# Photon Hall Pinwheel Radiation of Angular Momentum by a Diffusing Magneto-Optical Medium

B. A. van Tiggelen[1,*] and G. L. J. A. Rikken[2]
[1]*Univ. Grenoble Alpes, CNRS, LPMMC, 38000 Grenoble, France,*
*and Univ. Grenoble Alpes, CNRS, LNCMI, EMFL, Toulouse/Grenoble, France*
[2]*Univ. Grenoble Alpes, CNRS, LNCMI, EMFL, Toulouse/Grenoble, France*

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We present a new optical effect that exchanges angular momentum between light and matter. The matter consists of an optically thick spherical, rigid agglomerate of magneto-optical scatterers placed in a homogeneous magnetic field. The light comes from an unpolarized, coherent central light source. The photon Hall effect induces a *spiraling* Poynting vector, both inside and outside the medium. Optical orbital angular momentum leaks out and induces a torque proportional to the injection power of the source and the photon Hall angle.

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The photon Hall effect (PHE) [1,2] is the analogy in diffusive photon transport of the well-known electronic Hall effect, corresponding to a deflection of the diffusing photon flux in the direction perpendicular to both the flux gradient and the magnetic field. The PHE originates from Faraday rotation inside the dielectric particles [3] and emerges in multiple scattering. The PHE is also predicted to occur in light scattering by atoms [4,5]. Other related effects in wave diffusion have been identified, such as the photon spin Hall effect [6–8], the quantum spin Hall effect of light [9], the phonon Hall effect [10], the plasmon Hall effect [11], and other photon Hall-type effects [12].

In this Letter, we present a simplified model in which the PHE drives the transfer of angular momentum from light to matter. The latter is assumed to consist of a rigid, initially immobile, nonabsorbing, disordered agglomerate of spherical geometry, with random dielectric constant $\varepsilon(\boldsymbol{r})$ that is magnetoactive when exposed to a homogeneous magnetic field but with no magnetic susceptibility ($\mu = 1$). The scattering mean free path $\ell$ is assumed constant, small compared to the radius $a$ of the sphere, and large compared to the wavelength of the light. This picture is consistent with radiative diffusion theory [13] with longitudinal diffusion constant $D$ and a transverse PHE diffusion constant $D_H \ll D$, proportional to the Verdet constant of the average medium and to the magnetic field. An isotropic, coherent, light source in the center of the sphere injects light, escaping eventually after a random walk at the boundaries. The source is assumed to be unpolarized to avoid injecting photons with angular momentum and monochromatic since most optical parameters are wavelength dependent. The spherical geometry and the idealized point source are both not essential but simplify calculations. As scatterers, we typically have in mind powders of rare-earth compounds, used by us before [2] and known for their large magneto-optical activity, frozen solidly into a transparent inactive medium. Other media such as stellar atmospheres or atomic vapors may be relevant, though they come with specific technical problems not discussed here. We report a novel optical effect, namely the existence of a spiraling Poynting vector and optical angular momentum proportional to the applied magnetic field that is radiated away. The resulting torque on the sphere is within experimental reach.

The Poynting vector associated with the PHE is given by $\mathbf{S}_{\mathrm{PHE}} = D_H \hat{\boldsymbol{B}}_0 \times \nabla\rho(\boldsymbol{r}, t)$, with $\rho(\boldsymbol{r}, t)$ the electromagnetic energy density. The PHE angle is defined by $\phi_H = D_H/D$. Let us first consider an infinite medium and a point source $P(\boldsymbol{r}, t) = P(t)\delta(\boldsymbol{r})$ injecting a power $P$. For a single light pulse $W$, $P(t) = W\delta(t)$, we can insert the well-known solution of the diffusion equation [13] to obtain

$$\mathbf{S}_{\mathrm{PHE}}(\boldsymbol{r}, t) = -\frac{W D_H r}{16\pi^{3/2}(Dt)^{5/2}} \exp\left(-\frac{r^2}{4Dt}\right)\hat{\phi}, \quad (1)$$

