Fractional Angular Momentum and Anyon Statistics of Impurities in Laughlin Liquids

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The elementary excitations of a fractional quantum Hall liquid are quasiparticles or quasiholes that are neither bosons nor fermions, but are so-called anyons. Here we study impurity particles immersed in a quantum Hall liquid that bind to the quasiholes via repulsive interactions with the liquid. We show that the angular momentum of an impurity is given by the multiple of a fractional "quantum" of angular momentum, and can directly be observed from the impurity density. In a system with several impurities bound to quasiholes, their total angular momentum interpolates between the values for free fermions and for free bosons. This interpolation is characterized by the fractional statistical parameter of the anyons, which is typically defined via their braiding behavior.

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Introduction.—When quasiparticles emerge from strongly correlated quantum matter, their properties can be quite different from those of the matter particles. A paradigm are bulk excitations in fractional quantum Hall (FQH) liquids: the liquid is made of interacting electrons, but its excitations appear as fractional electrons, having fractional charge, fractional angular momentum, and fractional exchange statistics [1–4]. With this, they are neither bosons nor fermions, but are so-called anyons. To date, the best experimental evidence of fractional quasiparticles is obtained by determining the fractional charge via shot noise measurements [5]. Fabry-Perot interferometry [6–8] and beam splitter experiments [9] have provided signatures of fractional statistics. Strong efforts to improve the experimental evidence of anyons concern the implementation of FQH physics in highly controllable quantum systems such as cold atoms [10,11] or photonic quantum simulators [12]. Light-matter interactions can create and trap fractional quasiparticles in atomic gases [13] or electronic systems [14], and may facilitate braiding operations [13,15–17]. It has also been suggested to observe the fractional exclusion principle spectroscopically in atomic systems [18], graphene [19], or magnetic materials [20]. Moreover, signatures of fractional statistics are carried by the total angular momentum of a fractional quantum Hall system, which can be measured by time-of-flight imaging [21]. It has also been proposed to engineer anyonic systems through appropriately defined bath interactions [22,23]. Various works propose to use impurities that bind to fractional quasiparticles [24–28], and which then exhibit features such as fractional relative angular momentum [24], non-Abelian or Abelian statistics [25,26], or quantized transport properties [27,28].

Here, we take up the idea of binding impurities to quasiholes in a FQH liquid. First, we consider a single impurity and show that its angular momentum is fractional (in units $\hbar \equiv 1$). Then, by adding more impurities, taken as noninteracting fermions, we observe how the "anyon sea" is filled. Specifically, we show that the total angular momentum of the impurities matches neither the value from a fermionic construction—that is, by filling the single-particle levels—nor the value of bosonic condensation. Instead, the total angular momentum is reproduced by a linear interpolation between fermionic and bosonic distribution, proportional to $\alpha = 1 - \nu$. Here, ν is the filling factor of the FQH liquid, and α equals the anyons' statistical parameter.

While our results are obtained by numerically solving the underlying quantum Hall model, they can also be understood from fundamental theoretical concepts, and may thus serve as an illustration thereof. In fact, the relation of anyonic physics and fractional quantization of angular momentum dates back to earliest work on the subject: in Wilczek's picture of anyonic statistics [2], the fractional behavior emerges through the attachment of matter particles to fluxes, i.e., vortex lines. In Laughlin's wave function for FQH liquids [3], the matter particles appear as fluxes seen by the quasihole, and on a mean-field level, this flux attachment redefines the quasiholes' effective vacuum, i.e., the effective magnetic field seen by the quasiholes. In Ref. [24], this reasoning has already been employed to explain the fractional relative angular momentum between two anyons, which can be measured via the correlation function of impurities bound to quasiholes. In the present Letter, we demonstrate that the properties of the anyon vacuum and fractional angular momentum can even

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be probed with a single excitation. The fractionalization of angular momentum can directly be inferred from the density of impurities bound to quasiholes, making it easily accessible in experiment.

The characterization of the single-particle levels provides crucial information that we then use to reveal the anyonic quantum statistics of the impurities. Usually, the statistical behavior of anyons is defined by considering adiabatic braiding operations [4]. In contrast, this Letter examines how the many-body angular momentum of several impurities is composed of the single-particle values, which yields an immediate fingerprint of the distribution function describing the anyon statistics. In accordance with the general expectation [29], this distribution interpolates between Bose and Fermi statistics, and, strikingly, even for very small systems, the statistical parameter obtained from the interpolation agrees almost perfectly with the predictions from an effective impurity Hamiltonian [26].

