Collisions of Three-Component Vector Solitons in Bose-Einstein Condensates

Stefan Lannig,[1,*] Christian-Marcel Schmied,[1] Maximilian Prüfer,[1] Philipp Kunkel,[1] Robin Strohmaier,[1] Helmut Strobel,[1] Thomas Gasenzer,[1] Panayotis G. Kevrekidis,[2] and Markus K. Oberthaler[1]

[1]*Kirchhoff-Institut für Physik, Universität Heidelberg, Im Neuenheimer Feld 227, 69120 Heidelberg, Germany*
[2]*Department of Mathematics and Statistics, University of Massachusetts, Amherst, Massachusetts 01003-4515 USA*

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Ultracold gases provide an unprecedented level of control for the investigation of soliton dynamics and collisions. We present a scheme for deterministically preparing pairs of three-component solitons in a Bose-Einstein condensate. Our method is based on local spin rotations which simultaneously imprint suitable phase and density distributions. This enables us to observe striking collisional properties of the vector degree of freedom which naturally arises for the coherent nature of the emerging multicomponent solitons. We find that the solitonic properties in the quasi-one-dimensional system are quantitatively described by the integrable repulsive three-component Manakov model.

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Solitons, nondispersive wave packets in nonlinear systems, are realized in a broad variety of settings across nature—from optics and classical fluids to plasmas and ultracold atoms [1,2]. While an extensive effort has been invested in the understanding of single-component systems, the study of coupled multicomponent nonlinear models is far less developed, especially in settings involving more than two components. In the presence of well-defined phase relations between the constituent fields the concept of vector solitons arises and their internal degree of freedom leads to striking interaction features [3].

There are different platforms for investigating solitonic collisions, most notably nonlinear optics systems where polarization shifts have been demonstrated [4–6]. Nowadays, the unprecedented level of control available in ultracold atomic systems offers new perspectives. These systems do not only provide a variable number of internal states with long coherence times but also a wide variety of methods for manipulating and detecting the constituent fields. Single-component collisions have already been studied in great detail [7–9]. Recently, this has been extended to the experimental detection of two-component [10] and magnetic solitons [11,12], as well as three-component solitonic structures [13].

For our experiments on three-component solitons we employ a quasi-one-dimensional Bose-Einstein condensate (BEC) of ^{87}Rb trapped in a homogeneous magnetic field. We realize the different components with the magnetic sublevels $m_F = 0, \pm 1$ of the $F = 1$ hyperfine manifold. The soliton we are investigating is a coherent superposition of all m_F fields, where $m_F = 0$ features a density minimum accompanied by a phase jump. The bright components in $m_F = \pm 1$ feature density maxima at the same position. In the nonlinear physics context, this type of excitation is known as dark-bright-bright soliton [14]. The fixed phase relation between the bright components allows defining the associated polarization vector. This is a genuine feature for solitonic excitations with at least two bright components which we control and detect in our experiment.

To generate this type of nonlinear excitation, we use a spatially localized spin rotation based on the vector Stark shift [15] realized with a steerable laser beam (for details see [16]). This coherently transfers atoms from the initial $m_F = 0$ to the bright components [see Fig. 1(a)]. The Gaussian beam profile with a root mean square (rms) radius of approximately 4 μm leads to density distributions of the magnetic substates via the corresponding position-dependent Rabi coupling $\Omega(x)$ which is proportional to the modulation amplitude of the vector Stark shift and thus to the corresponding light intensity [15].

Simultaneously, a spatially dependent phase is imprinted which is close to the phase structure of the vector soliton, namely a phase step in the $m_F = 0$ field and constant phases in $m_F = \pm 1$. For this we use that a Rabi oscillation of duration τ induces field amplitudes according to $\psi_0 \propto \cos(\Omega\tau)$ in $m_F = 0$. Thus, rotation angles $\Omega\tau > \pi/2$ in the center of the light beam induce a region with flipped sign of ψ_0. In Fig. 1 we show the subsequent dynamics which is probed via Stern-Gerlach absorption images. For $\Omega\tau > \pi/2$ we find indeed that a pair of solitons is formed [Fig. 1(b), right column] while for shorter Rabi couplings the initial density distribution disperses [Fig. 1(b), left column].

