Dissipation-Time Uncertainty Relation

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We show that the entropy production rate bounds the rate at which physical processes can be performed in stochastic systems far from equilibrium. In particular, we prove the fundamental tradeoff $\langle \dot{S}_e \rangle \mathcal{T} \geq k_B$ between the entropy flow $\langle \dot{S}_e \rangle$ into the reservoirs and the mean time $\mathcal{T}$ to complete any process whose time-reversed is exponentially rarer. This dissipation-time uncertainty relation is a novel form of speed limit: the smaller the dissipation, the larger the time to perform a process.

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Despite operating in noisy environments, complex systems are capable of actuating processes at finite precision and speed. Living systems in particular perform processes that are precise and fast enough to sustain, grow, and replicate themselves. To this end, nonequilibrium conditions are required. Indeed, no process that is based on a continuous supply of (matter, energy, etc.) currents can take place without dissipation.

Recently, an intrinsic limitation on precision set by dissipation has been established by thermodynamic uncertainty relations [1–6]. Roughly speaking, these inequalities state that the squared mean-to-variance ratio of currents is upper bounded by (a function of) the entropy production. Despite producing loose bounds for some specific models [7,8], their fundamental importance is undeniable as they demonstrate that thermodynamics broadly constrains nonequilibrium dynamics [9].

For speed instead, an equivalent limitation set by dissipation can only be speculated. For example, we know from macroscopic thermodynamics that thermodynamic machines will produce entropy to deliver finite power. Yet, a constraint on par with thermodynamic uncertainty relations, only based on dissipation, is still lacking.

Efforts in this direction have appeared lately [10,11], inspired by research on quantum speed limits which are bounds on the time needed to transform a system from one state into another [12]. When extended to classical stochastic dynamics, these relations acquire a somewhat formal appearance [13–16]. In their most explicit form they bound the distance between an initial state and a final one at time t—technically, the 1-norm between the two probability distributions—in terms of the chosen time t, the dissipation, and other kinetic features of the system [13]. However, many systems of interest, especially biological ones, operate under stationary (or time-periodic) conditions [17]. They do not involve any (net) transformation in the system's state. The changes are confined to the reservoirs that fuel the nonequilibrium dynamics via mass or energy exchanges, for instance. Furthermore, the kinetic features of these systems are hardly known [18,19].

We show in this Letter that the dissipation alone suffices to bound the pace at which any stationary (or time-periodic) process can be performed. To do so, we set up the most appropriate framework to describe nontransient operations. Namely, we unambiguously define the process duration by the first-passage time for an observable O to reach a given threshold D [20–23]. We first derive a bound for the instantaneous rate of the process $r(t)$, uniquely specified by the survival probability, $p^s(t) = e^{-\int_0^t dt' r(t')}$, that the process is not yet completed at time t [24]. Then, for stationary (respectively, time-periodic) dynamics we obtain an uncertainty relation between the average duration of the process $\mathcal{T}$ and the mean (time-averaged) dissipation rate in the reservoirs $\langle \dot{S}_e \rangle$ (respectively, $\overline{\langle \dot{S}_e \rangle}$). This novel speed limit applies to any process that is exponentially more likely than its reverse, defined by the time-reversed observable $\tilde{O}$.

We start by considering stochastic trajectories ω_t of duration t—a list of states $x_{t'}$ with $t' \in [0, t]$—in a space Ω_t with a stationary probability measure $P(\omega_t) = P(\omega_t|x_0)\rho(x_0)$, with $\rho(x_0)$ the stationary probability density of state x_0. We can think of ω_t as a diffusion or a jump process describing a nonequilibrium system subjected to the action of nonconservative forces which may be mechanical or generated by reservoirs with different temperature or chemical potentials, for instance. Also non-Markovian dynamics [25–27] or unravelled quantum trajectories of open systems [28] may fit into the following framework. We introduce the stopping time $\tau := \inf\{t \geq 0 : O(\omega_t) \in D\}$ as the minimum time for an observable $O : \Omega_t \to \mathbb{R}$ to reach values belonging to a specific domain $D \subset \mathbb{R}$, loosely referred to as "threshold". The observable O, the threshold D, and the time τ, define the physical process and its duration. For concreteness, τ may be the minimum time to displace a mass, or to exchange a given amount of energy or particles with a reservoir. In general, it represents the time needed for a specific physical process to be carried out by the system.

