Universal Voltage Fluctuations in Disordered Superconductors

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The Aharonov-Casher effect is the analogue of the Aharonov-Bohm effect that applies to neutral particles carrying a magnetic moment. This effect can be manifested by vortices or fluxons flowing in trajectories that encompass an electric charge. These vortices have been predicted to result in a persistent voltage that fluctuates for different sample realizations. Here, we show that disordered superconductors exhibit reproducible voltage fluctuation, which is antisymmetrical with respect to the magnetic field, as a function of various parameters such as the magnetic field amplitude, field orientations, and gate voltage. These results are interpreted as the vortex equivalent of the universal conductance fluctuations typical of mesoscopic disordered metallic systems. We analyze the data in the framework of random matrix theory and show that the fluctuation correlation functions and curvature distributions exhibit behavior that is consistent with Aharonov-Casher physics. The results demonstrate the quantum nature of the vortices in highly disordered superconductors, both above and below T_c.

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The Aharonov-Casher (AC) effect [1] is the dual effect to the Aharonov-Bohm (AB) effect [2]. While in the AB effect a charged particle quantum mechanical phase is affected by the electromagnetic vector potential, in the AC effect, a neutral particle carrying a magnetic moment quantum mechanical phase is also affected by a “charge vector potential”, even when no force is exerted on the particle.

Magnetic vortices are an example of a neutral particle with a magnetic moment, and the AC effect has been discussed for vortices in type II superconductors [3–5]. A somewhat different realization of similar ideas is for a vortex (fluxon) in a two-dimensional (2D) Josephson-junctions array [see Fig. 1(a)]. Although it carries no local magnetic flux, the phase of such a fluxon is influenced by the charge encompassed by the array [6]. Indeed, oscillatory behavior has been observed for transport measurements of such arrays [7,8].

The phenomenon of persistent currents is an illuminating demonstration of the AB physics. Essentially, when a ring encircles a magnetic flux Φ, a persistent equilibrium current proportional to $\partial E/\partial \Phi$, where E is the energy, is predicted [9–14] and measured [15–18]. In Ref. [6], van Wees realized that for the dual situation of a vortex in a 2D ring-shaped array circling a charge Q, a persistent voltage between the inner and external circumferences of the ring proportional to $\partial E/\partial Q$ should appear. This prediction has not been experimentally verified yet.

A different direction from the neat realizations described above is to seek manifestations of AC physics in disordered samples [equivalent to mesoscopic disordered metals that exhibit universal conductance fluctuations (UCF)]. Here, one avoids difficult preparation of the sample but has much less control over the details [Fig. 1(b)]. Specifically, a disordered 2D Josephson array composed of irregularly placed and shaped superconducting islands is expected to exhibit AC physics manifested by reproducible voltage fluctuations for different sample realizations.

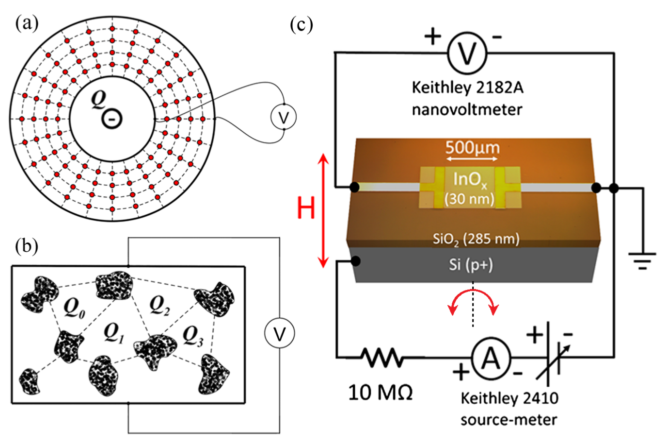

FIG. 1. Sketches of systems expected to exhibit persistent voltage. (a) 2D ring-shaped Josephson array circling a charge Q. The persistent voltage is measured between the inner and outer edges. In order for a vortex to appear in the ring, an external magnetic field perpendicular to the 2D plane must be applied. (b) Disordered 2D Josephson array composed of irregularly placed and shaped superconducting islands. Depending on the magnetic field and on the charges trapped between the islands, a reproducible, random, persistent voltage between two points on the edge of the sample is expected. (c) Schematic description of the sample geometry and measurement setup. The red arrow denotes the rotation axis in a magnetic field that is originally perpendicular to the substrate.