whose maximum spirals outward as $r \sim \sqrt{Dt}$ while decaying in time as $1/(Dt)^{3/2}$. Although widely accepted in electric transport, the existence of a *spiraling* light flux is surprising. Standard textbooks [14–16] insist on the ambiguity in the interpretation of $\mathbf{S} = c_0 \boldsymbol{E} \times \boldsymbol{H}$ as the energy current density. Based on the conservation law $\partial_t \rho + \nabla \cdot \mathbf{S} = P$, a freedom remains to add the curl of an arbitrary vector field to the current density $\mathbf{S}$, as is the case for the PHE. In homogeneous, conservative media, the directions of the Poynting vector and group velocity coincide, and the ambiguity is lifted [17]. In *absorbing* magneto-optical media, the addition of a curl turned out to be necessary [18]. In *inhomogeneous*, conservative media, this ambiguity has so far never been addressed. In our simple spherical model with elastic multiple scattering, the circulating light does not, *in fine*, move around energy, and arguably, there is no ambiguity. Yet this same light carries momentum that can be transferred to matter and is thus observable [19]. The interpretation of electromagnetic angular momentum (for $\mu = 1$), $\boldsymbol{K}(t) = c_0^{-2} \int d^3\boldsymbol{r}\,\boldsymbol{r} \times \mathbf{S}$, is not subject to this curl ambiguity, since adding any curl to $\mathbf{S}$ would produce an additional finite angular momentum. The PHE generates the angular momentum

$$\boldsymbol{K}(t) = -\frac{2D_H}{c_0^2}\hat{z} \int d^3\boldsymbol{r}\rho(\boldsymbol{r}, t) = -\frac{2D_H}{c_0^2}\hat{z} \int_0^t dt' P(t') \quad (2)$$

proportional to the total amount of energy injected and conserved in time after the injection stops. The conservation of total angular momentum can be expressed as

$$\frac{d}{dt}(\boldsymbol{K} + \boldsymbol{K}_{\text{mat}})_i = \lim_{r\to\infty} \int d^2A(r)\hat{r}_n\epsilon_{ijk}\hat{r}_j T_{nk}(\mathbf{r}, t) \quad (3)$$

with $\boldsymbol{K}_{\text{mat}}$ the angular momentum of the matter in the sphere and $\boldsymbol{T}$ the symmetrical electromagnetic stress tensor [14]. The term on the righthand side describes the flow of angular momentum to infinity and does not vanish in the presence of the PHE. Nevertheless, if we choose $r > c_0 t$, the leak of angular momentum is outside the light cone and vanishes at infinity. As a result, $\boldsymbol{K} + \boldsymbol{K}_{\text{mat}}$ must be conserved in time [20]. The corresponding torque on the diffuse medium is $N = -dK_{\text{mat}}/dt = 2D_H\hat{z}P(t)/c_0^2$. For the largest PHE angle reported of $D_H/D \sim 10^{-3}$ [21], a mean free path $\ell = 10\ \mu$m, and $P = 10$ W, Eq. (2) predicts a torque $N = 10^{-16}$ Nm.

Knowing that the flow of optical angular momentum follows from simple arguments, we next consider the more

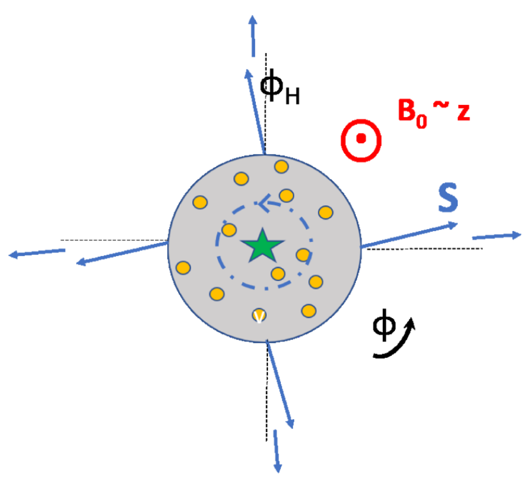


FIG. 1. The photon Hall pinwheel: The photon Hall effect inside a diffuse sphere emerges as a Poynting vector *spiraling* around the origin, not affecting *net* energy transport but carrying a finite orbital angular momentum. The radiation leaves the sphere under the photon Hall angle $\phi_H = D_H/D$ with the local normal (here drawn for $D_H < 0$), decreasing as $1/r$ with distance.

