrivial $\mathcal{S}$ matrix generally implies a production of entanglement: starting with an incoming pure state, the scattering generally returns an outgoing state with nonvanishing entanglement entropy. It is then interesting to ask if there exists a nontrivial $\mathcal{S}$ matrix that generates no entanglement. In this Letter, we argue that the answer is the $\mathcal{S}$-matrix for the scattering of classical black holes. We study the spin entanglement in the scattering of arbitrary spinning particles. Augmenting the $\mathcal{S}$-matrix with Thomas–Wigner rotation factors, we derive the entanglement entropy from the gravitational induced $2 \to 2$ amplitude. In the Eikonal limit, we find that the relative entanglement entropy, defined here as the *difference* between the entanglement entropy of the *in* and *out* states, is nearly zero for minimal coupling irrespective of the *in* state and increases significantly for any nonvanishing spin multipole moments. This finding suggests that minimal couplings of spinning particles, whose classical limit corresponds to a Kerr black hole, have the unique feature of generating near zero entanglement.

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Introduction.—One of the fascinating realizations in the interplay of gravitational scattering amplitudes and the dynamics of compact binary systems is the equivalence of minimally coupled spinning particles and rotating black holes. In the analysis of three-point amplitudes of particles with general spin, a unique amplitude for massive spin-s particles emitting a massless graviton was defined kinematically in [1] and termed *minimal coupling*. The term reflects its matching to minimal derivative coupling when taking the high energy limit for $s \leq 2$. Since massless particles have spins bounded by 2 in flat space, the role of minimal coupling with $s > 2$ was initially not clear.

Through a series of subsequent analyses [2–6], it was understood that the spin multipoles generated by minimal coupling are exactly those of a spinning black hole, i.e., the spin moments in the effective stress-energy tensor of the linearized Kerr solution. This was verified by reproducing the Wilson coefficients of the one-particle effective field theory [7,8] for a Kerr black hole [3] and the classical scattering angle at leading order in the Newton constant G to all orders in spin [4].

While the equivalence can be established through various direct matchings, the principle that underlies such correspondence remains unclear. In this Letter, we seek to resolve this uncertainty by studying spin entanglement entropy. We study the $2 \to 2$ Eikonal limit $\mathcal{S}$-matrix in the basis of two-particle spin states. By measuring the relative entanglement entropy for the final state, defined as

$$\Delta S \equiv -\mathrm{tr}[\rho^{\mathrm{out}} \log \rho^{\mathrm{out}}] + \mathrm{tr}[\rho^{\mathrm{in}} \log \rho^{\mathrm{in}}], \tag{1}$$

where $\rho^{\mathrm{in,out}}$ is the reduced density matrix for the in and out states, remarkably we find that $\Delta S \approx 0$ or, equivalently, when the effective field theory Wilson coefficients are set to the black hole value of unity. Any deviation from unity significantly increases the relative entropy.

Entanglement via $\mathcal{S}$ matrix.—The study of entanglement in scattering events has a long history (for recent developments, see [9–12]). After denoting the two-particle Hilbert space by $\mathcal{H} = \mathcal{H}_a \otimes \mathcal{H}_b$, for each subsystem we can further divide the space into spin and momentum degrees of freedom, e.g., $\mathcal{H}_{\mathrm{a}} = \mathcal{H}_{s_a} \otimes \mathcal{H}_{p_a}$. In computing the entanglement from scattering, there are two sources of difficulty. First, the trace over momentum states lead to divergences due to the infinite space-time volume, and introducing a cutoff leads to regulator dependent results (see, e.g., [13,14]). Second, under Lorentz rotations, the spin undergoes Thomas–Wigner rotation and thus one does not have a Lorentz invariant definition of the reduced density matrix [15,16] (see [17] for further discussions).