We confirm the preparation of three-component vector solitons by comparing our experimental observations with an appropriate analytical model. We expect our system to be well described by a repulsive three-component Manakov model with density interactions of equal coupling strength between all components. Beyond density coupling, our multicomponent ^{87}Rb Bose gas features also spin interactions, which lead to a small spin-dependent modification

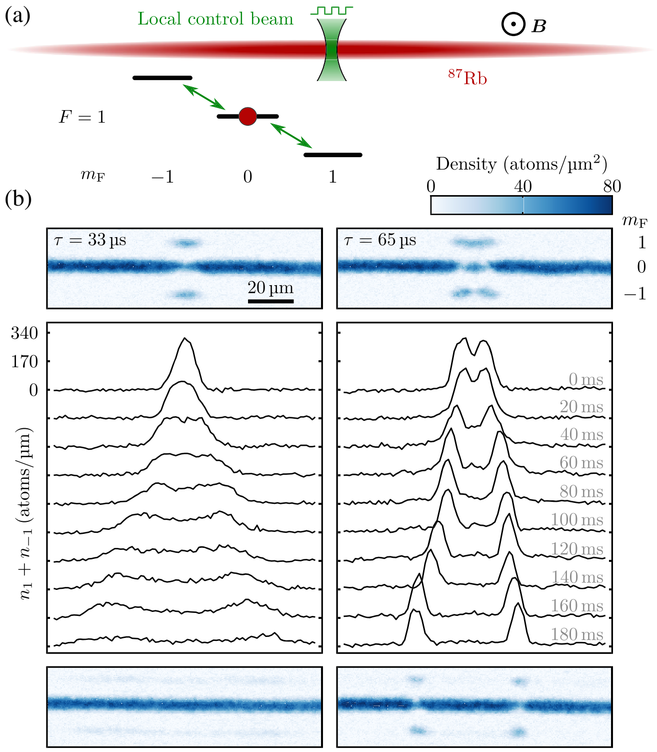

FIG. 1. Formation of three-component vector solitons. (a) An amplitude-modulated steerable laser beam (green) is used to implement local spin rotations in an elongated BEC (red) subject to a homogeneous magnetic field $\boldsymbol{B}$ along the z direction. This coherently transfers atoms from the initial state $m_F = 0$ (red disk) to $m_F = \pm 1$ (green arrows). (b) The upper panel shows absorption images after Stern-Gerlach separation revealing the density distributions of the three m_F states after local spin rotations of duration τ. For short Rabi coupling ($\tau = 33\ \mu$s) the population is transferred to $m_F = \pm 1$; and for longer pulse duration ($\tau = 65\ \mu$s) the population is coherently transferred back to $m_F = 0$ in the center of the laser beam, implying a sign change in ψ_0. The subsequent dynamics shown below (summed $m_F = \pm 1$ densities $n_{\pm 1}$) indicates the formation of a soliton pair as a result of the sign change (right column). Each soliton consists of shape-preserving bright components ($m_F = \pm 1$) and a corresponding density depletion in the $m_F = 0$ component (see lower absorption image). In contrast, without phase jump the initial density distribution disperses (left column).

