Enhancing Spin-Phonon and Spin-Spin Interactions Using Linear Resources in a Hybrid Quantum System

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Hybrid spin-mechanical setups offer a versatile platform for quantum science and technology, but improving the spin-phonon as well as the spin-spin couplings of such systems remains a crucial challenge. Here, we propose and analyze an experimentally feasible and simple method for exponentially enhancing the spin-phonon and the phonon-mediated spin-spin interactions in a hybrid spin-mechanical setup, using only linear resources. Through modulating the spring constant of the mechanical cantilever with a time-dependent pump, we can acquire a tunable and nonlinear (two-phonon) drive to the mechanical mode, thus amplifying the mechanical zero-point fluctuations and directly enhancing the spin-phonon coupling. This method allows the spin-mechanical system to be driven from the weak-coupling regime to the strong-coupling regime, and even the ultrastrong coupling regime. In the dispersive regime, this method gives rise to a large enhancement of the phonon-mediated spin-spin interactions between distant solid-state spins, typically two orders of magnitude larger than that without modulation. As an example, we show that the proposed scheme can apply to generating entangled states of multiple spins with high fidelities even in the presence of large dissipations.

Hybrid quantum systems combining completely different physical systems can realize new functionalities that the individual components can never offer [1,2]. The strong coupling regime of interactions between these different subsystems, where coherent interactions dominate dissipative processes, is at the heart of implementing more complex tasks such as quantum information processing. However, couplings between different physical systems are often extremely weak, and strong coupling has been actively pursued since the birth of hybrid quantum systems.

Recently, interfacing solid-state spins with quantum nanomechanical elements has attracted great interest [3–21]. This hybrid spin-mechanical system takes advantages of the long coherence time of solid-state spins [22–47] and the enormous Q factors of nanomechanical oscillators [48], and has wide applications ranging from quantum information processing to quantum sensing [49]. To construct a spin-mechanical setup, solid-state spin qubits like nitrogen-vacancy (NV) centers in diamond can couple to nanomechanical oscillators either through mechanical strain [3–10] or via external magnetic field gradients [11–16]. However, none of these existing systems have reached the strong coupling regime thus far and novel approaches are needed to improve the spin-phonon and the spin-spin interactions such that they can enter the strong coupling regime.

In this Letter, we introduce an experimentally feasible and simple approach that can exponentially enhance the spin-phonon, and the phonon-mediated spin-spin couplings in a spin-mechanical system using only linear resources. Through modulating the spring constant of the cantilever in time, we can acquire a tunable and two-phonon drive to the mechanical mode [50], thus amplifying the mechanical zero-point fluctuations [51–56]. This amplification directly enhances the spin-phonon magnetic or strain coupling but without the need to use additional nonlinear resources [57–60]. Thus, this proposal could implement nonlinear processes with only linear resources, and significantly simplifies the experimental realization. We show that the spin-mechanical system can be driven from the weak-coupling regime to the strong-coupling regime, and even the ultrastrong-coupling regime. When considering multiple solid-state spins coupled to the same cantilever in the dispersive regime [61,62], this method gives rise to a large enhancement of the spin-spin interactions between different spins, typically two orders of magnitude stronger than that without spring constant modulation. As an intriguing application, we show how this approach allows us to

generate spin squeezed states with high qualities even in the presence of large dissipations. The proposed method is general, and can apply to other defect centers or solid-state systems coupled to a quantum nanomechanical element. Related approaches using bosonic parametric driving for spin squeezing have been considered in the context of trapped ions [63] and cavity QED [64]. This work differs fundamentally from these proposals with a markedly different kind of hybrid spin-mechanical system.

The setup.—We consider the spin-mechanical setup, as illustrated in Fig. 1(a), where a single NV center is magnetically coupled to the mechanical motion of a cantilever with dimensions (l, w, t) via a sharp magnet tip attached to its end. By applying a periodic drive to modulate the spring constant of the cantilever [50], the zero-point fluctuations of the mechanical motion can be amplified. This effect can be realized experimentally by positioning an electrode near the lower surface of the cantilever and applying a tunable and time-varying voltage to this electrode [50]. The gradient of the electrostatic force from the electrode has the effect of modifying the spring constant [65].