On the other hand, the same difficulty also appears in the extraction of conservative Hamiltonians of binary systems from relativistic scattering amplitudes. In particular, in a $2 \to 2$ scattering process, the spin (little group) space of the incoming particles is invariably distinct from the outgoing space as the particles' momentum is distinct. However, by augmenting the $\mathcal{S}$ matrix with Thomas–Wigner rotation factors, the final state can be mapped back into the spin Hilbert space of the incoming state. Indeed, this *Hilbert space matching* procedure was used heavily in the computation of the spin-dependent part of the conservative Hamiltonian [18–20].

We thus consider elastic scattering in the spin Hilbert space $\mathcal{H} = \mathcal{H}_{s_a} \otimes \mathcal{H}_{s_b}$. With a given in state, we can obtain the out state via the amplitude as

$$|\text{out}\rangle = (U_a \otimes U_b)M|\text{in}\rangle, \tag{2}$$

where $U_{a,b}$ are the Hilbert space matching factors that will be discussed in the next section [21]. The total density matrix of the out state is then simply $\rho^{\text{out}}_{a,b} = |\text{out}\rangle\langle\text{out}|$, and the reduced density matrix is given by $\rho_a = \text{tr}_b \rho_{a,b}$. Equipped with ρ_a, we can consider a variety of entanglement quantifiers. A canonical choice is the entanglement entropy, i.e., the Von Neumann entropy of the reduced density matrix $S_{\text{VN}} = -\text{tr}_a[\rho_a \log \rho_a]$. Note that here, S_{VN} in principle depends on the in state. For a quantifier that is independent of the in state, we can consider the entanglement power [23] given by

$$\mathcal{E}_a = 1 - \int \frac{d\Omega_a}{4\pi}\frac{d\Omega_b}{4\pi}\text{tr}_a \rho_a^2, \tag{3}$$

where Ω represents the spin-s phase space.

In the following, we will consider the elastic $\mathcal{S}$ matrix acting on $|\text{in}\rangle = |s_a\rangle \otimes |s_b\rangle$, i.e., the in state is set up as a pure state. Thus, by computing the entanglement entropy of the out state, we obtain the entanglement enhancement of the scattering process.

The Eikonal amplitude in spin space.—In this section, we compute the leading order amplitude $ab \to a'b'$ for general massive spinning particles in the Eikonal limit. Working in the center of mass frame where $p_a = (E_a, 0, 0, \vec{p})$, $p_b = (E_b, 0, 0, -\vec{p})$ and the momentum transfer $q = p_a - p'_a = (0, \vec{q})$, the Eikonal limit corresponds to $q^2 \to 0$. Fourier transforming to the impact parameter space, we obtain the Eikonal phase, whose exponentiation yields to the $\mathcal{S}$ matrix in the Eikonal limit.

Spin-s amplitudes and Hilbert space matching: We begin with the scattering of spinning particles induced by gravitational interactions. At leading order in the Newton constant G, the four point amplitude for the $ab \to a'b'$ illustrated in Fig. 1, can be written as [19]

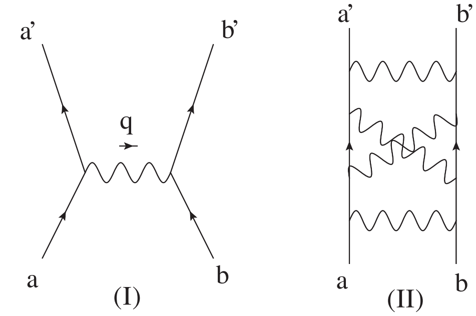


FIG. 1. $2 \to 2$ scattering of two spinning objects exchanging gravitons. (I) Process in the leading order of the Newton constant G. (II) Eikonal approximation, which resums the ladder diagrams.