In this Letter, we describe measurements of spontaneous voltage in amorphous indium oxide (a-InO) films where the disorder is tuned so that the samples are close to the superconductor-insulator transition (SIT). Indeed, it has recently been shown that the physics in the vicinity of this transition is determined by the Aharonov-Bohm-Casher effect [19]. The samples, despite being morphologically uniform, have been shown to include "emergent granularity" in the form of superconducting puddles embedded in an insulating matrix [20–31]; hence, they are perfect candidates for detection of AC-effect signatures. For these films, we find reproducible voltage fluctuations as a function of the magnetic field amplitude, field orientation, and gate voltage. We analyze the results in terms of random matrix theory and show that they exhibit universal features expected for the AC effect.

The studied samples were a-InO films of thickness 30 nm that were e-beam evaporated on MEMpax(TM) borosilicate glass or gateable doped Si/SiO of thickness 0.4 mm [see Fig 1(c)]. For these films, the superconducting coherence length is 10–30 nm [32], which places our films in the quasi-2D regime. The O_2 partial pressure during evaporation (in the range $1 - 8 \times 10^{-5}$ Torr) determines the initial state of the sample, superconductor or insulator. The results presented in this Letter represent measurements performed on seven samples spanning the SIT with sheet resistance $R_{T=5K}$ ranging from 500 Ω to 10 kΩ. For more experimental details, see Supplemental Material [33].

The natural parameter to vary for obtaining different sample realizations for the AC effect is the gate voltage, which controls the charges trapped between the islands and the various loops of vortex trajectories. However, for our 2D disordered samples on conventional substrates, obtaining detailed enough structure for analysis requires unattainably large voltages. Figure 2(a) shows that varying the gate voltage results in voltage fluctuations. However, utilizing the variation of a magnetic field as a driving parameter gives rise to a much richer structure; hence, we prefer this knob for generating AC voltage fluctuations. Depending on the magnetic field, which determines the vortex arrangement in the sample, a reproducible random persistent voltage between two points on the edge of the sample can be expected due to several different origins, the most obvious being the variation of the number of fluxons. The field may also change the superconducting properties of the islands as well as the charge distribution in the normal metal areas.

Figure 2(b) depicts the voltage as a function of a perpendicular magnetic field for a number of a-InO films spanning the SIT. We note that for any realistic experimental setup, one cannot avoid some stray voltage V_0, which is present even at zero magnetic field. By subtracting this stray voltage, we obtain an antisymmetric-with-magnetic-field $[V(H) = -V(-H)]$ reproducible structure. The antisymmetric nature of the fluctuations indicates that they originate from vortex motion. The structure is a true "fingerprint" of the sample microscopics in the sense that it is very reproducible for different magnetic field scans and scan rates (see Supplemental Material [33]) of a single sample, but it changes from sample to sample. Hence, we attribute these results to the AC equivalent of the UCF originating from the AB effect in disordered metallic systems.