realistic case of a finite medium out of which the light ultimately escapes, in particular the rigid disordered sphere described earlier (see Fig. 1). At times $t > a^2/4D$ after injection, the photons start leaving the sphere. In a first approximation, we can imagine the PHE at the boundaries making the photons *all* leave the sphere at the photon Hall angle $\phi_H$ with the local normal. With $P(a, t) = 4\pi a^2 S(a, t)$, the optical power leaving the sphere, the rate of optical orbital angular momentum (with direction $\hat{z}$) flowing out of the sphere is $-a \times \phi_H P(a, t)/c_0$. This loss rate must equal the opposite of the mechanical torque $N(t)$ on the sphere. Energy conservation imposes that $P(a, t) = P(t) - \partial_t W$, with $W$ the electromagnetic energy inside the sphere. If we assume the latter is stationary, then

$$\boldsymbol{N}(t) = \hat{\mathbf{z}}\phi_H \frac{a}{c_0} P(t). \quad (4)$$

If freely suspended, this torque will make the sphere, assumed rigid, start to rotate homogeneously, thus resembling the pinwheel effect of certain fireworks. This simple argument can be straightforwardly extended to nonspherical bodies.

For a quantitative picture of how angular momentum radiates outside the sphere, we follow the standard method to deal with boundaries of the diffusion equation and calculate the electromagnetic angular momentum outside the sphere. A good treatment of the skin layer [22] is essential to respect flux conservation. The magneto-optics of the effective medium [13] is described by a uniform complex dielectric constant $\varepsilon_\sigma(\boldsymbol{k})$ in terms of which wave vector $k_\sigma$ and extinction length $\ell_\sigma$ are defined as $\varepsilon_\sigma(\boldsymbol{k})\omega^2/c_0^2 = (k_\sigma + i/2\ell_\sigma)^2$ that here depend both on circular polarization and wave vector direction. In radiative transport, the electric field in and outside the medium can be imagined to be emitted by a diffuse random secondary source as $E_i(\boldsymbol{r}) = \int d^3\boldsymbol{r}_s G_{ij}(\boldsymbol{r}, \boldsymbol{r}_s)s_j(\boldsymbol{r}_s)$. Consequently, the “ensemble-averaged" field correlation function is

$$\langle E_i(\boldsymbol{r})\bar{E}_j(\boldsymbol{r}')\rangle = \int d^3\boldsymbol{r}_S \int d^3\boldsymbol{x}\langle G_{ik}(\boldsymbol{r}, \boldsymbol{r}_s^+)\rangle\langle \bar{G}_{lj}(\boldsymbol{r}', \boldsymbol{r}_s^-)\rangle \times \langle s_k(\boldsymbol{r}_s^+)\bar{s}_l(\boldsymbol{r}_s^-)\rangle \quad (5)$$

with $\boldsymbol{r}_s^\pm = \boldsymbol{r}_s \pm \boldsymbol{x}/2$. In the far field outside the sphere $G_{ij}(\boldsymbol{r}^\pm) \to (\delta_{ij} - \hat{r}_i\hat{r}_j)\exp(iKr)\exp(ik\hat{\boldsymbol{r}}\cdot\boldsymbol{x}/2)/(-4\pi r)$, and this expression simplifies to a variant of the Van Cittert-Zernike theorem in coherence theory [15],

$$\langle E_\sigma(\boldsymbol{r})\bar{E}_{\sigma'}(\boldsymbol{r})\rangle = \frac{1}{(4\pi)^2} \int d^3\boldsymbol{r}_s \frac{e^{-i(K_\sigma - \bar{K}_{\sigma'})b(r_s)}}{|\boldsymbol{r} - \boldsymbol{r}_s|^2} C_{\sigma\sigma'}(\boldsymbol{k}, \boldsymbol{r}_s) \quad (6)$$

with $b(\boldsymbol{r}_s)$ the length traversed by the light from the source to its exit point (see Fig. 2). Because $K_\sigma$ is complex valued inside the sphere, the integral is restricted to the skin layer