$$M_{\text{tree}}(q^2) = -8\pi G \frac{m_a^2 m_b^2}{q^2} \times \sum_{\eta=\pm1} e^{2\eta\Theta}[\varepsilon^*_{a'} W_a(\eta\tau_a)\varepsilon_a][\varepsilon^*_{b'} W_2(\eta\tau_b)\varepsilon_b] + \mathcal{O}(q^0), \tag{4}$$

where q^μ is the transfer momenta, ε_i is the polarization tensor of the spinning particle, $\tau_{a,b} = q \cdot S/m_{a,b}$, and the exponential parameters are defined as $\cosh\Theta \equiv p_a \cdot p_b/m_a m_b$ and $\eta = \pm1$ labeling the exchanged graviton's helicity. The function $W(\eta\tau)$ is defined as

$$W_{a,b}(\eta\tau_{a,b}) = \left[\sum_{n=0}^{2s_{a,b}} \frac{C_n}{n!}\left(\eta\frac{q \cdot S}{m_{a,b}}\right)^n\right], \tag{5}$$

where S is the Pauli–Lubanski spin vector and $C_{a,n}$, $C_{b,n}$ parameterizes the possible distinct couplings for particle a, b. These are the $2s$ multipole moments carried by a spin-s particle and can be directly matched to the Wilson coefficients of the one-particle effective action (see [24] for the all order in spin action). For rotating black holes $C_{a,n} = C_{b,n} = 1$, and the classical spin is recovered in the limit $s \to \infty$, $\hbar \to 0$ while keeping the classical spin $S \equiv s\hbar$ fixed (see [25] for a more detailed discussion).

As shown in Ref. [18], we can transform the spin vector S in an operator acting in the little group space through the insertion of a complete set of polarization tensors associated with the incoming particles:

$$\mathbb{S}_{a,b} \equiv \varepsilon^*_{a,b,\{I_s\}} S^\mu \varepsilon^{\{J_s\}}_{a,b}, \tag{6}$$

where $\{I_s\}, \{J_s\}$ are the $SU(2)$ indices of particle a, b. In components, we have that

$$\mathbb{S}^\mu_{a,b} = \left[\frac{\vec{p}_{a,b} \cdot \vec{\Sigma}}{m_{a,b}}, \vec{\Sigma} + \frac{\vec{p}_{a,b} \cdot \vec{\Sigma}}{m_{a,b}(m_{a,b} + E_{a,b})}\vec{p}_{a,b}\right], \tag{7}$$

where $\vec{\Sigma}$ is the spin-s rest frame spin operator satisfying the commutation relation $[\Sigma_i, \Sigma_j] = i\epsilon_{ijk}\Sigma_k$. For detailed derivation of the spin-vectors please see Ref. [29]. Then the operator τ in the little group space is given by

$$\mathbb{T}_{a,b} \equiv \varepsilon^{*}_{a,b,\{I_s\}} \frac{q \cdot S}{m_{a,b}} \varepsilon^{\{J_s\}}_{a,b} \equiv \frac{q \cdot \mathbb{S}_{a,b}}{m_{a,b}}. \tag{8}$$

Writing Eq. (4) in terms of $\mathbb{T}$ leads to an amplitude that corresponds to an operator acting on states in a distinct little group space as the momenta of a, b are distinct from a', b'. This can be rectified by the so-called *Hilbert space matching* procedure that uses the Lorentz transformation that relates the momenta of the in states to the out states to convert the out states' Hilbert space back to the in states [18,19]. The result is the additional Thomas–Wigner rotation factors for each of the two particles. For example, for particle a this factor, in leading order in q^2, is written as

$$U_a = \exp\left[-i \frac{m_a m_b \mathbb{E}_a}{(m_a + E)E}\right], \tag{9}$$

where $\mathbb{E}_a \equiv \epsilon(q, u_a, u_b, a_a) = \epsilon_{\mu\nu\rho\sigma} q^\mu u_a^\nu u_b^\rho a_a^\sigma$, $a_a = \mathbb{S}_a/m_a$, $u_{a,b} = p_{a,b}/m_{a,b}$, and $E = E_a + E_b$. In summary, the amplitude after the Hilbert space matching, denoted by $\bar{M}$, is given by

$$\begin{aligned}\bar{M}_{\text{tree}}(q^2) &= -8\pi G \frac{m_a^2 m_b^2}{q^2} \\ &\times \sum_{\eta=\pm 1} e^{2\eta\Theta} W_a(\eta \mathbb{T}_a) W_b(\eta \mathbb{T}_b) U_a U_b + \mathcal{O}(q^0).\end{aligned} \tag{10}$$