We next identify the space of "survived" trajectories at time t, Ω_t^s, such that if $\omega_t \in \Omega_t^s$ then $O(\omega_t) \notin D$. They correspond to trajectories in which the process is not completed. The associated probability that the process is not yet completed at time t is expressed by the survival probability $p^s(t) \coloneqq \mathrm{Prob}(\tau > t)$, which satisfies $p^s(0) = 1$ by definition. We can formally write it as

$$p^s(t) = \sum_{\omega_t \in \Omega_t^s} P(\omega_t) = \sum_{\omega_t \in \Omega_t} \chi(O(\omega_t)) P(\omega_t) \tag{1}$$

where $\chi(O(\omega_t))$ equals 1 if $O(\omega_{t'}) \notin D$ for all $t' \leq t$ and zero otherwise.

We then consider the (involutive) transformation $\omega_t \mapsto \tilde{\omega}_t$ that time reverses the order of the states $x_{t'}$ (possibly changing sign, according to their parity). This allows us to define the log ratio

$$k_B \log \frac{P(\omega_t)}{P(\tilde{\omega}_t)} \eqqcolon \int_0^t dt' \dot{\Sigma}(\omega_t). \tag{2}$$

Here $P(\tilde{\omega}_t) = P(\tilde{\omega}_t | \tilde{x}_0) \rho(\tilde{x}_0)$ is the probability measure of time-reversed trajectories evolving with the original dynamics and starting from the stationary probability distribution $\rho(\tilde{x}_0)$. If the dynamics obeys local detailed balance [29,30], $\dot{\Sigma} = \dot{S}_e + dS/dt'$ is the entropy production rate at time t', which splits into the entropy flux in the reservoirs, $\dot{S}_e$, plus the time derivative of the system entropy $S = -k_B \log \rho(x_{t'})$.

Applying the time reversal to Eq. (1) and using Eq. (2) we find

$$p^s(t) = \sum_{\omega_t \in \Omega_t} \chi(O(\tilde{\omega}_t)) e^{-\int_0^t dt' \dot{\Sigma}(\omega_t)/k_B} P(\omega_t) \tag{3}$$

after the relabeling $\tilde{\omega}_t \to \omega_t$. Notice that the sum in Eq. (3) is restricted by $\chi(O(\tilde{\omega}_t))$ to a subset of trajectories $\tilde{\Omega}_t^s$ which differs from Ω_t^s if $\tilde{O}(\omega_t) \coloneqq O(\tilde{\omega}_t) \neq O(\omega_t)$. This defines a different process, named the reverse process, whose associated survival probability is $\tilde{p}^s(t) \coloneqq \sum_{\omega_t \in \Omega_t} \chi(\tilde{O}(\omega_t)) P(\omega_t)$. Hence, we arrive at the modified integral fluctuation relation

$$p^s(t) = \tilde{p}^s(t) \langle e^{-\int_0^t dt' \dot{\Sigma}/k_B} \rangle_{\tilde{s}}. \tag{4}$$

Hereafter, $\langle F \rangle_{\tilde{s}} \coloneqq \sum_{\omega_t \in \tilde{\Omega}_t^s} F(\omega_t) P(\omega_t) / \tilde{p}^s(t)$ denotes the normalized average of the generic observable F on the set of survived trajectories $\tilde{\Omega}_t^s$. One should note that Eq. (4) appears in implicit form in Refs. [31–33] as a generalized fluctuation theorem holding when a subset of forward trajectories have no time-reversal equivalent [34]. Our crucial new ingredient is to define Ω^s and $\tilde{\Omega}^s$ via the choice of an observable and a threshold, and to assign stopping times to trajectories in that subset.

As a consequence of Jensen's inequality, Eq. (4) yields

$$p^s(t) \geq \tilde{p}^s(t) e^{-\int_0^t dt' \langle \dot{\Sigma} \rangle_{\tilde{s}} / k_B}, \tag{5}$$

which gives a bound on the pace at which the two processes proceed. Note that the theory does not impose any constraint on processes defined by time symmetric observables, i.e., $\tilde{O} = O$. For them, p^s and $\tilde{p}^s$ coincide and they drop out of Eq. (5). Still Eqs. (4) and (5) give nontrivial results, i.e., the integral fluctuation theorem and the positivity of entropy production for survived trajectories, respectively.