System.—The system consists of two types of particles a and b: majority particles (a type) form a FQH liquid with Landau filling fraction $\nu = 1/q$, in which impurity particles (b type) are immersed. For simplicity, we assume similar single-particle physics for both species: they have equal mass M, are trapped in the xy plane by harmonic potentials of frequencies ω_a and ω_b, and are brought into the lowest Landau level by a sufficiently strong gauge potential $\mathbf{A} = (B/2)(-y, x, 0)$. In this gauge, the Fock-Darwin functions, $\varphi_m(z) = (2\pi m! 2^m)^{-1/2} z^m e^{-|z|^2/4}$, are characterized by an angular momentum quantum number m. The corresponding single-particle energies are $\epsilon_{m,s} = \hbar m \Omega_s$, with $s \in \{a, b\}$, and $\Omega_s \equiv \sqrt{\omega_B^2 + \omega_s^2} - \omega_B$. Here, $\omega_B = eB/M$ is the cyclotron frequency, with e as the electric or synthetic charge of the particles. The coordinates $z = (x + iy)/l_B$ are given in units of the oscillator length $l_B = (\hbar/M\Omega_s)^{1/2}$, which depend on the trapping frequency. We assume $\omega_a \approx \omega_b \ll \omega_B$, such that $\Omega_a \approx \Omega_b$, and the length scale l_B takes the same value for both a and b.

To make the a particles form a FQH liquid, we consider repulsive interactions. Conveniently, interactions are expressed by Haldane pseudopotentials U_ℓ, which parametrize the strength of interactions for pairs of particles with fixed relative angular momentum ℓ [30]. By truncating the pseudopotential expansion at $\ell = q$ (i.e., by setting $U_\ell = 0$ for $\ell \geq q$), we obtain a parent Hamiltonian for the Laughlin liquid at $\nu = 1/q$. Its ground state is exactly given by the Laughlin wave function $\Psi_q \sim \prod_{i<j\in a}(z_i - z_j)^q e^{-\sum_i |z_i|^2/4}$, and it has zero interaction energy. The total angular momentum of the Laughlin ground state is $L_q = (q/2)N_a(N_a - 1)$, with N_a as the number of a particles. No eigenstates of zero interaction energy are possible for $L_a < L_q$, and, within the Hilbert space with $L_a = L_q$, the Laughlin wave function Ψ_q is nondegenerate. Laughlin liquids can be formed either by fermionic or bosonic a particles, depending on whether q is odd or even.

TABLE I. Number $\mathcal{N}_d$ of edge modes in the Laughlin liquid of degree d, and number $\mathcal{N}_{d,\mathrm{imp}}$ of zero-energy modes of degree d in the presence of an impurity.

d	0	1	2	3	4	5	6
$\mathcal{N}_d$	1	1	2	3	5	7	11
$\mathcal{N}_{d,\mathrm{imp}}$	1	2	4	7	12	19	30

When the angular momentum of the liquid is increased above L_q, i.e., for $L_a = L_q + d$ with $d > 0$, the liquid can accommodate a characteristic number $\mathcal{N}_d$ of zero-energy modes. Their wave function is of the form $\Psi^\alpha_{q,d} = f^\alpha_d(\{z_i\})\Psi_q$, where $f^\alpha_d(\{z_i\})$ is an arbitrary symmetric polynomial of degree d. The index α runs from 1 to $\mathcal{N}_d$, and $\mathcal{N}_d$ equals the number of partitions of the positive integer d. These states describe deformations at the edge, when $d \sim 1$, but for $d \sim N_a$ they may also describe quasiholes in the bulk. Specifically, the function $\Psi_{q,\mathrm{qh}} \sim \prod_i(w - z_i)\Psi_q$ describes a quasihole at position w. For $w = 0$, the factor $\prod_i(w - z_i)$ becomes a symmetric polynomial of degree $d = N_a$, and the state belongs to the manifold of zero-energy solutions at $L_a = L_q + N_a$.