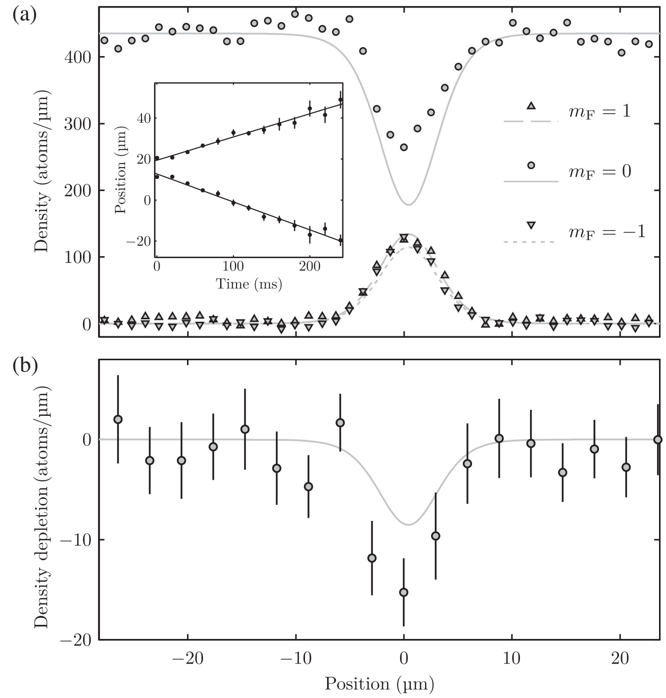

FIG. 2. Quantitative comparison with a repulsive three-component Manakov vector soliton solution. (a) Experimentally extracted density profiles (markers) of a single realization at $t = 100$ ms compared to the analytical prediction (solid and dashed lines) of Eq. (1) with independently extracted parameters from the experimental observations (see main text). The spreading of the atomic absorption signal induced by the imaging setup is taken into account by convolving the model densities with a Gaussian with rms radius of $1.2\ \mu$m [16,21]. The inset shows the experimentally extracted soliton positions (solid lines are linear fits). (b) The total density of the three-component Manakov soliton features a small depletion [solid line for parameters used in (a)] which we confirm by taking the difference between the total densities with and without solitons measured by omitting the Stern-Gerlach separation of the components and averaging over 20 realizations. All error bars mark the 1 s.d. interval of the mean.

of the density interactions (less than one percent), and to spin changing collisions redistributing the population between the components (see [16] for details). We strongly suppress the latter process by working in a regime of positive quadratic Zeeman energy, more than 20 times larger than the spin interaction energy (i.e., deep in the polar phase [19]). The size of the solitons of $\sim 6\ \mu$m is larger than the transverse extent of $\sim 4\ \mu$m of the atomic cloud; therefore, a one-dimensional model is adequate to describe our system.

For the three-component variant of the Manakov model, it is well known that dark-bright-bright solitons exist for the system subject to nonzero boundary conditions [20]. Recently, for solutions of the form

$$\psi_{\pm 1}(x) = c_{\pm 1}\eta \sin\alpha\, \mathrm{sech}[\kappa(x - x_0)],$$
$$\psi_0(x) = e^{i\varphi_S}\{i\cos\alpha + \sin\alpha\, \tanh[\kappa(x - x_0)]\}, \quad (1)$$

an inverse-scattering analysis has been developed in order to predict the change of their characteristics upon collision [22]. In Eq. (1) the indices label the different m_F states, $c_{\pm 1}$ the entries of the polarization vector $\boldsymbol{c}$ associated with the bright soliton components, κ the inverse soliton width, $x_0 = \tilde{x}_0 + vt$ the position of the soliton propagating with velocity v, and φ_S a relative phase between $m_F = 0$ and $m_F = \pm 1$. The remaining quantities for our atomic system are given by $\eta = \sqrt{1 - (\hbar^2\kappa^2/m + mv^2)/\mu}$ and $\tan\alpha = \hbar\kappa/(mv)$, where m denotes the atomic mass and μ is the chemical potential of the total background density (see [16]).