Expanding Eq. (10) up to order $\mathcal{O}(\mathbb{S}^{2s_i})$ gives

$$\begin{aligned}\bar{M}_{\text{tree}}(q^2) = -&\frac{16\pi G m_a^2 m_b^2}{q^2} \\ &\times \left\{ \sum_{m=0}^{\lfloor s_a \rfloor} \sum_{n=0}^{\lfloor s_b \rfloor} A_{2m,2n} (\mathbb{T}_a^{2m} \otimes \mathbb{T}_b^{2n}) \right. \\ &+ \frac{m_a^2 m_b}{E} \sum_{m=0}^{\lceil s_a \rceil -1} \sum_{n=0}^{\lfloor s_b \rfloor} A_{2m+1,2n} (\text{Sym}[\mathbb{E}_a \mathbb{T}_a^{2m}] \otimes \mathbb{T}_b^{2n}) \\ &+ \frac{m_a m_b^2}{E} \sum_{m=0}^{\lfloor s_a \rfloor} \sum_{n=0}^{\lceil s_b \rceil -1} A_{2m,2n+1} (\mathbb{T}_a^{2m} \otimes \text{Sym}[\mathbb{E}_b \mathbb{T}_b^{2n}]) \\ &+ \left. \sum_{m=0}^{\lceil s_a \rceil -1} \sum_{n=0}^{\lceil s_b \rceil -1} A_{2m+1,2n+1} (\mathbb{T}_a^{2m+1} \otimes \mathbb{T}_b^{2n+1}) \right\},\end{aligned} \tag{11}$$

where we used the shorthand notation $\mathbb{T}_{a,b} \equiv (q \cdot a_{a,b})$ and

$$\begin{aligned}&\text{Sym}[\mathbb{E}_i \mathbb{T}_i^{2n}] \\ &\equiv \frac{1}{2n+1} [\mathbb{E}_i \mathbb{T}_i^{2n} + \mathbb{T}_i \mathbb{E}_i \mathbb{T}_i^{2n-1} + \cdots + \mathbb{T}_i^{2n} \mathbb{E}_i]\end{aligned} \tag{12}$$

for $i = a$, b. The explicit form of the coefficients $A_{m,n}$ in Eq. (11), up to m, $n = 2$, is given by