For single NV centers, the ground-state energy level structure is shown in Fig. 1(b), with the ground triplet states $|m_s = 0, \pm 1\rangle$, and the zero-field splitting $D = 2\pi \times 2.87$ GHz between the degenerate sublevels $|m_s = \pm 1\rangle$ and $|m_s = 0\rangle$. We apply a homogeneous static magnetic field B_{static} to remove the degenerate states $|m_s = \pm 1\rangle$ with the Zeeman splitting $\delta = 2g_e\mu_B B_{\text{static}}$, where $g_e \simeq 2$ and $\mu_B = 14$ MHz/mT are the NV's Landé factor and Bohr magneton, respectively. We further apply dichromatic microwave classical fields $B_x^{\pm}(t) = B_0^{\pm}\cos(\omega_{\pm}t + \phi_{\pm})$ polarized in the x direction to drive the transitions between

the states $|0\rangle$ and $|\pm 1\rangle$. In the rotating frame with the microwave frequencies $\omega_{\pm}$, we obtain the Hamiltonian $\hat{H}_{\text{NV}} = \sum_{j=\pm 1} -\Delta_j |j\rangle\langle j| + (\Omega_j/2)(|0\rangle\langle j| + |j\rangle\langle 0|)$, where $\Delta_{\pm} \equiv |D - \omega_{\pm} \pm \delta/2|$ and $\Omega_{\pm} \equiv g_e\mu_B B_0^{\pm}/\sqrt{2}$. In the following, we restrict the discussion to symmetric conditions: $\Delta_{\pm} = \Delta$ and $\Omega_{\pm} = \Omega$.

The Hamiltonian for the nanomechanical resonator with a modulated spring is $\hat{H}_{\text{mec}} = \hat{p}_z^2/2M + \frac{1}{2}k(t)\hat{z}^2$, where $\hat{p}_z$ and $\hat{z}$ are the cantilever's momentum and displacement operators, with effective mass M and fundamental frequency ω_m. The spring constant of the cantilever is modified (pumped) at a frequency $2\omega_p$ by the electric field from the capacitor plate, $k(t) = k_0 + k_r(t)$, where $k_0 = M\omega_m^2$ is the fundamental spring constant, and the time-dependent correction item $k_r(t) = \partial F_e/\partial z = \Delta k \cos(2\omega_p t)$ [65]. Here, $F_e = \partial(C_r V^2)/(2\partial z)$ is the tunable electrostatic force exerted on the cantilever by the electrode [50], with C_r the electrode-cantilever capacitance, and $V(t)$ the time-dependent voltage, which is assumed to have the form $V(t) = V_0 + V_p \cos 2\omega_p t$. Then, we can obtain $\Delta k = (\partial^2 C_r/\partial z^2)V_0 V_p$. Expressing the momentum operator $\hat{p}_z$ and the displacement operator $\hat{z}$ with the oscillator operator $\hat{a}$ of the fundamental oscillating mode and the zero field fluctuation $z_{\text{zpf}} = \sqrt{\hbar/2M\omega_m}$, i.e., $\hat{p}_z = -i(M\hbar\omega_m/2)^{1/2}(\hat{a} - \hat{a}^{\dagger})$ and $\hat{z} = z_{\text{zpf}}(\hat{a}^{\dagger} + \hat{a})$, we obtain ($\hbar = 1$) [65]

$$\hat{H}_{\text{mec}} = \omega_m \hat{a}^{\dagger}\hat{a} - \Omega_p \cos(2\omega_p t)(\hat{a}^{\dagger} + \hat{a})^2, \quad (1)$$

where $\Omega_p = -\Delta k z_{\text{zpf}}^2/2$ is the classical drive amplitude.