Since survival probabilities are positive and monotonically decreasing, one can define the instantaneous rate $r(t)$ of the process as [35]

$$r(t) \coloneqq -\frac{1}{p^s(t)} \frac{dp^s}{dt}(t), \tag{6}$$

and analogously for $\tilde{r}(t)$, which satisfy the bound

$$\frac{1}{k_B} \int_0^t dt' \langle \dot{\Sigma} \rangle_{\tilde{s}} \geq \int_0^t dt' (r - \tilde{r}). \tag{7}$$

Further assuming $r(t) \gg \tilde{r}(t)$, for $t \ll \min_t 1/\tilde{r}(t)$ we can set $\tilde{p}^s(t)$ equal to 1 and approximate the entropy production rate with the (constant) mean entropy flux of the stationary dynamics, $\langle \dot{\Sigma} \rangle_{\tilde{s}} \simeq \langle \dot{\Sigma} \rangle = \langle \dot{S}_e \rangle$. Therefore, Eq. (7) gives our first key result: the entropy flux is an upper bound on the time-averaged rate $\bar{r}(t) \coloneqq (1/t) \int_0^t dt' r$ of the process,

$$\langle \dot{S}_e \rangle / k_B \geq \bar{r}(t). \tag{8}$$

Furthermore, when the mean time of the process exists, i.e., when $\langle \tau \rangle = \int_0^\infty dt p^s(t) \eqqcolon \mathcal{T}$ is finite [24], Eq. (8) can be turned into a bound on $\mathcal{T}$. Assuming that the reverse process is negligible for all times in which $p^s(t)$ decays, integration of Eq. (5) yields our main result: the dissipation-time uncertainty principle,

$$\langle \dot{S}_e \rangle \mathcal{T} \geq k_B. \tag{9}$$

The fundamental implication of this result is that to realize a nonequilibrium process in a given (average) time $\mathcal{T}$ at least $k_B/\mathcal{T}$ must be dissipated in the reservoirs. The only requirement is that the reverse process is much rarer. This can happen even close to equilibrium for large thresholds, i.e., when the domain D can be reached only by atypical fluctuations; or in the presence of weak noise, which is typically the case for problems described by transition state theory, or by macroscopic fluctuation theory where fluctuations are exponentially suppressed in the system size. We will illustrate these two cases on paradigmatic models.

The first example represents overdamped particle transport in one spatial dimension. The dynamics follows the Langevin equation

$$\dot{x} = -U'(x) + \sqrt{2/\beta}\xi \tag{10}$$

with periodic boundary conditions in $x \in [0, 2\pi]$. Here, $(k_B\beta)^{-1}$ denotes temperature, ξ is a zero-mean Gaussian white noise of unit variance, and $U(x) = a\cos(x) - fx$ is a periodic potential superimposed to a constant nonconservative tilt $f > 0$. This model describes a wealth of transport processes ranging from loaded molecular motors [36] to electrons across Josephson junctions [37]. Equations (8) and (9) apply in the stationary regime for the process of transporting the coordinate x over $N > 0$ periods, i.e.,

$$O = \int_0^t \dot{x}_{t'} dt', \qquad D = [2\pi N, \infty). \tag{11}$$

The reverse process, actuated by the trajectories realizing a negative current $\tilde{O} = -2\pi N$, is much rarer for large N or β on the time scales relevant to the processes Eq. (11).

For large N, the rates $r(t)$ and $\tilde{r}(t)$ are both negligibly small at short time, when the processes can only be realized by large fluctuations of ξ, whereas $r(t) \gg \tilde{r}(t)$ at times $t \gtrsim 2\pi N/f$ thanks to the ballistic transport induced by the tilt f. Thus, Eqs. (8) and (9) apply with the stationary entropy flow given by $\langle \dot{S}_e \rangle = k_B f \beta \langle \dot{x} \rangle$, where the mean velocity $\langle \dot{x} \rangle$ and the mean time $\mathcal{T}$ are analytically known [38] (see Fig. 1).