$$\begin{aligned}A_{0,0} &= c_{2\Theta}, \qquad A_{1,0} = \frac{i(2Er_a c_\Theta - m_b c_{2\Theta})}{m_a^2 m_b r_a}, \\ A_{1,1} &= \frac{c_{2\Theta} s_\Theta^2 m_a m_b}{E^2 r_a r_b} + c_{2\Theta} - \frac{2m_b c_\Theta s_\Theta^2}{E r_a} - \frac{2m_a c_\Theta s_\Theta^2}{E r_b}, \\ A_{2,0} &= \frac{C_{a,2} c_{2\Theta}}{2} + \frac{m_b^2 c_{2\Theta} s_\Theta^2}{2E^2 r_a^2} - \frac{2m_b c_\Theta s_\Theta^2}{E r_a}, \\ A_{2,1} &= i\left(\frac{E C_{a,2} c_\Theta}{m_a m_b^2} - \frac{C_{a,2} c_{2\Theta}}{2m_b^2 r_b} + \frac{c_{2\Theta}}{4E^2 r_a^2 r_b} \right. \\ &\quad - \frac{c_{4\Theta}}{8E^2 r_a^2 r_b} - \frac{c_\Theta}{2E r_a m_b r_b} + \frac{c_{3\Theta}}{2E r_a m_b r_b} \\ &\quad \left. - \frac{c_{2\Theta}}{m_a r_a m_b} - \frac{1}{8E^2 r_a^2 r_b} - \frac{c_\Theta}{4E m_a r_a^2} + \frac{c_{3\Theta}}{4E m_a r_a^2} \right), \\ A_{2,2} &= \frac{C_{a,2} C_{b,2} c_{2\Theta}}{4} - \frac{C_{a,2} c_\Theta s_\Theta^2 m_a}{E r_b} - \frac{C_{b,2} c_\Theta s_\Theta^2 m_b}{E r_a} \\ &\quad + \frac{c_{2\Theta} s_\Theta^4 m_a^2 m_b^2}{4E^4 r_a^2 r_b^2} - \frac{c_\Theta s_\Theta^4 m_a m_b^2}{E^3 r_a^2 r_b} - \frac{c_\Theta s_\Theta^4 m_a^2 m_b}{E^3 r_a r_b^2} \\ &\quad + \frac{c_{2\Theta} s_\Theta^2 m_a m_b}{E^2 r_a r_b} + \frac{C_{b,2} c_{2\Theta} s_\Theta^2 m_b^2}{4E^2 r_a^2} + \frac{C_{a,2} c_{2\Theta} s_\Theta^2 m_a^2}{4E^2 r_b^2},\end{aligned} \tag{13}$$

where $C_{a,2}$ and $C_{b,2}$ are the Wilson coefficients for each particle, $(c_\Theta, s_\Theta) \equiv (\cosh\Theta, \sinh\Theta)$, and $r_{a,b} \equiv 1 + E_{a,b}/m_{a,b}$. We can see that the Wilson coefficients $C_{a,n}$ and $C_{b,n}$ start to appear at $A_{2,0}$, which means that we need to go to at least spin-1 to compare the difference between black holes and other objects.

Eikonal phase: The Eikonal phase, at order $\mathcal{O}(G)$, is given simply by the Fourier transform of the tree-level amplitude in Eq. (10) to the impact parameter space:

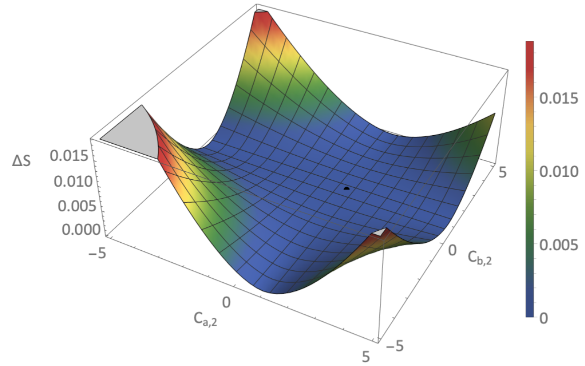
$$\chi(b) = \frac{1}{4|\vec{p}|E} \int \frac{d^2\vec{q}}{(2\pi)^2} e^{i\vec{q}\cdot\vec{b}} \bar{M}_{\text{tree}}(q^2). \tag{14}$$

Since $q^2 \to 0$ in the Eikonal limit, we have $\vec{q} \cdot \vec{p} = q^2/2 \to 0$. This orthogonality between $\vec{q}$ and $\vec{p}$ defines the impact parameter space, which is the plane perpendicular to the incoming momentum, i.e., $\vec{b} = (b_x, b_y, 0)$. Note that, in this limit, we can simply replace all $\mathbb{S}$ in Eq. (11) by $\vec{\Sigma}$, which is the rest frame spin operator. The $\mathcal{S}$ matrix in the Eikonal approximation is then the exponential of the phase