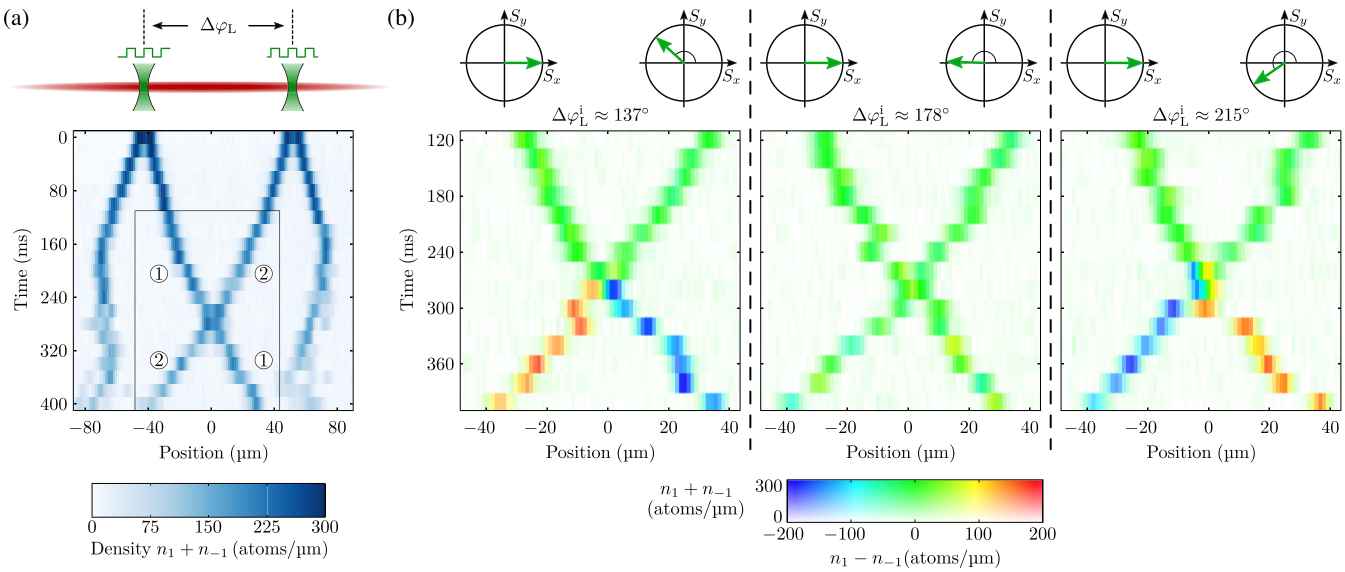

FIG. 3. Collision of three-component vector solitons. (a) We generate two pairs of solitons by applying two local spin rotations. The resulting evolution of the bright components $n_1 + n_{-1}$, averaged over 6 realizations, is shown in the lower panel. Encircled numbers label the solitons. (b) Detailed view of the collision area marked by the rectangle in (a) for three different relative Larmor phases $\Delta\varphi_L = \varphi_L^{(2)} - \varphi_L^{(1)}$ imprinted by corresponding phases of the laser beam modulation. The orientation of the pseudospin 1/2 in the x-y plane is indicated by the green arrows. The color-coded density difference $n_1 - n_{-1}$ between the two bright components of the vector soliton after collision reveals a strong dependence on the initial $\Delta\varphi_L^{\mathrm{i}}$. The saturation of the color indicates the density $n_1 + n_{-1}$. We confirm that the shape of the solitons remains unaltered and that the polarization features a $\Delta\varphi_{\mathrm{L}}$-dependent change after the collision.

We parametrize the soliton polarization

$$\boldsymbol{c} = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{1+S_z}e^{-i\varphi_L/2} \\ \sqrt{1-S_z}e^{+i\varphi_L/2} \end{pmatrix}, \tag{2}$$

motivated by the collective pseudospin 1/2 representation of the bright components, in terms of $S_z = (N_{+1} - N_{-1})/(N_{+1} + N_{-1})$, with $N_{\pm 1}$ representing the atom numbers in the corresponding bright components. The Larmor phase φ_L is given by the transversal spin $S_x + iS_y = |S_\perp|e^{i\varphi_L}$ with $S_x \propto \int(\psi_{+1}^*\psi_{-1} + \text{c.c.})dx$ and $S_y \propto \int(-i\psi_{+1}^*\psi_{-1} + \text{c.c.})dx$. In this language the coherence is given by the length $|\boldsymbol{S}| = \sqrt{S_z^2 + |S_\perp|^2}$ and is equal to 1 in the theoretical framework of the model.