The Hamiltonian $\hat{H}_{\text{int}} = \mu_B g_e G_m \hat{z}\hat{S}_z$ describes the magnetic interaction between the NV spin and the cantilever's vibrating mode, with G_m the magnetic field gradient. We switch to the dressed state basis $\{|d\rangle = 1/\sqrt{2}(|+1\rangle - |-1\rangle), |g\rangle = \cos\theta|0\rangle - \sin\theta|b\rangle, |e\rangle = \cos\theta|b\rangle + \sin\theta|0\rangle\}$, with $|b\rangle = (|+1\rangle + |-1\rangle)/\sqrt{2}$, and $\tan(2\theta) = -\sqrt{2}\Omega/\Delta$, as shown in Fig. 1(c). We assume that the transition frequency between the dressed states $|g\rangle$ and $|d\rangle$ becomes comparable with the oscillator frequency, i.e., $\omega_{dg} \sim \omega_m$. The total Hamiltonian for this hybrid system under the rotating-wave approximation by dropping the high frequency oscillation and the constant items can be simplified as [65]

$$\hat{H}_{\text{Total}} \simeq \delta_m \hat{a}^{\dagger}\hat{a} + \frac{\delta_{dg}}{2}\hat{\sigma}_z - \frac{\Omega_p}{2}(\hat{a}^{\dagger 2} + \hat{a}^2)$$
$$+ \lambda(\hat{a}^{\dagger}\hat{\sigma}_- + \hat{a}\hat{\sigma}_+), \quad (2)$$

where the coefficients are $\delta_m = \omega_m - \omega_p$, $\delta_{dg} = \omega_{dg} - \omega_p$, $\lambda = -\mu_B g_e G_m z_{\text{zpf}}\sin\theta$, $\hat{\sigma}_z \equiv (|d\rangle\langle d| - |g\rangle\langle g|)$, $\hat{\sigma}_+ \equiv |d\rangle\langle g|$, and $\hat{\sigma}_- \equiv |g\rangle\langle d|$ [11]. Note that the above model Hamiltonian can also be realized for the case where NV spins are coupled to a nanomechanical cantilever via mechanical strain [65].
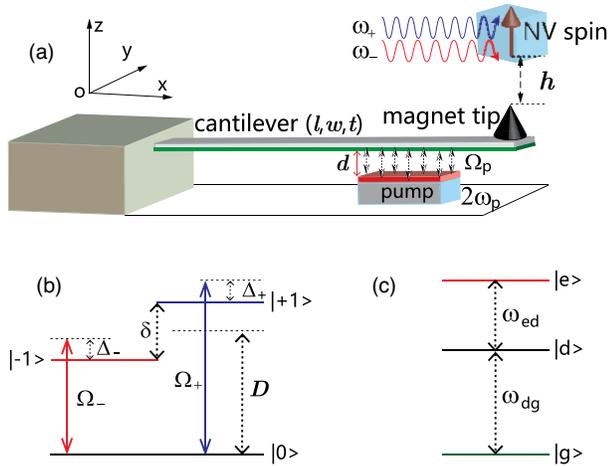
FIG. 1. (a) Schematic of a single NV center magnetically coupled to a silicon cantilever mechanical resonator, whose spring constant is periodically modulated. The NV center is set just on the top of the corresponding magnet tip with a distance $h \sim 25$ nm. (b) Level diagram of the driven NV center electronic ground states $|m_s = 0, \pm 1\rangle$. (c) Level diagram of the dressed spin basis states.

Enhancing the spin-phonon interaction.—Considering the Hamiltonian (2), we can diagonalize the mechanical part of $\hat{H}_{\text{Total}}$ by the unitary transformation $\hat{U}_s(r) = \exp[r(\hat{a}^2 - \hat{a}^{\dagger 2})/2]$, where the squeezing parameter r is defined via the relation $\tanh 2r = \Omega_p/\delta_m$. Then, we can obtain the Rabi Hamiltonian in this squeezed frame [65]

$$\hat{H}_{\text{Rabi}}^S = \Delta_m \hat{a}_s^\dagger \hat{a}_s + \frac{\delta_{dg}}{2}\hat{\sigma}_z + \lambda_{\text{eff}}(\hat{a}^\dagger + \hat{a})(\hat{\sigma}_+ + \hat{\sigma}_-). \quad (3)$$