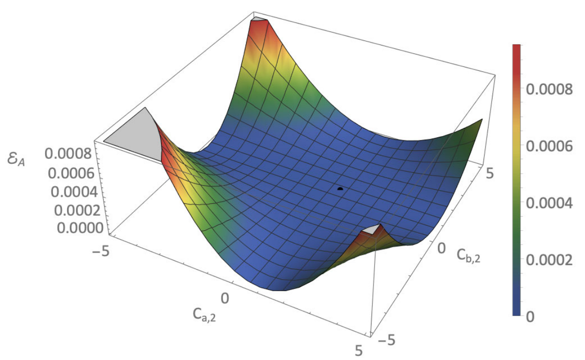
$$\mathcal{S}_{\text{Eikonal}} = e^{i\chi(b)}. \tag{15}$$

This allows us to write the out state in the Eikonal approximation, replacing the matrix element of $U_a U_b \mathcal{S}$ by the ones of $\mathcal{S}_{\text{Eikonal}}$ in Eq. (2):

$$|\text{out}\rangle = \mathcal{S}_{\text{Eikonal}} |\text{in}\rangle. \tag{16}$$



(I) Relative Von Neumann entropy.



(II) Entanglement power.

FIG. 2. Change in spin-entanglement for spin-1 massive particle. (I) Relative entanglement entropy ΔS and (II) the entanglement power $\mathcal{E}_a$ for massive spin-1 particles. The initial state is set to $|\mathrm{in}\rangle = |\uparrow\uparrow\rangle$, and the kinematic parameters are given by $|\vec{p}_a| = |\vec{p}_b| = |\vec{p}|$, $m_a = m_b = m$, $\vec{b} = (b,0,0)$, $Gm^2 = 10^{-4}$, $|\vec{p}|b = 1000$, $|\vec{p}|/m = 100$. The minimum, represented by the black point, corresponds to the Wilson coefficient value $(C_{a,2}, C_{b,2}) = (1,1)$, $\Delta S \approx 1.54 \times 10^{-9}$, and $\mathcal{E}_a \approx 1.10 \times 10^{-10}$.

Entanglement entropy of binary systems.—We now have all the ingredients necessary to compute the entanglement entropy and the entanglement power for the out state in the Eikonal approximation. We first compute the entanglement entropy for spin-1 particles, which corresponds to keeping operators of at most degree two in spin-vectors for each particle in the Eikonal phase. Starting with a pure state $|\mathrm{in}\rangle = |\uparrow\uparrow\rangle$, the entanglement entropy for the resulting out state yields directly to the relative entropy ΔS in Eq. (1). The result is plotted in Fig. 2 against the Wilson coefficients pair $(C_{a,2}, C_{b,2})$. Remarkably, the minimum is exactly at the Kerr black hole value $C_{a,2}, = C_{b,2} = 1$ and deviating from this point raises the entropy of the system. This is unchanged for a different choice of in states, which is illustrated by the computation of entanglement power given by Eq. (3) and shown in Fig. 2. We've also obtained a similar result with mixed in states.

In order to show that this is indeed a robust result, we also consider higher spins. Using the same setup, we calculate the relative Von Neumann entropy for spin-3