For comparison with the analytical model we independently determine the polarization parameters φ_L and S_z where we estimate $N_{\pm 1}$ by summing over three times the fitted rms width. The position x_0 and the inverse width κ are extracted as the mean from independent fits to the $m_F = \pm 1$ components. The velocity v is obtained from the position assuming linear motion of the solitons [cf. inset of Fig. 2(a)], and μ from the background density n (see [16] for details). In Fig. 2(a) we compare the individual densities $n_{0,\pm 1} \propto |\psi_{0,\pm 1}|^2$ with the solution Eq. (1) and find good quantitative agreement. We attribute the remaining deviations in amplitude and width of the $m_F = 0$ profile to the filling up of the density minimum during time of flight for spatially separating the hyperfine levels and imaging.

An additional feature of the multicomponent soliton is a maximal depletion $\delta n/n = \hbar^2\kappa^2/(m\mu)$ of the total density relative to the background density n. For our parameters we expect approximately 60 atoms to be missing in the total number of particles. This is on the order of the atomic shot noise of the total atom number over the size of the soliton. To achieve this precision we image without Stern-Gerlach separation and subtract total density profiles without solitons, each averaged over 20 realizations [23]. The result in Fig. 2(b) is close to the expectation and we find a depletion of ~100 atoms which corresponds to $\delta n/n \approx 0.03$ of the background density.

We now turn to the study of collisions, a defining characteristic of solitons. For this we consecutively generate two soliton pairs by applying two separate local spin rotations where the experimental control allows modifying the soliton polarization. Here we tune the initial Larmor phase difference $\Delta\varphi_L^{\mathrm{i}} = \varphi_L^{(2)} - \varphi_L^{(1)}$ of the colliding solitons by adjusting the relative phase of the amplitude modulation of the laser beams [see Fig. 3(a)]. After approximately $t \approx 260$ ms the two central solitons collide without significantly changing their shape. However, we observe a strong variation of the outgoing soliton polarization as a function of the initial polarization difference, exemplified for three settings of $\Delta\varphi_L^{\mathrm{i}}$ shown in Fig. 3(b). While for $\Delta\varphi_L^{\mathrm{i}} \approx 180°$ the polarization is not altered, we

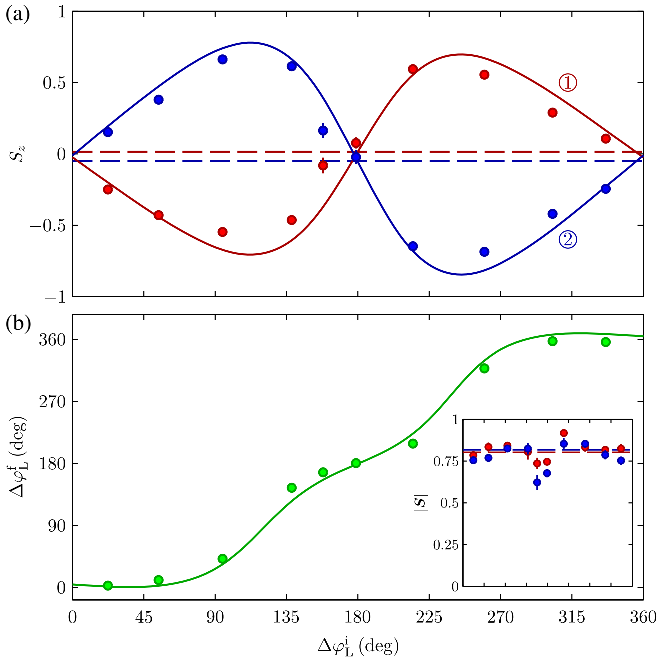

FIG. 4. Quantitative comparison of the experimental soliton polarization dynamics with the analytical solution. We compare the measured polarization after collision (circles, averaged over times $t = 320$–400 ms) with the predictions using independently determined model parameters (solid lines) for different $\Delta\varphi_L^i$ measured before collision. (a) S_z of both solitons (red and blue). As a reference the dashed lines show the experimental S_z before collision averaged over times $t = 40$–220 ms and all measured phases. We attribute the different amplitudes to the differences in initial velocities, widths, and S_z of the two solitons. (b) The measured Larmor phase difference $\Delta\varphi_L^f$ after collision matches the analytical solution. The inset shows the experimentally measured pseudospin 1/2 length before [dashed line, averaged as in (a)] and after (circles) collision, revealing the conservation of coherence. The ticks on all x axes correspond to the same values indicated at the bottom of (b) and all error bars indicate 1 s.d. interval of the mean.