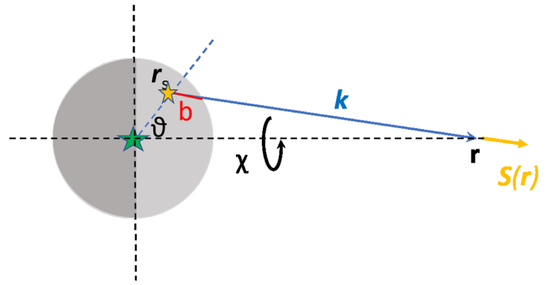


FIG. 2. Geometry of the PHE in the far field of the sphere. The diffuse light arriving from the source in the center initiates secondary random sources at $\boldsymbol{r}_s$ within a skin layer of depth $\ell$ near the surface, that contribute to the Poynting vector $\mathbf{S}$ in the far field. The induced PHE at $\boldsymbol{r}$ is largest when the magnetic field is perpendicular to the plane.

of depth $\ell_\sigma$ below the surface and the far field correlation function is proportional to the Fourier transform $C(\boldsymbol{k},\boldsymbol{r}_s)$ of the source correlation function $\langle s(\boldsymbol{r}_s^+)\bar{s}(\boldsymbol{r}_s^-)\rangle$, integrated over the surface of the sphere. A *second* fundamental result from transport theory is that in the diffuse regime

$$C_{\sigma\sigma'}(\boldsymbol{k}_\sigma,\boldsymbol{r}_s)=2\pi k\delta_{\sigma\sigma'}\mathrm{Im}\varepsilon_\sigma(\boldsymbol{k}_\sigma)\left(1-\frac{3}{c_0}\hat{\boldsymbol{k}}\cdot\boldsymbol{D}\cdot\nabla\right)\rho(\boldsymbol{r}_s). \quad (7)$$

This is basically a manifestation of the fluctuation dissipation theorem, with the first term standing for detailed balance and the gradient term causing the diffuse, *anisotropic* energy flow. Both depend on magnetic field, the second containing the PHE via $D_{ij}=D\delta_{ij}+D_H\epsilon_{ijz}$. In the far field outside the medium the Poynting vector at $\boldsymbol{r}$ from emission at $\boldsymbol{r}_s$ is just $\mathbf{S}(\boldsymbol{r})=c_0\hat{\boldsymbol{k}}(\hat{\boldsymbol{r}}_s)|E(\boldsymbol{r})|^2$, with $\hat{\boldsymbol{k}}$ the unit vector along the vector $\boldsymbol{r}-\boldsymbol{r}_s$. Combining Eqs. (6) and (7), the total Poynting vector at $\boldsymbol{r}$ becomes

$$\mathbf{S}\left(\boldsymbol{r},t+\frac{r}{c_0}\right)=\frac{c_0}{8\pi}\sum_\sigma\int d^2\hat{\boldsymbol{r}}_s\int_0^\infty dz e^{-z/\mu\ell_\sigma}\frac{\hat{\boldsymbol{k}}(\boldsymbol{r}_s)}{|\boldsymbol{r}-\boldsymbol{r}_s|^2}\times\frac{1}{\ell_\sigma}\left[1-\frac{3}{c_0}\hat{\boldsymbol{k}}(\boldsymbol{r}_s)\cdot\boldsymbol{D}\cdot\nabla\right]\rho(a\hat{\boldsymbol{r}}_s) \quad (8)$$

with $\mu=\cos\theta$, $b\approx z/\mu$ (for $r\gg a$) and the integral of $\mathbf{r}_S$ restricted to the surface visible from $\boldsymbol{r}$. Hence,