Here, $\Delta_m = \delta_m/\cosh 2r$. We have neglected the undesired correction to the ideal Rabi Hamiltonian in the large amplification regime. This term (with coefficient $\lambda e^{-r}/2$) is explicitly suppressed when we increase the squeeze parameter r, and is negligible in the large amplification regime $1/e^r \sim 0$. More importantly, we can obtain the exponentially enhanced spin-phonon coupling strength $\lambda_{\text{eff}} = \lambda e^r/2$, which can be orders of magnitude larger than the original coupling strength as shown in Fig. 2(a), and comparable with Δ_m and δ_{dg}, or even stronger than both of them.

To quantify the enhancement of the spin-phonon coupling [76], we exploit the cooperativity $C = \lambda^2/\Gamma_m\gamma_{\text{NV}}$. Here, Γ_m and γ_{NV} correspond to the effective mechanical dissipation rate and the dephasing rate of the spin, respectively. To circumvent the detrimental effect of amplified mechanical noises, a possible strategy is to use the dissipative squeezing approach [53,77,78], in which an additional optical or microwave mode is added to the system, and is used as an engineered reservoir to keep the
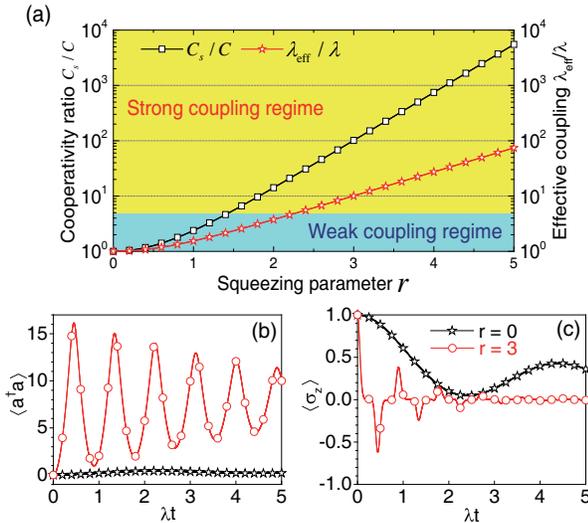

FIG. 2. (a) Cooperativity enhancement C_S/C and the spin-phonon coupling enhancement $\lambda_{\text{eff}}/\lambda$ versus the squeezing parameter r, with $\delta_m = 10\lambda$. (b),(c) Quantum dynamics of the mechanical resonator and NV spin with the two-phonon drive ($r = 3$) or without it ($r = 0$), where the parameters are $\delta_m = \lambda$, $\delta_{dg} = 0$, $\Gamma_m^S = \gamma_{\text{NV}} = 0.1\lambda$. The mechanical mode and NV center are initially prepared in the vacuum state $|0\rangle_m$ and dressed state $|d\rangle$, respectively.

Bogoliubov mode in its ground state [65]. This steady-state technique has already been implemented experimentally [53,77,78]. In this case, the squeezed phonon mode equivalently interacts with the thermal vacuum reservoir, and we can obtain the master equation in the squeezed frame [65] $\dot{\hat{\rho}} = i[\hat{\rho}, \hat{H}_{\text{Rabi}}^S] + \Gamma_m^S \mathcal{D}[\hat{a}]\hat{\rho} + \gamma_{\text{NV}}\mathcal{D}[\hat{\sigma}_z]\hat{\rho}$, where Γ_m^S is the engineered effective dissipation rate resulting from the coupling of the mechanical mode to the auxiliary bath. Therefore, we can also define the effective cooperativity $C_S = \lambda_{\text{eff}}^2/\Gamma_m^S\gamma_{\text{NV}}$.