observe a significant change of S_z for other angles. For the cases shown the collision redistributes the populations in $m_F = \pm 1$ such that the outgoing solitons mainly contain one dominant bright component (population ratio of $\sim 0.8/0.2$).

For further characterization we apply a detection scheme for simultaneous readout of orthogonal transversal projections of the pseudospin degree of freedom [24] with which we access initial and final $\Delta\varphi_L$ as well as the transversal spin length $|S_\perp|$ (see [16] for the readout sequence). This allows the quantitative comparison of experimental data and theoretical predictions. In Fig. 4 we show the experimentally extracted polarization parameters S_z and relative phase $\Delta\varphi_L^f$ after collision as a function of the initial phase difference $\Delta\varphi_L^i$. For the repulsive three-component Manakov model the postcollision polarizations of soliton 1 and 2 are given by

$$\begin{aligned} \boldsymbol{c}_1^{\mathrm{f}} &= \chi(\boldsymbol{c}_1^{\mathrm{i}} + A_{12}\langle \boldsymbol{c}_2^{\mathrm{i}} | \boldsymbol{c}_1^{\mathrm{i}} \rangle \boldsymbol{c}_2^{\mathrm{i}}), \\ \boldsymbol{c}_2^{\mathrm{f}} &= \chi(\boldsymbol{c}_2^{\mathrm{i}} + A_{21}^{*}\langle \boldsymbol{c}_1^{\mathrm{i}} | \boldsymbol{c}_2^{\mathrm{i}} \rangle \boldsymbol{c}_1^{\mathrm{i}}), \end{aligned} \quad (3)$$

where $\langle \cdot | \cdot \rangle$ denotes the complex inner product, with the polarization vectors $\boldsymbol{c}^{\mathrm{i}}$ and $\boldsymbol{c}^{\mathrm{f}}$ before and after collision, respectively. The normalization factor χ and the coupling parameters A_{jk} depend on the velocities and widths of the colliding solitons as well as on the background density (for the theoretical analysis see [22], for the connection to our experiment see [16]). The outgoing polarization can be seen as a superposition of the transmitted part and an admixture of the reflected part weighted with the overlap between the two polarization vectors of the incoming solitons. We calculate the parameters of Eq. (3) from the experimental quantities and find good quantitative agreement between experiment and analytical predictions (see Fig. 4). The initial experimental asymmetries in the soliton polarization, width, and velocity cause slight amplitude differences in the postcollisional S_z which are also captured by the theory. Our measurements reveal that the collisions conserve the pseudospin length $|\boldsymbol{S}|$ [see inset Fig. 4(b)] which confirms the coherence-preserving nature of the collisions.

Summarizing, we present a novel method for controlled generation of coherent multi-component solitons and verify the key features of three-component vector-solitonic propagation and interactions experimentally. We find quantitative agreement with analytical predictions of collision-induced polarization shifts in the repulsive three-component Manakov model. The scalability of the technique provides the means for direct generation of solitonic lattices or random soliton gases [25–28] with a spin degree of freedom. Combined with the observed long lifetime of the solitons the regime of multiple soliton collisions can be investigated. Notably, the collisional properties can also be described by an attractive two-component Manakov model [22] for the bright components. Thus our work paves the way for the study of bright-soliton collisions in the robust environment of repulsive BECs. Combined with the decoupled spin degree of freedom this leads to long coherence times—a new route to quantum solitons entangled in the spin degree of freedom after solitonic collision.

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*vectorsolitons@matterwave.de

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