The b species are taken as noninteracting fermions. To bind to quasiholes of the Laughlin liquid, we consider a sufficiently strong repulsive contact interaction between a and b particles. This interaction allows for the exchange of angular momentum between the species, but the joint angular momentum $L = L_a + L_b$ remains a conserved quantity. For the case of a single impurity, the quasihole state $\Psi_{q,\mathrm{imp}} \sim \prod_i(w - z_i)\Psi_q$ is a state of zero interaction energy, where the dynamical variable w represents the position of the impurity. The interspecies repulsion makes this state nondegenerate at $L = L_q + N_a$, and no zero-energy states exist at $L < L_q + N_a$. Degenerate zero-energy solutions exist at $L = L_q + N_a + d$ with $d > 0$, of the form $\Psi^\alpha_{q,m_1,m_2} \sim w^{m_1} f^\alpha_{m_2}(\{z_i\}) \prod_i(w - z_i)\Psi_q$, where m_1 and m_2 are positive integers with $m_1 + m_2 = d$. Thus, the number of zero-energy modes at $L = L_q + N_a + d$ is given by $\mathcal{N}_{d,\mathrm{imp}} = \sum_{m_2=0}^{d} \mathcal{N}_{m_2}$, see Table I.

Results for a single impurity.—The Laughlin state Ψ_q can be seen as an effective impurity vacuum, and the states $\Psi_{q,\mathrm{imp}}$ and Ψ^α_{q,m_1,m_2} define the ground state and excited states of a single impurity. These states have total angular momentum $L = L_q + N_a$ and $L = L_q + N_a + m_1 + m_2$, but it is not immediately clear how the angular momentum is distributed between the two species. Let L^0_b denote the average angular momentum of the impurity in its ground state, i.e., $L^0_b \equiv \langle \Psi_{q,\mathrm{imp}} | \hat{L}_b | \Psi_{q,\mathrm{imp}} \rangle$ with $\hat{L}_b$ the angular momentum operator for the b particle. Naïvely, one may expect that the angular momentum L^m_b of an impurity in its mth excited state, i.e., $L^m_b \equiv \langle \Psi^\alpha_{q,m,d-m} | \hat{L}_b | \Psi^\alpha_{q,m,d-m} \rangle$, is given by $L_b^m = L_b^0 + m$. However, as we show below, this is not the case. Instead, the angular momenta of impurity levels differ by multiples of a fractional value, suggesting the interpretation of fractional quantization.

Analytical arguments for this behavior are based on the notion that the impurity at w "sees" the majority particles at z_i as fluxes, reducing the effective gauge field for the impurity to $B^* = B - 2\pi\ell_B^2\rho_a B = B(1-\nu)$, where ρ_a is the density of the majority particles [24]. This leads to an increased magnetic length scale $l_B^* = l_B/\sqrt{1-\nu}$. Thus, the renormalized wave functions for a single impurity are given by

$$\tilde{\varphi}_m(w) = \sqrt{\frac{(1-\nu)^{m+1}}{2\pi 2^m m!}} w^m e^{-(1-\nu)|w|^2/4}. \tag{1}$$

In the limit of $\nu = 0$, this wave function is identical to the unrenormalized wave function $\varphi_m(w)$. The density corresponding to $\tilde{\varphi}_m$ is given by

$$\begin{aligned}\tilde{\rho}_m(w) = |\tilde{\varphi}_m(w)|^2 &= \frac{(1-\nu)^{m+1}}{2\pi 2^m m!}|w|^{2m} e^{-(1-\nu)|w|^2/2} \\ &= \sum_{n=0}^{\infty} \rho_{m+n}(w)\nu^n(1-\nu)^{m+1}\frac{(m+n)!}{m!n!}. \end{aligned} \tag{2}$$

In the second line, we have expanded the renormalized density $\tilde{\rho}_m$ in terms of unrenormalized densities $\rho_{n+m} = |\varphi_{n+m}|^2$, corresponding to angular momentum $n+m$. Thus, the average angular momentum L_b^m of an impurity in level m is given by

$$L_b^m = \sum_{n=0}^{\infty}(n+m)\nu^n(1-\nu)^{m+1}\frac{(m+n)!}{m!n!} = \frac{m+\nu}{1-\nu}. \tag{3}$$

In its ground state ($m=0$), the impurity has average angular momentum value $L_b^0 = \nu/(1-\nu)$, and exciting the impurity by one unit (from m to $m+1$) changes the average angular momentum by $\Delta L_b = 1/(1-\nu) > 1$. The standard deviation is $\delta L_b^m = \sqrt{\nu(m+1)}/(1-\nu)$, so the relative error $\delta L_b^m / L_b^m \to 0$ for large m.