In Fig. 2(a) we plot the cooperativity enhancement $C_S/C \sim e^{2r}/4$, as well as the spin-phonon coupling enhancement $\lambda_{\text{eff}}/\lambda$, versus the squeezing parameter r. We find that increasing the parameter r enables an exponential enhancement in the spin-phonon coupling, thus directly giving rise to the cooperativity enhancement. Figures 2(b) and 2(c) show the quantum dynamics of the spin-mechanical system for the cases when the spring constant is modulated or not. As the spring constant is modulated, the system can be pumped and driven from the weak-coupling regime to the strong-coupling, or even the ultrastrong-coupling regime.

Enhancing the phonon-mediated spin-spin interaction.—We now consider multiple NV spins coupled to the cantilever through either magnetic or strain coupling. When the spring constant of the cantilever is modulated, we can obtain the following Hamiltonian describing the coupled system [65]

$$\hat{H}_{\text{Rabi}}^N = \Delta_m \hat{a}^\dagger \hat{a} + \sum_{j=1}^N \left[\frac{\delta_{dg}^j}{2}\hat{\sigma}_z^j + \lambda_{\text{eff}}^j(\hat{a}^\dagger + \hat{a})\hat{\sigma}_x^j\right]. \quad (4)$$

In the following, we set $\delta_{dg}^j = 0$ for simplicity. We apply the unitary polaron transformation $\hat{U} = e^{-i\hat{Z}}$ to $\hat{H}_{\text{Rabi}}^N$, with $\hat{Z} = i\sum_{k=1}^N \eta_k(\hat{a}^\dagger - \hat{a})\hat{\sigma}_x^k$ and the Lamb-Dicke condition $\eta_k = \lambda_{\text{eff}}^k/\Delta_m \ll 1$. In this case, the phonons are only virtually excited and can mediate effective interactions between the otherwise decoupled solid-state spins [3,13]. Then we can obtain the effective spin-spin interactions [65] $\hat{H}_{\text{eff}} = \sum_{j,k=1}^N \Lambda^{jk}\hat{\sigma}_x^j\hat{\sigma}_x^k$, where $\Lambda^{jk} = (1 + \exp 4r)(\lambda^j\lambda^k/8\delta_m)$ is the effective coupling strength between the jth and the kth NV spins via the exchange of virtual phonons. Here the effective coupling strength for the phonon-mediated spin-spin interactions has an amplification factor e^{4r}, and can be orders of magnitude larger than that without mechanical amplification. In the case of homogeneous coupling, we have

$$\hat{H}_{\text{OAT}} = \Lambda \hat{J}_x^2, \quad (5)$$

where $\Lambda = (1 + \exp 4r)(\lambda^2/8\delta_m)$, and $\hat{J}_x = \sum_{j=1}^N \hat{\sigma}_x^j$. This Hamiltonian corresponds to the one-axis twisting interaction [79] or equivalently belongs to the well-known Lipkin-Meshkov-Glick (LMG) model [80,81].
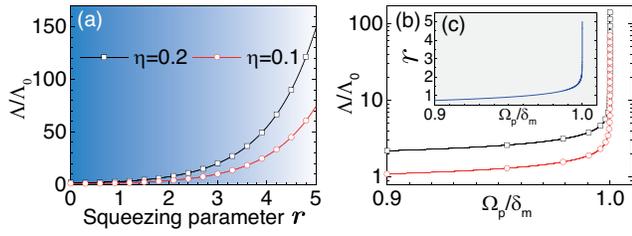
FIG. 3. The enhanced effective spin-spin coupling strength Λ varying with the squeezing parameter $r \in [0, 5]$ (a) and the pump amplitude $\Omega_p/\delta_m \in [0.9, 1)$ (b), with different constraint $\eta = 0.2$ and $\eta = 0.1$. (c) The squeezing parameter r versus Ω_p/δ_m. Here we assume that the effective spin-spin coupling strength without the two-phonon drive ($r = 0$) is $\Lambda_0 \simeq 0.1\lambda$.