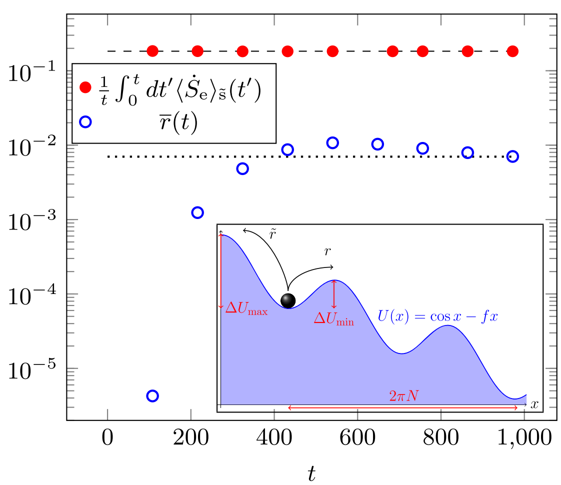

FIG. 1. Speed limit for the dynamics Eq. (10) with process Eq. (11), obtained by numerical averages over 10^4 trajectories with time steps $\Delta t = 10^{-2}$, $a = 1$, $\beta^{-1} = 0.7, f = 0.5$, $N = 11$, $k_B = 1$. Dashed and dotted lines correspond to the analytical value of $\langle \dot{S}_e \rangle$ and $1/\mathcal{T}$, respectively. Inset: sketch of the dynamics Eq. (10).

For β large with respect to the smaller energy barrier $\Delta U_{\min}$, escaping from the tilted potential well is a weak noise problem [39]. Namely, the process Eq. (11) unfolds with a constant rate, roughly estimated as the product of N Arrhenius factors, i.e., $r \sim e^{-N\beta\Delta U_{\min}}$ (see Fig. 2). Similarly, the reverse process takes place with rate $\tilde{r} \sim e^{-N\beta\Delta U_{\max}} \ll r$ (where $U_{\max}$ is the larger energy barrier), which is negligible for sufficiently large f. Even though the weak-noise estimation of r breaks down for $f \to 1$ (the value for which $\Delta U_{\min} \to 0$), the bounds Eqs. (8) and (9) hold true and they become tighter as the distance from equilibrium increases (see inset in Fig. 2). Note that in both cases, i.e., large (but finite) N and β, $\tilde{r} \to r$ as $f \to 0$, i.e., for detailed balance dynamics, so that Eq. (7) cannot be simplified to Eq. (9).

The second example represents energy transfer between two heat baths (at inverse temperatures $k_B\beta_h$ and $k_B\beta_c$, respectively) mediated by a two-level system. The latter performs Markovian jumps (corresponding to Poisson processes $dN^{\nu}_{i\to j}$) between the two states $i = \{1, 2\}$ of energy ϵ_i with rates $w^{\nu}_{i\to j} = e^{-\beta_\nu(\epsilon_j - \epsilon_i)/2}$ associated with the baths $\nu = \{h, c\}$. We define the process as the transfer of an energy E in a fixed time δ into the cold bath $\nu = c$:

$$O = (\epsilon_2 - \epsilon_1) \int_0^\delta dt' \left[\frac{dN^c_{2\to 1}}{dt'} - \frac{dN^c_{1\to 2}}{dt'}\right], \qquad D = [E, \infty). \tag{12}$$

One may think of E as an activation energy (e.g., of reaction [40]) and of δ as the timescale over which it may be dissipated. For large δ^{-1} and/or $E > 0$ (with respect to the

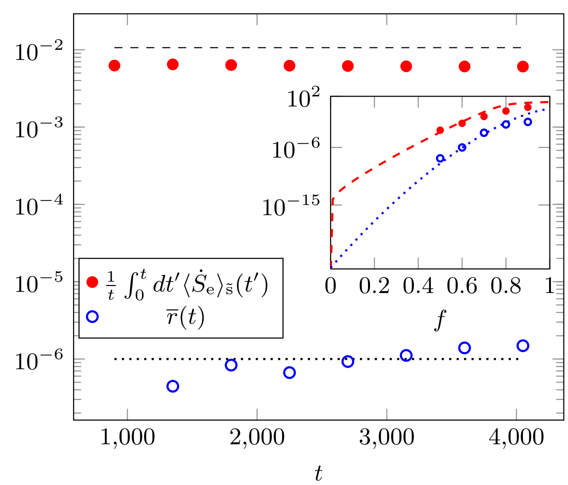

FIG. 2. Speed limit for the dynamics Eq. (10) with process Eq. (11), obtained by numerical averages over 10^4 trajectories with time steps $\Delta t = 10^{-2}$, $a = 1$, $\beta^{-1} = 0.07, f = 0.6, N = 2$, $k_B = 1$. Dashed and dotted lines correspond to the weak-noise estimation of $\langle \dot{S}_e \rangle$ and $r = 1/\mathcal{T}$, respectively. Inset: the time averaged values of $\langle \dot{S}_e \rangle_s$ (filled) and r (empty) as a function of the tilt f. At short times $t \lesssim 10^3$ and small tilt $f \lesssim 0.4$ the numerical estimation of r is impeded by the finite statistics.