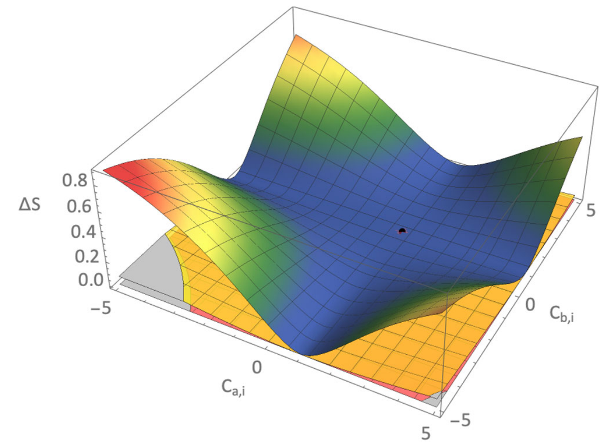


FIG. 3. Relative entanglement entropy for massive spin-3 particles. The initial state is set to $|\mathrm{in}\rangle = |\uparrow\uparrow\rangle$, and the kinematic parameters are give by $|\vec{p}_a| = |\vec{p}_b| = |\vec{p}|$, $m_a = m_b = m$, $\vec{b} = (b,0,0)$, $Gm^2 = 10^{-4}$, $|\vec{p}|b = 1000$, $|\vec{p}|/m = 100$. The Wilson coefficients $(C_{a,i\neq 2}, C_{b,j\neq 2})$ are set to 1. The minimum, represented by the black point, is at $\Delta S \approx 1.26 \times 10^{-8}$ and corresponds to the Wilson coefficient value $(C_{a,2}, C_{b,2}) = (1,1)$.

massive particles, which has a total of $5 + 5 = 10$ Wilson coefficients. In our extensive scan, we find that the black hole value, $C_{a,i} = C_{b,i} = 1$ for $i = 2, \ldots, 6$, is the unique point that gives the minimum value. As an illustrative example, we set all Wilson coefficients to 1 except the pair $(C_{a,2}, C_{b,2})$ and plot ΔS with respect to $(C_{a,2}, C_{b,2})$ in Fig. 3. The results show the minimum at (1,1), while the two orthogonal valleys represent keeping only one of the coefficients at 1. In Fig. 4, we plot $C_{a,2} = C_{b,2} = C_2$ and $C_{a,3} = C_{b,3} = C_3$, while keeping all remaining coefficients at 1. Once again, the corresponding black hole point gives near zero entanglement. For plots with respect to other choices of Wilson coecients, see Ref. [29].

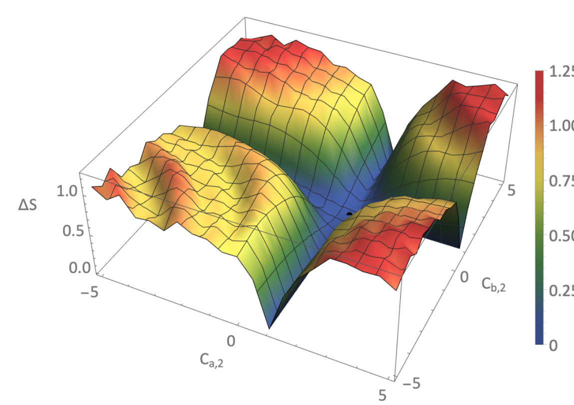


FIG. 4. Relative entanglement entropy for massive spin-3 particles. The initial state is set to $|\mathrm{in}\rangle = |\uparrow\uparrow\rangle$, and the kinematic parameters are given by $|\vec{p}_a| = |\vec{p}_b| = |\vec{p}|$, $m_a = m_b = m$, $\vec{b} = (b,0,0)$, $Gm^2 = 10^{-4}$, $|\vec{p}|b = 1000$, $|\vec{p}|/m = 100$, $C_{a,2} = C_{b,2} = C_2$, $C_{a,3} = C_{b,3} = C_3$. All others' Wilson coefficients are set to 1. The minimum is at $\Delta S \approx 1.26 \times 10^{-8}$ and corresponds to the Wilson coefficient value $(C_2, C_3) = (1,1)$.