FIG. 4. (a) Metrological spin squeezing parameter ξ_R^2 and spin squeezing parameter ξ_S^2, and (b) metrological gain $(\Delta\theta_{SQL}/\Delta\theta)^2$ versus time under different values for the mechanical amplification parameter r, with the initial state of the NV spins $|g \cdots g\rangle$ and the coefficients $\delta_{dg} = 0$, $\Lambda_0 \simeq 0.1\lambda$, $\gamma_{NV} = 0.001\lambda$, and $N = 6$.

Figure 3 shows the ratio of the enhanced spin-spin coupling strength Λ to the bare coupling Λ_0 as a function of the parameter r as well as the pump amplitude Ω_p/δ_m. Increasing the mechanical parametric drive gives rise to a large enhancement of the phonon-mediated spin-spin interaction, typically two orders of magnitude larger than the bare coupling. Note that since the phonon modes have been adiabatically eliminated, this amplified spin-spin coupling does not rely on the specific frame of phonons. This large, controllable phonon-mediated interaction between NV spins is at the heart of realizing many quantum technologies such as quantum computation and simulation.

Applications.—We now consider generating entangled states with this setup in the presence of dissipations. Here, we focus on entangling multiple separated NV spins through exchanging virtual phonons [65]. The one-axis twisting Hamiltonian (5) can be used to produce spin squeezed states which generally exhibit many-body entanglement. Taking into account the effect of spin dephasing, the system is described by the following master equation $\dot{\hat{\rho}} = i[\hat{\rho}, \hat{H}_{OAT}] + \sum_{j=1}^{N} \gamma_{NV} \mathcal{D}[\hat{\sigma}_z^j]\hat{\rho}$. Here, we investigate the metrological spin squeezing parameter ξ_R^2, the spin squeezing parameter ξ_S^2 [82], and the metrological gain (the gain of phase sensitivity relative to the standard quantum limit) $(\Delta\theta_{SQL}/\Delta\theta)^2$ [83]. When $\xi_{S/R}^2 < 1$, the states can be shown to be entangled, and have direct implications for spin ensemble-based metrology applications [$(\Delta\theta_{SQL}/\Delta\theta)^2 > 1$] [83].

Figures 4(a) and 4(b) show the time evolution of the spin squeezing parameter $\xi_{S/R}^2$ and metrological gain under different r. For a fixed interaction time and in the presence of spin dephasing, the spin squeezing parameter $\xi_{S/R}^2$ and metrological gain can be improved significantly by increasing r. Without mechanical amplification, the spin-squeezed state is seriously spoiled by the detrimental decoherence. However, when modulating the spring constant of the mechanical cantilever and increasing the pump amplitude Ω_p to a critical value, the quality of the produced state and the speed for generating it can be greatly improved.
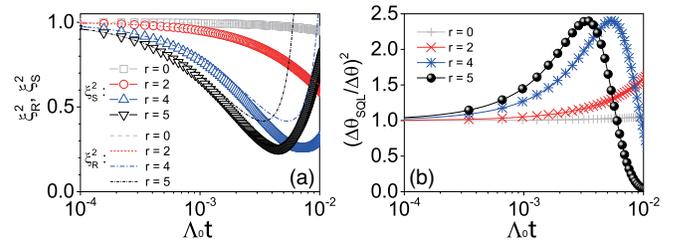
Experimental feasibility.—To examine the feasibility of this proposal for experiments, we consider a silicon cantilever with dimensions $(l = 6, w = 0.1, t = 0.05)$ μm. The fundamental frequency and the zero-field fluctuation can be expressed as $\omega_m \sim 3.516 \times (t/l^2)\sqrt{E/12\varrho} \sim 2\pi \times$ 11 MHz (with its quality factor Q about 10^5–10^6) and $z_{zpf} = \sqrt{\hbar/2M\omega_m} \sim 2.14 \times 10^{-13}$ m, with Young's modulus $E \sim 1.3 \times 10^{11}$ Pa, the mass density $\varrho \sim 2.33 \times 10^3$ kg/m^3, and effective mass $M \sim \varrho lwt/4$. Assuming an environmental temperature 10 mK in a dilution refrigerator, the thermal phonon number is about $n_{th} \sim 100$. Thus the effective mechanical dissipation rate is $\Gamma_m = n_{th}\omega_m/Q \sim 2\pi \times 1$ kHz. It is worth noting that the strain coupling scheme is particularly suitable for the case of multiple NV centers simultaneously coupled to the same cantilever. For the case of magnetic coupling, we assume that the magnetic tip has a transverse width of 50 nm, longitudinal height of 100 nm, and a radius of curvature of the tip $\sim$20 nm. An array of NV centers are placed homogeneously and sparsely in the vicinity of the upper surface of the diamond sample, just under the magnet tips one by one with the same distance $h \sim 25$ nm. Note that individual, optically resolvable NV centers can be implanted determinately at a single spot 5–10 nm below the surface of the diamond sample by targeted ion implantation [17,18], in direct analogy to the excellent control over the locations and distances between the ions in trapped ions.