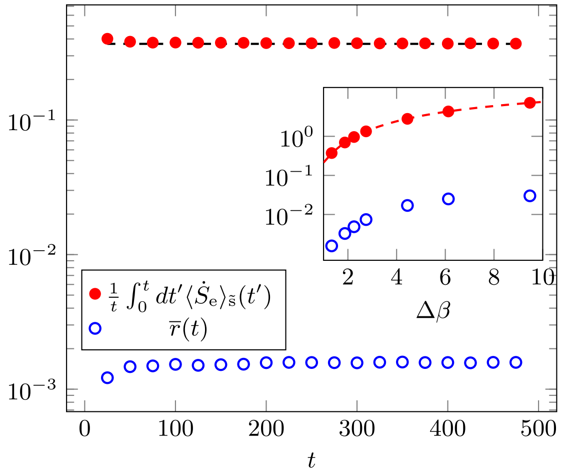

FIG. 3. Speed limit for the two-state system of the main text with process Eq. (12), obtained by numerical averages over 10^5 Gillespie trajectories with $1/\beta_h = 1.5, 1/\beta_c = 0.5, \epsilon_2 = 1, \epsilon_1 = 0, E = 5, \delta = 6, k_B = 1$. The dashed line corresponds to $\langle \dot{S}_e \rangle$. At short times $t \lesssim 50$ the numerical estimation of $r(t)$ is impeded by the finite statistics. Inset: time-averaged value of $\langle \dot{S}_e \rangle_{\tilde{s}}$ (filled) and r (empty) as function of $\Delta\beta \coloneqq \beta_c - \beta_h$ at fixed average temperature $(1/\beta_c + 1/\beta_h)/2 = 1$.

rates $w^{\nu}_{i\to j}$ and the energy gap $\Delta\epsilon \coloneqq |\epsilon_1 - \epsilon_2|$, respectively) the process is realized by large fluctuations and, therefore, is rare. The rate of the reverse process, defined by extracting an energy larger than E in the time δ from the cold reservoir, is in comparison negligible. Equation (9) holds true for rate r and the entropy flow $\langle \dot{S}_e \rangle_{\tilde{s}} \simeq \langle \dot{S}_e \rangle = k_B(1/\beta_c - 1/\beta_h)(e^{(\Delta\epsilon\beta_h/2)} - e^{(\Delta\epsilon\beta_c/2)})(e^{(\Delta\epsilon\beta_h/2)+(\Delta\epsilon\beta_c/2)} + 1)^{-1}$, which are found to be constant (Fig. 3).

These two examples suggest that currents are natural observables to which our theory applies. For integrated currents, i.e., $\tilde{O}(\omega_t) = -O(\omega_t)$, Eq. (9) can be directly derived from the steady state fluctuation theorem [34]. For such observables, Eq. (9) can be retrieved as a special case of a bound derived in Ref. [41] for the mean time required to identify time-reversal breaking in stationary *Markovian* dynamics. Also, a result complementary to ours based on large deviations theory is known, which holds when O is an integrated current [20] (respectively, counting observable [21]) scaling linearly with t. It *lower* bounds $\mathcal{T}^2$ by the variance of τ times the entropy flux (respectively, the dynamical activity). Our theory, instead, provides an upper bound only based on the entropy flux that is not restricted to Markovian dynamics or to currents extensive in the trajectory duration t.