We have used different methods to verify these results numerically: (i) applying numerical diagonalization to the pseudopotential Hamiltonian at fixed total angular momentum L, the analytical construction of the zero-energy modes can be verified, and in particular the counting of Table I. We lift the ground state degeneracy $\mathcal{N}_{d,\mathrm{imp}}$ at $L = L_q + N_a + d$ by choosing the trap frequency ω_a slightly larger than ω_b. The states within the quasidegenerate manifold are then energetically ordered decreasingly with the excitation level m of the impurity: the unique ground state is $\Psi_{q,d,0}$, followed by $\Psi_{q,d-1,1}$, and subsequently two degenerate states $\Psi^\alpha_{q,d-2,2}$, etc. The corresponding impurity angular momentum $\langle \hat{L}_b \rangle$ is immediately obtained from the numerical solution, and for each $m \leq d$, we find $\mathcal{N}_{d-m}$ degenerate states, in which the impurity's angular momentum matches very well with the theoretically expected value $L_b^m = L_b^0 + m\Delta L_b$. This behavior is exemplified in Fig. 1 for two cases corresponding to Laughlin filling factors $\nu = 1/3$ and $\nu = 1/5$. In this example, we have chosen $d = 4$ yielding 12 quasidegenerate states (left of the red-dotted vertical line).

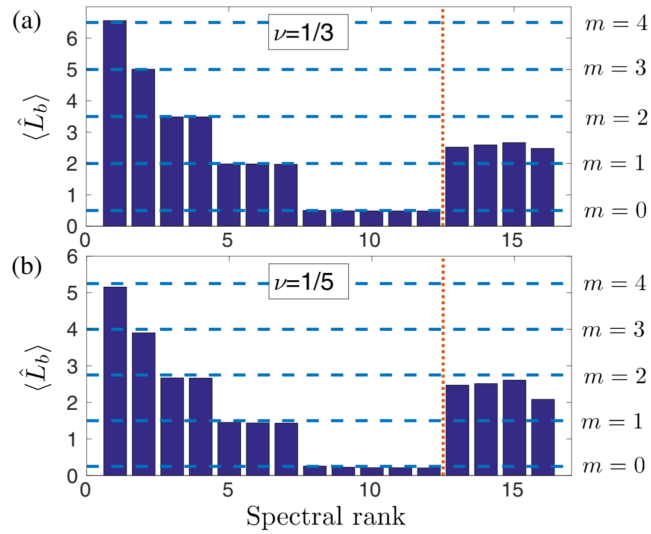

FIG. 1. We plot the angular momentum $\langle \hat{L}_b \rangle$ of an impurity in a Laughlin liquid at (a) $\nu = 1/3$ ($N_a = 8$ particles at $L = L_3 + N_a + 4 = 96$), and at (b) $\nu = 1/5$ ($N_a = 6$ particles at $L = L_5 + N_a + 4 = 85$). The 12 lowest states (on the left to the red-dotted line) are states of zero interaction energy. On average, the impurity takes fractionally quantized values $L_b = [(m+\nu)/(1-\nu)]$ (indicated through the blue-dashed lines).

(ii) Equation (3) can also be verified by evaluating the impurity angular momentum from the first-quantized wave functions, either by symbolical operations (cf. Ref. [31]), or by attaining much larger system sizes (e.g., $N_a \sim 40$), numerically via the Monte Carlo integration method. We have used this method to determine the impurity angular momentum of $\Psi_{q,1,0}$ for $2 \leq q \leq 6$, which is accurately given by L_b^0.

The fractional "quantization" of angular momentum is reflected by the impurity density, plotted in Fig. 2(a). Higher orbitals correspond to larger angular momenta and are characterized by broader density profiles. More quantitatively, there is a linear relation between the mean square of the radial position, $\langle r^2 \rangle$, and the angular momentum m. In the absence of a liquid (i.e., for $\nu = 0$), we have $\langle r^2 \rangle_m \equiv \int_0^\infty dr r^3 |\varphi_m(r)|^2 = 2m + 2$. As we find numerically, the slope of this curve changes at finite ν [see Fig. 2(b)]. In this case, $\langle r^2 \rangle_{m,q} \equiv \int_0^\infty dr r^3 \rho_b^{m,q}(r)$, where the impurity density $\rho_b^{m,q}(r)$ corresponds to a many-body state $\Psi^\alpha_{q,m,d-m}$ and is essentially independent from the choice of d and α. Specifically, at $\nu = 0$, the slope of value