$$\mathbf{S}\left(\boldsymbol{r},t+\frac{r}{c_0}\right)=\frac{c_0}{4\pi}\sum_\sigma\int d^2\hat{\boldsymbol{r}}_s\frac{\mu\hat{\boldsymbol{k}}(\boldsymbol{r}_s)}{|\boldsymbol{r}-\boldsymbol{r}_s|^2}\times\left[\rho(a,t)-\frac{3}{c_0}\hat{\boldsymbol{k}}(\boldsymbol{r}_s)\cdot\boldsymbol{D}\cdot\hat{\boldsymbol{r}}_s\partial_r\rho(a,t)\right]. \quad (9)$$

The only magneto-optical effect that has survived in this expression for the Poynting vector is the PHE contained in $\boldsymbol{D}$. The dominant radial flow of energy is obtained by putting $\hat{\boldsymbol{k}}\approx\hat{\boldsymbol{r}}$ and becomes

$$\mathbf{S}_r\left(\boldsymbol{r},t+\frac{r}{c_0}\right)=\frac{c_0a^2}{2r^2}\hat{\boldsymbol{r}}\int_0^1 d\mu\mu\left[\rho(a,t)-\frac{3}{c_0}\mu D\partial_r\rho(a,t)\right]=\frac{c_0a^2}{4r^2}\hat{\boldsymbol{r}}\left[\rho(a,t)-\frac{2}{c_0}D\partial_r\rho(a,t)\right] \quad (10)$$

with $D=\hat{\boldsymbol{r}}\cdot\boldsymbol{D}\cdot\hat{\boldsymbol{r}}$ in which the PHE drops out. The usual radiative boundary condition at the surface is $\rho+(2/c_0)D\partial_r\rho=0$ and identifies $z_0=2D/c_0$ as the extrapolation length [22]. The total current leaving the sphere is therefore $J(a,t)=4\pi a^2S_r(a)=-4\pi a^2D\partial_r\rho(a,t-a/c_0)$. To find the PHE, we write more accurately $\hat{\boldsymbol{k}}=\hat{\boldsymbol{r}}(1+\mu r_s/r)-\mathbf{r}_S/r+\mathcal{O}(1/r^2)$. The second term gives

$$\mathbf{S}(\boldsymbol{r},t)=\frac{c_0a^2}{4\pi r^2}\int_0^1 d\mu\int_0^{2\pi}d\chi\mu\left(\frac{-\mathbf{r}_S}{r}\right)\hat{\boldsymbol{r}}\cdot(\hat{\boldsymbol{r}}_s\times\hat{\boldsymbol{z}})\times\frac{-3D_H}{c_0}\partial_r\rho\left(a,t-\frac{r}{c_0}\right)=\frac{3}{16}\frac{a^2}{r^3}D_H\partial_r\rho\left(a,t-\frac{r}{c_0}\right)(\hat{\boldsymbol{z}}\times\hat{\boldsymbol{r}}) \quad (11)$$

This expression corresponds to a PHE outside the sphere with a Hall angle $S_\phi/S_r=(3/16)(D_H/D)a/r$ that decays slowly with distance. The angular momentum is

$$\boldsymbol{K}=\frac{1}{c_0^2}\int d^3\boldsymbol{r}\,\boldsymbol{r}\times\mathbf{S}=-\hat{\boldsymbol{z}}\frac{D_H}{D}\frac{a}{8c_0^2}\int_a^{c_0t}dr\,J\left(a,t-\frac{r}{c_0}\right) \quad (12)$$

in terms of the total energy current $J(a,t)$ leaving the sphere. Since $J(t<0)=0$, the radial integral extends until $r=c_0t$. The angular momentum is directed along the vector $-D_H\boldsymbol{B}_0$ and propagates outwardly. This conclusion can also be obtained by considering the righthand side of Eq. (3). The total angular momentum grows with time as longs as energy is injected into the sphere, so that we can identify the torque on the sphere as

$$\boldsymbol{N}=-\frac{d\boldsymbol{K}}{dt}=\hat{\boldsymbol{z}}\frac{D_H}{D}\frac{a}{8c_0}J\left(a,t-\frac{a}{c_0}\right). \quad (13)$$