In order to ensure that the magnetic dipole interactions between adjacent centers can be ignored, we assume that the distance between the adjacent NV centers (or the adjacent magnetic tips) is about 50 nm. Furthermore, the distance between the adjacent magnetic tips and NV centers is also about 50 nm. Therefore, for each NV spin, the influence caused by the adjacent magnetic tips can be ignored. The first-order gradient magnetic field caused by the sharp magnetic tip is about $G \sim 1.7 \times 10^7$ T/m. We can obtain the magnetic coupling strength between the cantilever and the NV spin as $\lambda/2\pi \sim 100$ kHz. We expect the variations in the size and spacing of the nanomagnets and NV centers give rise to a degree of disorder in the

system [62]. The disorder makes the coupling λ cannot be the same for all of the NV centers. However, as analyzed [62,65], when the disorder factor is less than 5%, its effective influence on the system can be neglected.

We assume that the pump frequency and the amplitude are respectively $\omega_p/2\pi \sim 10$ MHz and $\Omega_p/2\pi \sim 1$ MHz [84–90]. In this protocol, the squeezing parameter satisfies $r \in [0, 5]$, and then we can obtain the effective spin-phonon coupling $\lambda_{\text{eff}} \sim 100\lambda$ and the effective spin-spin coupling $\Lambda \sim 100\Lambda_0 \sim 10\lambda$. On the other hand, the single NV spin decoherence in diamond is mainly caused by the coupling of the surrounding electron or nuclear spins, such as the electron spins P1 centers, the nuclear spins ^{14}N spins and ^{13}C spins. With the development of the dynamical decoupling techniques [91–96], the dephasing time for a single NV center in diamond is about $T_2 \sim 1/\gamma_{\text{NV}} \sim 1$ ms. Based on the above parameters, we have the magnified cooperativity $C_S > 10^6$ with this spin-mechanical hybrid system, much larger than that (about $C_S > 10^2$) achieved in a cavity QED or circuit QED system [59,60].

Another issue that should be considered is the noise suppression for this system. In the presence of the mechanical amplification, the noise coming from the mechanical bath is also amplified. As discussed above, to circumvent such undesired noises, a possible strategy is to use the dissipative squeezing approach. In order to generate the desired squeezed-vacuum reservoir, the mechanical mode should be prepared in the squeezed state with the squeezing parameter $r \sim 1.5$ through the dissipative squeezing method. Note that recent experiments have already demonstrated the generation of squeezed phonon states with the squeezing parameter $r \sim 1.45$ by dissipative squeezing [97], which corresponds to a 12 dB reduction below the standard quantum limit.

Conclusion.—In this work, we propose an experimentally feasible and simple scheme for exponentially enhancing the spin-phonon and the spin-spin interactions in a spin-mechanical system with only linear resources. We show that, by modulating the spring constant of the mechanical cantilever with a time-dependent pump, the mechanical zero-point fluctuations can be amplified, giving rise to a large enhancement of the spin-phonon and the phonon-mediated spin-spin interactions. The proposed method is general, and can apply to other defect centers or solid-state systems such as silicon-vacancy center, germanium-vacancy center, and tin-vacancy center in diamond [98–102] coupled to a quantum nanomechanical element.

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