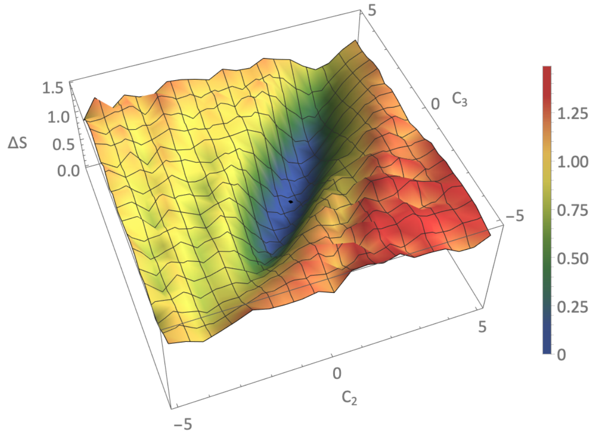


FIG. 5. Relative entanglement entropy for massive spin-2 particles. The initial state is set to $|\mathrm{in}\rangle = |\uparrow\uparrow\rangle$, and the kinematic parameters are given by $|\vec{p}_a| = |\vec{p}_b| = |\vec{p}|$, $m_a = m_b = m$, $\vec{b} = (b, 0, 0)$, $Gm^2 = 10^{-4}$, $|\vec{p}|b = 1000$, $|\vec{p}|/m = 100$. The planes correspond respectively to $(C_{a,2}, C_{b,2})$, $(C_{a,3}, C_{b,3})$, and $(C_{a,4}, C_{b,4})$, while all others' Wilson coefficients are set to 1. In any of the cases, the minimum, represented by the black point, corresponds to the Wilson coefficients set to 1 and $\Delta S \approx 5.84 \times 10^{-9}$.

While the deformation of each Wilson coefficient away from unity raises the entanglement entropy, comparatively, the effect of C_2 is dominant. This is illustrated in Fig. 5, which compares ΔS for deforming the three different pairs of Wilson coefficients in the spin-2 system. We can observe that deforming $(C_{a,2}, C_{b,2})$ has the dominant effect in generating entanglement.

Finally, we expect that including higher spins do not change the main result. The minimum of the relative entropy is always at the Kerr black hole Wilson coefficient point. Moving away from this point quickly increases the entanglement entropy. A comparison of the spin-1, spin-2, and spin-3 cases, keeping all Wilson coefficients at 1 except $C_{a,2}$, can be seen in Fig. 6.

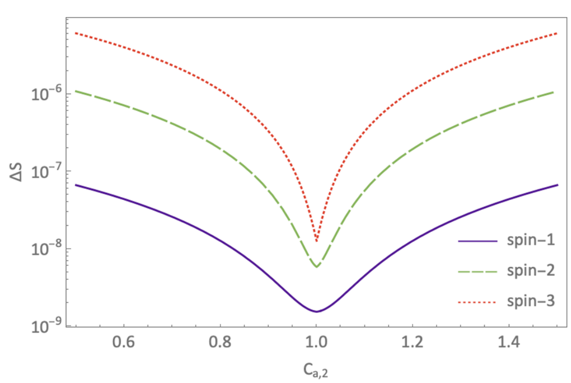


FIG. 6. Comparison between the relative entanglement entropy for spin-1, spin-2, and spin-3. All Wilson coefficients are set to 1 except $C_{a,2}$.

Conclusions and outlook.—In this Letter, we consider the entanglement entropy generated by gravitationally coupled binary systems. By considering the Hilbert space of spin states, we demonstrate that minimal coupling for massive arbitrary spin particles has the unique feature of generating nearly zero entanglement in the scattering process. Given the correspondence between minimal coupling and rotating black holes, the result suggests that this feature can also be attributed to the entanglement properties of spinning black holes. Note that this phenomenon is reminiscent of what was found in strong interactions, where entanglement suppression is associated with symmetry enhancement [10].

While the relative entropy is near zero, it is not zero, which may be an artifact of confining ourselves to leading order in Eikonal approximation. This makes clear investigation at nonlinear optics desirable. As mentioned in the introduction, there is a general correspondence between minimal coupling and black-hole-like solutions in four dimensions. This includes Reissner Nordstrom, Kerr Newman [30,31], Taub–NUT [32] and Kerr–Taub–NUT. Furthermore, gravitationally induced spin multipoles have also been studied recently in the context of fuzzball microstates [26–28]. For Kerr Newman, there are additional electromagnetic spin multipoles, while for Kerr–Taub–NUT and fuzzballs, the minimal couplings are dressed with additional complex phase factors. It will be fascinating to explore their features through the prism of spin entanglement. Finally, it will also be interesting to understand quantum corrections, in particular whether they generate anomalous gravitational multipole moments.

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