Moreover, our approach can be extended beyond stationarity. Systems subject to time-dependent driving (with period t_d) can be treated with a slight modification of the above derivation which includes time reversal of the driving protocols $\lambda(t') \mapsto \lambda(t - t')$ both in the dynamics and in the initial probability of time-reversed trajectories [42], which we take as the periodic steady state $\rho_{\lambda(t)}(x_t)$ at the final value of the protocol $\lambda(t)$. Then, Eq. (7) generalizes to

$$\frac{1}{k_B}\int_0^t dt' \langle \dot{\Sigma} \rangle^B_{\tilde{s}} \geq \int_0^t dt' (r_F - \tilde{r}_B), \tag{13}$$

which bounds the difference of the rates of the forward (r_F) and backward (r_B) process with the dissipation in the backward dynamics. Assuming again that the backward process is much rarer, Eq. (13) integrated over n driving periods gives $\overline{\langle \dot{S}_e \rangle^B}/k_B \geq \overline{r_{Fn}}$ which bounds the time-averaged rate $\overline{r_{Fn}} \coloneqq (1/nt_d)\int_0^{nt_d} dt' r_F(t')$ in terms of the time-averaged entropy flux $\overline{\langle \dot{S}_e \rangle^B} \coloneqq (1/t_d)\int_0^{t_d} dt' \langle \dot{S}_e \rangle^F$. When the driving period t_d is much shorter than the inverse of the time-averaged rate, i.e., $t_d \ll 1/\overline{r_{Fn}}$ for all n, it follows the uncertainty relation

$$\overline{\langle \dot{S}_e \rangle^B}\mathcal{T} \geq k_B, \tag{14}$$

for the mean time of the forward process $\mathcal{T}$. Note that for time-symmetric protocols, $\lambda(t) = \tilde{\lambda}(t)$, forward and backward dynamics coincide.

The derivation still holds in the most general case of driven transient dynamics choosing the solution of the forward dynamics at time t as the initial probability of the time-reversed trajectories. For times in which the backward process is negligibly rare, we find

$$\frac{1}{k_B}\int_0^t dt' (\langle dS/dt' \rangle^B + \langle \dot{S}_e \rangle^B) \geq \int_0^t dt' r_F. \tag{15}$$

The time derivative of the Shannon entropy appearing in Eq. (15) is essential to bound the instantaneous rate during transients. For example, the rate of the process Eq. (11) for the dynamics Eq. (10) with $a = 0$, starting from a sharply peaked probability, has the long-time asymptotics $r(t) \sim 1/(2t) + f^2\beta/4 = (dS_G/dt + \langle \dot{S}_e \rangle/4)/k_B$, with $S_G = \log(4\pi e t/\beta)/2$ the Shannon entropy of the transient Gaussian probability of drifted diffusion [34]. This result shows that our most general bound [Eq. (15)] can be rather tight. In fact, while our speed limits assert that a large dissipation allows for a fast process, they do not imply that increasing dissipation will necessarily speed up the process. As for thermodynamic uncertainty relations [7,8], kinetics aspects of the dynamics are essential to determine the tightness of the bound [43].

Finally, our method can be generalized to processes such that $\Omega^s = \tilde{\Omega}^s$, by using an appropriate auxiliary dynamics in Eq. (3). This general strategy yields system-specific bounds for processes that break symmetries other than time reversal (such as reflection). For example, for the dynamics Eq. (10) with $a = 0$ (not invariant under $f \mapsto -f$) between two absorbing boundaries (for which $\Omega^s = \tilde{\Omega}^s$), the bound

$r \leq \langle \dot{S}_e \rangle / 4k_B$ can be derived by using an auxiliary dynamics with zero drift f [34]. This retrieves a known result for open Hamiltonian systems in which large-scale particle leakage is compatible with a drifted diffusive process [44–46] and thus suggests an interesting link to the escape-rate theory of deterministic dynamical systems.

In summary, resorting to the concept of first passage time we have defined the physical process that a stochastic system can perform, and irrespective of many details of the stochastic dynamics, we have shown that its rate is upper bounded by the entropy production rate. In particular, the integrated rate of stationary (respectively, periodic) processes, that do not involve (respectively, any net) transformation of states, is bounded solely by the (time-averaged) entropy flux in the reservoirs. These results call for an extension of stochastic thermodynamics to systems with escape, also given the recent work on the second law at stopping times [23], and the renewed fundamental interest in unstable dynamics [47,48]. Also, they show manifestly, together with the recent thermodynamic uncertainty relations, how thermodynamics constrains nonequilibrium dynamics and they hint at a general emerging tradeoff between speed, precision, accuracy, and dissipation [49–52], in which the role of information [53–56] only awaits to be explicitly uncovered.

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