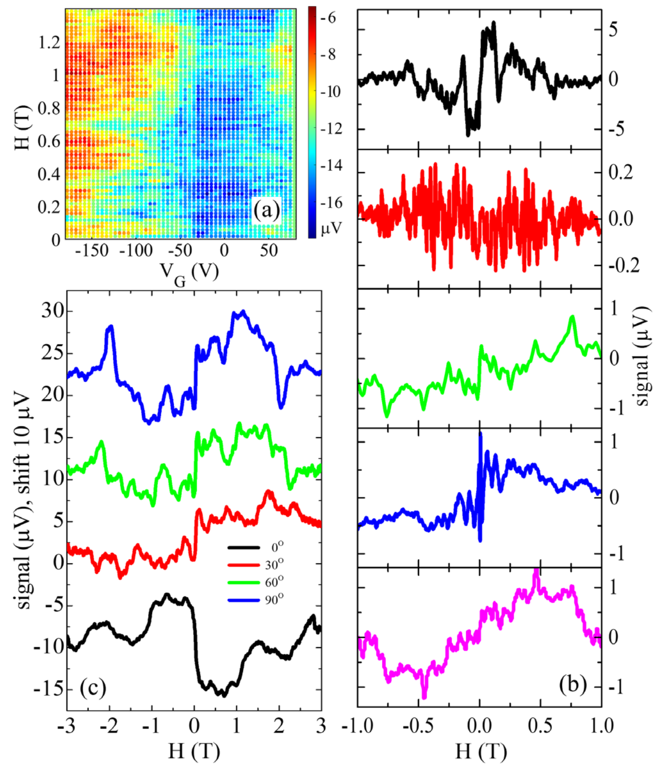

FIG. 2. (a) Spontaneous voltage as a function of gate voltage V_G and out-of-plane magnetic field H at 2 K for a sample of $T_c = 3.2$ K. Quasiperiodicity arises in response to both the magnetic field and the enclosed charge on the islands, controlled by V_G. (b) Spontaneous voltage as a function of out-of-plane magnetic field for a few a-InO films. Note that $V_g = 0$. (c) Raw $V(H)$ curves as a function of angle of the applied field relative to the film plane for one of our films. Note that $V_g = 0$. Voltage fluctuations with respect to the magnetic field are present even when the field is applied along the sample plane, indicating that processes other than the orbital effect are at work.

It should be noted that unlike the AB effect, AC physics does not depend directly on the magnetic flux penetrating the sample and thus is not expected to be washed out when the orientation of the field is varied from perpendicular to parallel to the film. Indeed, Fig. 2(c), which shows the $V(H)$ curves (before antisymmetrizing) as a function of magnetic field angle, demonstrates that, though the voltage measurements depend on the magnetic field orientation, the amplitude and the typical field scale of the fluctuations do not vary much as a function of field orientation.

Figure 3 shows the monotonous suppression of the effect with growing temperature. As with the AB effect,

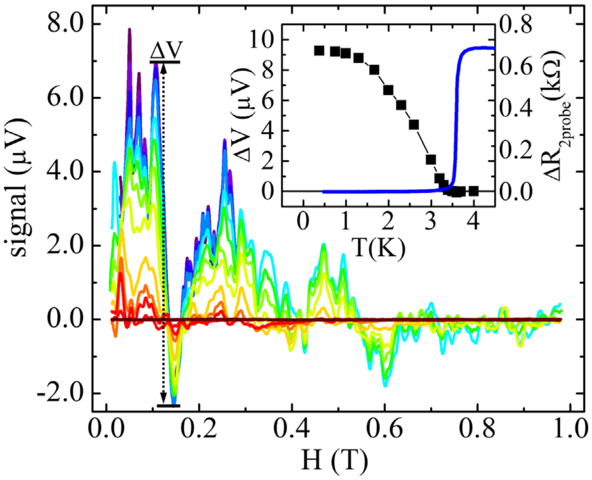

FIG. 3. Temperature dependence of the positive field regime of $V - H$ curves for an a-InO sample in the range $\mathrm{T} = 0.39$ K (purple) to 4.00 K (brown) (the full curve at base temperature is shown in the Supplemental Material [33]). Here, $V_g = 0$. Inset: Temperature dependence of the "amplitude" ΔV (black symbols) and resistance (blue line).

decoherence, as result of, e.g., temperature, should reduce the observed effect. Nevertheless, the interpretation of the effects of temperature on coherence should be done carefully since it also affects the superconductivity of the grains composing the network and thus the vortices. A similar situation also occurs for the magnetic field.

Since under these conditions one cannot model (nor control) the details of the sample, one must look for global and statistical properties of the reproducible voltage in order to verify its origin. Mesoscopic physics has taught us that even for disordered systems, certain universal properties can be teased out of the data, attesting to the physics of the system [36]. As in other cases of disordered mesoscopic systems [37–40], we assume that the universal part of the behavior of the system may be captured by a random matrix Hamiltonian. This assumption works well for the description of diverse phenomena from correlations in the conductance at different magnetic fields [41] to the fluctuations in position and height of conductance peaks in the Coulomb blockade regime [42–45].

The basic logic is similar here. We start with a rather general random matrix Hamiltonian (for more details, see Supplemental Material [33]) describing a disordered 2D Josephson array composed of irregularly placed and shaped superconducting islands. Depending on the magnetic field and on the charges trapped between the islands, a reproducible random persistent voltage between two points on the edge of the sample is expected. Keeping track of all the influences of magnetic field on the vortex motion is a rather herculean task. Nevertheless, we can exploit the theory developed for correlations [46–49] and curvature distribution [50–57] of the spectral response to external parameters, which show universal behavior of the derivative of the energies of the system with respect to an external parameter. Specifically, for the ith eigenvalue $\epsilon_i(x)$ (where x is the value of the external parameter) of the Hamiltonian, one may define a "velocity" $j_i = \partial_x \epsilon_i(x)/\delta$ (where δ is the mean level spacing, i.e., a system-dependent parameter) and a "curvature" $K_i = \partial_x \mathrm{j}_i(x)$ characterizing the response of the spectrum to the external parameter. This case corresponds to the identification of the persistent voltage with the current and its derivative with the curvature. The correlation is