Equation (13) is valid for $a\gg\ell$. It is a refinement of the result obtained in Eq. (4). The extra factor 1/8 stems from the fact that not all photons leave the sphere at the photon Hall angle. For $\phi_H=10^{-3}$ (corresponding to frozen glycerol doped with a 10% volume fraction of 2 $\mu$m $EuF_2$ particles, cooled to 4.2 K and for $B_0=1$ $T$ [21]), $a=10$ mm and $P=10$ W, this yields $N=4\times10^{-14}$ Nm. The observation of such a value is within experimental reach

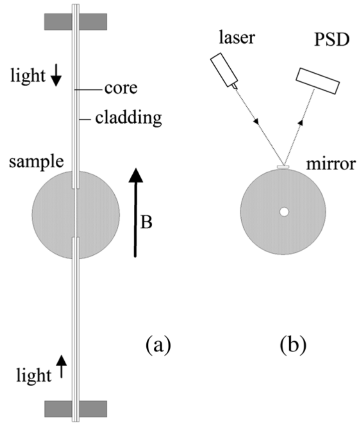


FIG. 3. Proposed experimental setup for the observation of the photon pinwheel effect. (a) Side view: The sample consists of a sphere of radius $\boldsymbol{a}$ of polydisperse particles with a large Faraday effect in a transparent rigid matrix, mounted on a optical fiber. The fiber *also* serves as a torsion wire, with the fiber cladding removed near the center of the sphere. The light injected into the two fiber ends is chopped at the torsion pendulum resonance frequency. (b) Mid-plane top view: Measurement of the torsion angle by deflection of a laser onto a position sensitive detector, readout with phase sensitive detection at the chopping frequency. By measuring only the deflection that changes sign with the sign of "B," all artifacts related to the geometrical imprecision of the sample and setup are eliminated and only the photon Hall torque is measured.

using a torsion pendulum [23]. Figure 3 shows a possible implementation for an experiment to observe this effect.

Under constant flux during a time $T$, the angular momentum radiates continuously into space and therefore the mechanical angular momentum acquired by the diffuse sphere will accumulate. The moment of inertia of a solid sphere is $I = (8\pi/15)\rho_m a^5$, so that the angular velocity of the sphere grows as $\Omega = N \times T/I$. After an exposure time of one hour and a mass density $\rho_M = 1.5\ \mathrm{g/cm^3}$, we find $\varpi = 10^{-3}\ \mathrm{rad/s}$. This is not small. The rotation can be further increased by decreasing $a$ while staying in the optically thick regime. The photon pinwheel effect is not small either from the perspective of a single photon. From Eq. (12), the total angular momentum is proportional to $\phi_H \times a/c_0 \times W$. The total electromagnetic energy is $W = n\hbar\omega$ with $n$ the number of injected photons, so that the total angular momentum carried away per photon, and measured in units of $\hbar$, is $K/n\hbar = \phi_H ka/8$. For $\phi_H \approx 10^{-3}$, a sphere of 1 mm in size has $ka = 12\,000$ at optical frequencies, and each photon leaving the sphere takes away roughly one $\hbar$ of angular momentum.

In summary, we have demonstrated the existence of a spiraling Poynting vector induced by the photon Hall effect in a medium with magneto-optical scatterers illuminated by a central optical source and placed in a homogenous magnetic field. This induces an orbital angular momentum along the magnetic field that propagates away outside the medium and exerts a torque on the medium that is within experimental reach. An analogy can be argued to exist with the Einstein-De Haas effect well-known in magnetism that establishes the link between magnetism and angular momentum. Related diffusive magnetotransport effects mentioned above may show similar pinwheel effects. The photon pinwheel effect may also manifest itself in other areas. A stellar atmosphere may experience a torque as a result of magneto-optical effects in Thompson and Compton scattering induced by the magnetic dipole field of the star.

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*Corresponding author.
bart.van-tiggelen@grenoble.cnrs.fr

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