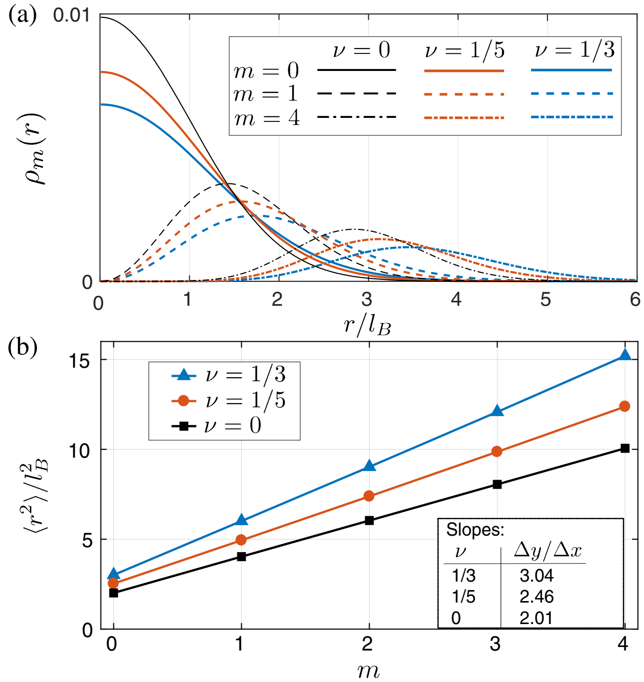

FIG. 2. (a) We plot the radial density $\rho_b^{m,q}(|w|)$ of an impurity, which is excited to the mth level ($m = 0$, 1, 4), and which is immersed in a FQH liquid at $\nu = 1/q$ (for $q = 3$ and $q = 5$). For concreteness, we have assumed a liquid of $N_a = 8$ particles at $L = L_3 + N_a + 4 = 96$ for $\nu = 1/3$, and a liquid of $N_a = 6$ particles at $L = L_5 + N_a + 4 = 85$ at $\nu = 1/5$, and different m levels correspond to different edge modes. We also plot the density $\rho_m(|w|) = |\varphi_m(w)|^2$ of a single impurity in the absence of a liquid ($\nu = 0$). (b) For different levels m, we plot the mean square $\langle r^2\rangle_m$ of the radial position of the impurity in the presence of a liquid at $\nu = 1/3$, $\nu = 1/5$, and in the absence of the liquid. The slope of the linear relation between m and $\langle r^2\rangle_m$ characterizes the quantization of angular momentum.

2 corresponds to integer quantization of angular momentum, whereas at $\nu = 1/3$ and $\nu = 1/5$, the slopes are increased by factors 3/2 and 5/4, in full accordance with the expected "quantization" of angular momentum.

Generalization to Moore-Read liquid.—The fractionalization of impurity angular momentum, as described by Eq. (3), does not only apply to impurities in a Laughlin liquid, but it is also in the non-Abelian Moore-Read liquid incorporating the pairing of particles. Such liquid allows for two types of quasiholes [32]: a "Laughlin"-like quasihole of charge νe, which is anticorrelated with all liquid particles, and a "Pfaffian"-like quasihole, of charge $\nu e/2$, which is anticorrelated only with one particle of each pair. By Monte Carlo integration of their wave functions (see also the Supplemental Material [33]), we verify that Eq. (3) holds for an impurity bound to a "Laughlin"-type quasihole in the Moore-Read liquid. In contrast, for an impurity bound to a "Pfaffian"-type quasihole, the formula has to be modified by replacing ν with $\nu/2$, that is, $L_b^m = [(2m + \nu)/(2 - \nu)]$. This modification