$$C_i(\delta x) = \langle j_i(x) j_i(x + \delta x) \rangle, \tag{1}$$

where $\langle \ldots \rangle$ is an average over different systems and ranges of x. After a renormalization of the parameter $X = \sqrt{C_i(0)}x$, a universal behavior emerges [46–49], where

$$C_i(\delta X) = -\frac{2}{(\beta \pi^2 \delta X)^2}, \tag{2}$$

as long as δX is larger than some nonuniversal value, and $\beta = 1$ for the orthogonal ensemble (GOE) and $\beta = 2$ for the unitary one (GUE). It is important to note that since x depends on a nonuniversal parameter of the system δ, which is hard to obtain, one can determine X only up to a factor. This process leads to an unusual correlation curve since the correlation should be maximum at $\delta X = 0$, and it approaches zero from below for large δX, which means that there is a negative minimum of the correlation at some intermediate value of δX. For the curvature, a universal distribution is expected [50–53]. Defining $k = K_i / |\langle K_i \rangle|$, one obtains the distribution

$$P(k) = \frac{A_\beta}{(1 + k^2)^{(2+\beta)/2}}, \tag{3}$$

where $A_1 = 1$, $A_2 = (4/\pi)$. Corrections to this distribution, which are especially notable at low values of k as a result of nonuniversal features, have also been discussed [55,57]. Since here one normalizes the curvature by its absolute averaged value, the dependence on δ disappears. The negative minimum of the correlation, as well as the distribution that has no adjustable fit parameters, is "smoking gun" evidence for the random matrix description of the physics of the system, as well as the identification of the fluctuation in voltage with the derivative of the energy, in agreement with the Ahronov-Casher picture.

Here, we analyze the results of seven different samples; for one of them, we analyze the data for seven different angles of the magnetic field with respect to the sample plane, and for another sample, at a range of temperatures between $T = 0.42$ K and 3.26 K. After subtracting the stray voltage background V_0, we antisymmetrize the data in the following way:

$$V_{AS}(H) = \frac{V(H) - V(-H)}{2} - \bar{V}, \qquad (4)$$

where $\bar{V}$ is the average over $(V(H) - V(-H))/2$ for the whole range of measurement, so $\langle V_{AS}(H) = 0 \rangle$.

In order to substantiate these observations, we calculate the correlation

$$C(\delta H) = \frac{\langle V_{AS}(H + \delta H) V_{AS}(H) \rangle_H}{\langle V_{AS}^2(H) \rangle_H}, \qquad (5)$$

where $\langle \ldots \rangle_H$ denotes an average over the available values of the magnetic fields. The samples show a peak in the correlation, which then turns into negative values and returns to values around zero. Rescaling the correlations such that $\delta X = a \delta H$, with a a sample specific constant, results in a very similar correlation curve for all samples [see Fig. 4(a)]. It is important to note that since a depends on a microscopic scale that is unknown to us, here it is not possible to distinguish between GOE and GUE ($\beta = 1$ and $\beta = 2$). Nevertheless, the width of the correlation for all samples is similar after rescaling, and the correlation follows the expected behavior proportional to $(\delta X)^{-2}$.

In Fig. 4(b), the experimental distributions for several different ensembles are plotted. Since here we can avoid sample-specific fit parameters by dividing the curvature by its sample average over different magnetic fields, we can pool together data from several samples, angles of the external magnetic field, and temperatures. All of these distributions have several common features. They follow quite closely the GOE distribution, especially if one compares the distribution from numerically generated $1/f$ noise sequences. The deviations are most pronounced for small values of k as is expected from nonuniversal corrections to the distribution [55,57]. It is also notable that the measurement of seven different angles treated as independent samples yields similar results as for seven different samples. This result lends further support to the fluxon interpretation, which is not based on orbital effects.

Although, as seen in Fig. 3, the temperature suppresses the amplitude of the voltage fluctuation, it hardly wipes out the curvature distribution, although at temperatures above $T > 2.3$ K, the global superconductance is strongly suppressed. This case hints at the fact that there is no need for a global coherent superconductivity, and remnant local superconductivity suffices. Similarly, we obtain repeatable voltage structure for samples in the insulating phase, though these have not been analyzed here. Indeed, local superconductivity, manifested by a finite gap, was observed both above T_c [58] and in the insulating phase [59] of a-InO films. This observation may also address another puzzle. A magnetic field breaks time-reversal symmetry, and therefore, one expects the appropriate random matrix ensemble to be GUE. Here, the distribution is GOE, which can be understood if the mechanism for the voltage fluctuation is short-ranged, and therefore, time-reversal symmetry is not broken on that length scale.

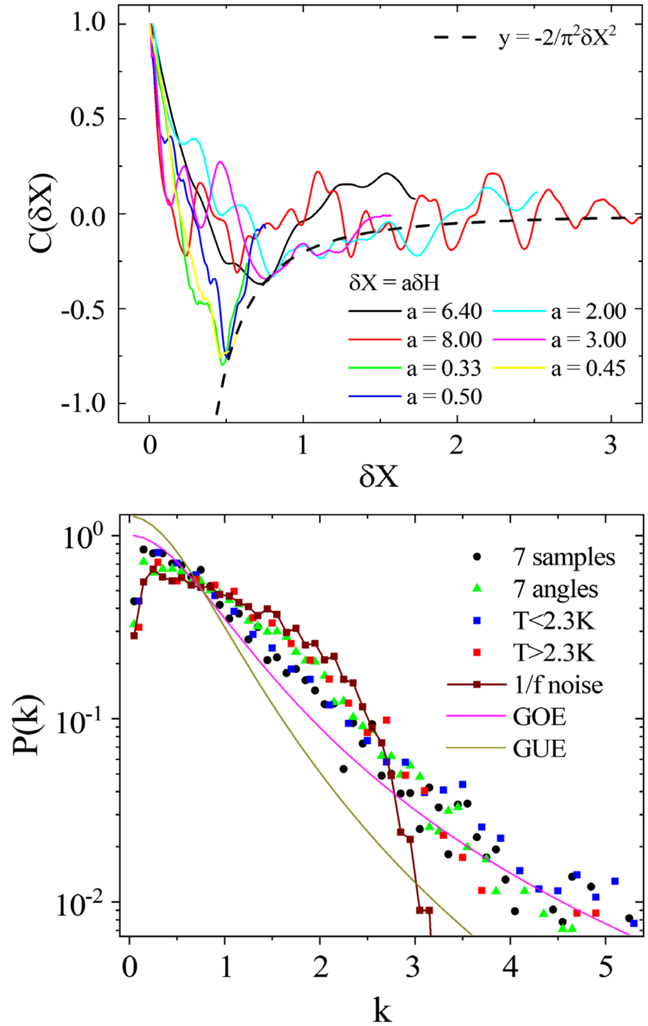

FIG. 4. Top panel: Correlation $C(\delta X)$ for seven different samples, where $\delta X = a \delta H$ and a is a sample dependent rescaling constant. The dashed curve corresponds to $-c/(\delta X)^2$, with $c = 2/\pi^2$ (GOE). As discussed in the text, plotting the GUE curve ($c = 1/\pi^2$) will result in the same figure with rescaled values of a by half. Bottom panel: Distribution $P(k)$ aggregated over seven different samples, as well as the distribution for seven different angles of the external magnetic field, and for low $T < 2.3$ K and high $T > 2.3$ K (T_c, defined at the temperature at which the resistance drops to 10% of its normal value, equals 2.3 K). For comparison, the distribution for a numerical sequence of $1/f$ noise is presented. The GOE ($\beta = 1$) and GUE ($\beta = 2$) distributions are plotted.

In summary, granular samples around the superconducting transition are promising candidates for experimental studies of additional features of random matrix theory. In our disordered superconductors, which incorporate “electronic granularity”, the correlations and curvature of the voltage fluctuations fit the expectation for a system that follows the universal features of random matrix theory much better than those of noncorrelated noise (e.g., $1/f$ noise). Nevertheless, it would be strange if nonuniversal features did not appear in realistic samples and open avenues for further study. Although some features, such as level spacings and wave-function properties, have been extensively studied experimentally in systems such as quantum dots [40], features such as level velocity and curvature, to the best of our knowledge, have not. The identification of the voltage fluctuations as an indication of the AC effect emphasizes the quantum nature of the vortices close to the SIT, below and above T_c.

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