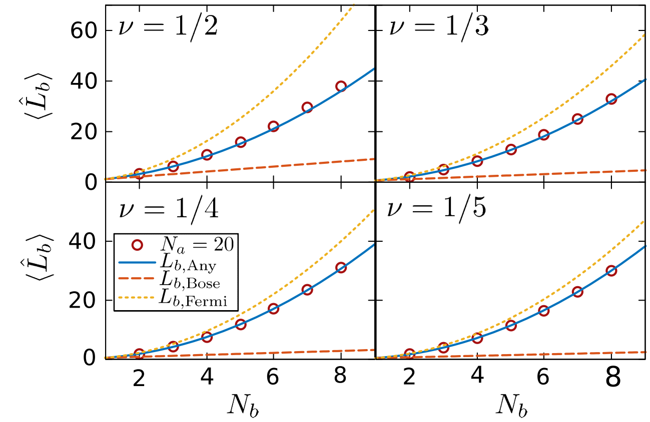

FIG. 3. The impurity angular momentum $\langle \hat{L}_b\rangle$ is plotted as a function of impurity number N_b, for different filling factors ν of the majority liquid in the Laughlin state. The numerical results are obtained from Monte Carlo sampling in the wave function Eq. (4) for $N_a = 20$ majority particles. We also plot $L_{b,\rm Bose}(N_b, \nu)$ and $L_{b,\rm Fermi}(N_b, \nu)$, the values expected if free bosons or fermions would fill the effective single-particle levels for impurities bound to quasiholes, as well as the anyonic interpolation between both curves, $L_{b,\rm Any}(N_b, \nu)$, defined in Eq. (5). The numerical data is found to match very well the anyonic prediction.

accounts for the fact that the "Pfaffian" quasihole only "sees" half of the liquid particles.

Results for multiple impurities.—Having established the angular momentum levels of a single impurity, Eq. (3), we now ask how $\langle \hat{L}_b\rangle$ behaves in the presence of N_b impurities. To obtain states of zero interaction energy, the total angular momentum needs to accommodate the anticorrelations of the majority liquid, the presence of N_b quasiholes, and, for fermionic impurities, a Vandermonde determinant $\prod_{i<j}(w_i - w_j)$. Thus, the zero-energy ground state occurs at $L = L_q + N_b N_a + \frac{1}{2}N_b(N_b - 1)$, and its wave function reads:

$$\Psi_{F,\rm qhs} \sim \left[\prod_{i<j}^{N_b}(w_i - w_j)\right]\left[\prod_{i=1}^{N_a}\prod_{j=1}^{N_b}(z_i - w_j)\right]\Psi_q. \quad (4)$$

Naïvely, one might expect that the total angular momentum of the impurities is equal to the value obtained from filling the single-particle levels, $L_{b,\rm Fermi}(N_b, \nu) = \sum_{m=0}^{N_b-1}[(m + \nu)/(1 - \nu)] = [1/(q - 1)][(q/2)N_b(N_b - 1) + N_b]$. However, this expectation is not correct: Fig. 3 shows our numerical results for $\langle \hat{L}_b\rangle$ as a function of the number N_b of fermionic impurities, which interact with a bosonic or fermionic liquid ($N_a = 20$) at different filling factors ν. For comparison, we also plot $L_{b,\rm Fermi}(N_b, \nu)$ as well as the angular momentum expected for Bose condensation in the lowest impurity level, $L_{b,\rm Bose}(N_b, \nu) = N_b L_b^0 = N_b[\nu/(1 - \nu)]$. The numerical value is intermediate, $L_{b,\rm Bose} < \langle \hat{L}_b\rangle < L_{b,\rm Fermi}$. More precisely, it matches extremely well with the following interpolation formula:

$$L_{b,\mathrm{Any}}(N_b,\nu) = (1-\nu)L_{b,\mathrm{Fermi}}(N_b,\nu) + \nu L_{b,\mathrm{Bose}}(N_b,\nu). \tag{5}$$

This formula suggests that the statistical parameter α, which interpolates from Bose statistics ($\alpha = 0$) to Fermi statistics ($\alpha = 1$), is given by $\alpha = 1 - \nu$. This is in agreement with the effective Hamiltonian derived in Ref. [26] for impurities coupled to fractional quasiholes (see also Refs. [36,37]), and with the general expectation for a Laughlin quasihole ($\alpha = -\nu$) bound to a fermion ($\alpha = 1$). Importantly, we note that similar results, as shown in Fig. 3 (with $N_a = 20$), can already be obtained for extremely small Laughlin liquids ($N_a < 10$), enabling the detection of anyon statistics in microscopic quantum simulators.

Summary and outlook.—We have shown that (i) the effective single-particle states for impurities bound to anyons can be characterized by their fractional angular momentum, and (ii) the filling of these levels is governed by fractional statistics. Our findings provide a way to detect anyonic properties without braiding via the density of impurity particles. This eases anyon detection, possibly also compared to existing schemes based on local density of state measurements [18,19], pair-correlation function of two impurities [24], or liquid density [21]. A key difference of our approach to other proposals involving impurities [19,24] is the fact that it keeps all impurity particles fully dynamical. This realizes a noninteracting gas of anyons, and an anyonic distribution function governs the impurity degrees of freedom. With this, the setup is also suited to study, in future work, the intricate thermodynamics of anyon gases.

The implementation of our ideas is possible either in microscopic quantum simulators using atoms or photons [10,12], or in macroscopic electronic samples with optically created impurities such as excitons or trions. Signatures of excitons bound to fractional quasiparticles have been reported in [38], and the exciton density can be detected via scanning-transmission-electron microscopy [39]. Additional information covering long-range Hamiltonians is presented in the Supplemental Material [33]. Future work shall explore the potential of our scheme for detecting non-Abelian anyons, and for studying thermodynamics of anyons, including interacting anyons which might themselves form FQH liquids.

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[1] J. M. Leinaas and J. Myrheim, On the theory of identical particles, Il Nuovo Cimento B **37**, 1 (1977).

[2] F. Wilczek, Magnetic Flux, Angular Momentum, and Statistics, Phys. Rev. Lett. **48**, 1144 (1982).

[3] R. B. Laughlin, Anomalous Quantum Hall Effect: An Incompressible Quantum Fluid with Fractionally Charged Excitations, Phys. Rev. Lett. **50**, 1395 (1983).

[4] D. Arovas, J. R. Schrieffer, and F. Wilczek, Fractional Statistics and the Quantum Hall Effect, Phys. Rev. Lett. **53**, 722 (1984).

[5] L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, Observation of the $e/3$ Fractionally Charged Laughlin Quasiparticle, Phys. Rev. Lett. **79**, 2526 (1997).

[6] F. E. Camino, W. Zhou, and V. J. Goldman, $e/3$ Laughlin Quasiparticle Primary-Filling $\nu = 1/3$ Interferometer, Phys. Rev. Lett. **98**, 076805 (2007).

[7] R. L. Willett, L. N. Pfeiffer, and K. W. West, Measurement of filling factor 5/2 quasiparticle interference with observation of charge $e/4$ and e/2 period oscillations, Proc. Natl. Acad. Sci. U.S.A. **106**, 8853 (2009).

[8] N. Ofek, A. Bid, M. Heiblum, A. Stern, V. Umansky, and D. Mahalu, Role of interactions in an electronic Fabry–Perot interferometer operating in the quantum Hall effect regime, Proc. Natl. Acad. Sci. U.S.A. **107**, 5276 (2010).

[9] H. Bartolomei, M. Kumar, R. Bisognin, A. Marguerite, J.-M. Berroir, E. Bocquillon, B. Plaçais, A. Cavanna, Q. Dong, U. Gennser, Y. Jin, and G. Fève, Fractional statistics in anyon collisions, Science **368**, 173 (2020).

[10] N. Gemelke, E. Sarajlic, and S. Chu, Rotating few-body atomic systems in the fractional quantum Hall regime, arXiv:1007.2677.

[11] N. Goldman, G. Juzeliūnas, P. Öhberg, and I. B. Spielman, Light-induced gauge fields for ultracold atoms, Rep. Prog. Phys. **77**, 126401 (2014).

[12] L. W. Clark, N. Schine, C. Baum, N. Jia, and J. Simon, Observation of Laughlin states made of light, Nature (London) **582**, 41 (2020).

[13] B. Paredes, P. Fedichev, J. I. Cirac, and P. Zoller, $\frac{1}{2}$-Anyons in Small Atomic Bose-Einstein Condensates, Phys. Rev. Lett. **87**, 010402 (2001).

[14] T. Graß, M. Gullans, P. Bienias, G. Zhu, A. Ghazaryan, P. Ghaemi, and M. Hafezi, Optical control over bulk excita-

tions in fractional quantum Hall systems, Phys. Rev. B **98**, 155124 (2018).

[15] E. Kapit, P. Ginsparg, and E. Mueller, Non-Abelian Braiding of Lattice Bosons, Phys. Rev. Lett. **108**, 066802 (2012).

[16] T. Graß, B. Juliá-Díaz, and M. Lewenstein, Topological phases in small quantum Hall samples, Phys. Rev. A **89**, 013623 (2014).

[17] S. Dutta and E. J. Mueller, Coherent generation of photonic fractional quantum Hall states in a cavity and the search for anyonic quasiparticles, Phys. Rev. A **97**, 033825 (2018).

[18] N. R. Cooper and S. H. Simon, Signatures of Fractional Exclusion Statistics in the Spectroscopy of Quantum Hall Droplets, Phys. Rev. Lett. **114**, 106802 (2015).

[19] Z. Papić, R. S. K. Mong, A. Yazdani, and M. P. Zaletel, Imaging Anyons with Scanning Tunneling Microscopy, Phys. Rev. X **8**, 011037 (2018).

[20] S. C. Morampudi, A. M. Turner, F. Pollmann, and F. Wilczek, Statistics of Fractionalized Excitations through Threshold Spectroscopy, Phys. Rev. Lett. **118**, 227201 (2017).

[21] R. O. Umucalılar, E. Macaluso, T. Comparin, and I. Carusotto, Time-of-Flight Measurements as a Possible Method to Observe Anyonic Statistics, Phys. Rev. Lett. **120**, 230403 (2018).

[22] E. Yakaboylu and M. Lemeshko, Anyonic statistics of quantum impurities in two dimensions, Phys. Rev. B **98**, 045402 (2018).

[23] E. Yakaboylu, A. Ghazaryan, D. Lundholm, N. Rougerie, M. Lemeshko, and R. Seiringer, A quantum impurity model for anyons, arXiv:1912.07890.

[24] Y. Zhang, G. J. Sreejith, N. D. Gemelke, and J. K. Jain, Fractional Angular Momentum in Cold-Atom Systems, Phys. Rev. Lett. **113**, 160404 (2014).

[25] Y. Zhang, G. J. Sreejith, and J. K. Jain, Creating and manipulating non-Abelian anyons in cold atom systems using auxiliary bosons, Phys. Rev. B **92**, 075116 (2015).

[26] D. Lundholm and N. Rougerie, Emergence of Fractional Statistics for Tracer Particles in a Laughlin Liquid, Phys. Rev. Lett. **116**, 170401 (2016).

[27] F. Grusdt, N. Y. Yao, D. Abanin, M. Fleischhauer, and E. Demler, Interferometric measurements of many-body topological invariants using mobile impurities, Nat. Commun. **7**, 11994 (2016).

[28] A. Camacho-Guardian, N. Goldman, P. Massignan, and G. M. Bruun, Dropping an impurity into a Chern insulator: A polaron view on topological matter, Phys. Rev. B **99**, 081105(R) (2019).

[29] A. Khare, *Fractional Statistics and Quantum Theory* (World Scientific Publishing, Singapore, 1997).

[30] F. D. M. Haldane, Fractional Quantization of the Hall Effect: A Hierarchy of Incompressible Quantum Fluid States, Phys. Rev. Lett. **51**, 605 (1983).

[31] B. Juliá-Díaz and T. Graß, Strongdeco: Expansion of analytical, strongly correlated quantum states into a many-body basis, Comput. Phys. Commun. **183**, 737 (2012).

[32] X. Wan, Z.-X. Hu, E. H. Rezayi, and K. Yang, Fractional quantum Hall effect at $\nu = 5/2$: Ground states, non-Abelian quasiholes, and edge modes in a microscopic model, Phys. Rev. B **77**, 165316 (2008).

[33] See the Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.125.136801 for a discussion of impurities in Moore-Read liquids, long-ranged Hamiltonians, as well as some modifications and limitations of our scheme. The Supplemental Material includes Refs. [34,35].

[34] Z. Papić, R. Thomale, and D. A. Abanin, Tunable Electron Interactions and Fractional Quantum Hall States in Graphene, Phys. Rev. Lett. **107**, 176602 (2011).

[35] C. Fey, P. Schmelcher, A. Imamoglu, and R. Schmidt, Theory of exciton-electron scattering in atomically thin semiconductors, Phys. Rev. B **101**, 195417 (2020).

[36] N. Rougerie, Some contributions to many-body quantum mathematics, arXiv:1607.03833.

[37] D. Lundholm, Many-anyon trial states, Phys. Rev. A **96**, 012116 (2017).

[38] M. Byszewski, B. Chwalisz, D. K. Maude, M. L. Sadowski, M. Potemski, T. Saku, Y. Hirayama, S. Studenikin, D. G. Austing, A. S. Sachrajda, and P. Hawrylak, Optical probing of composite fermions in a two-dimensional electron gas, Nat. Phys. **2**, 239 (2006).

[39] H. C. Nerl, K. T. Winther, F. S. Hage, K. S. Thygesen, L. Houben, C. Backes, J. N. Coleman, Q. M. Ramasse, and V. Nicolosi, Probing the local nature of excitons and plasmons in few-layer MoS2, npj 2D Mater. Appl. **1**, 2 (2017).