Quantum Noise Correlations of an Optical Parametric Oscillator Based on a Nondegenerate Four Wave Mixing Process in Hot Alkali Atoms

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We present the first measurement of two-mode squeezing between the twin beams produced by a doubly resonant optical parameter oscillator (OPO) in an above threshold operation based on parametric amplification by nondegenerate four wave mixing with rubidium (^{85}Rb) . We demonstrate a maximum intensity difference squeezing of −2.7 dB (−3.5 dB corrected for losses) with a pump power of 285 mW and an output power of 12 mW for each beam, operating close to the D1 line of Rb atoms. The use of open cavities combined with the high gain media can provide a strong level of noise compression and the access to new operation regimes that could not be explored by crystal based OPOs. The spectral bandwidth of the squeezed light is broadened by the cavity dynamics, and the squeezing level is robust for strong pump powers. Stable operation was obtained up to 4 times above the threshold. Moreover, operation of the OPO close to the atomic resonances of alkali atoms allows a natural integration into quantum networks, including structures such as quantum memories.

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The generation of quantum correlated fields is a fundamental resource for developing a quantum network for quantum communication and quantum computation [\[1\]](#page-4-1). In the context of continuous variables, optical parametric oscillators (OPOs) have become the keystone to engineer correlated and entangled light beams on solid-state platforms, both below [\[2\]](#page-4-2) and above [\[3\]](#page-4-3) the OPO oscillation threshold. Further interest also comes from the development of quantum correlated light sources in optical chips [\[4\]](#page-4-4). OPOs have also been used to create large ensembles of multimode entangled fields [\[5](#page-4-5)–8]. Moreover, it would be interesting to have these sources operating at wavelengths that are compatible with alkali atoms, which are good candidates for quantum memories or registers [\[9\].](#page-4-6)

On the other hand, it was shown that alkali atoms in vapor cells can generate quantum correlated beams at atomic wavelengths with high quantum correlations [\[10\]](#page-4-7) by parametric amplification using four wave mixing (4WM) based on the third order nonlinearity $\chi^{(3)}$. The operation requires a relatively strong pump power beam and an extra seed produced by an acousto-optical modulator. The high gain (from 2 to 20 fold) of this parametric process is used on the phase insensitive amplification of the Stokes and anti-Stokes fields [\[11\].](#page-4-8) It is much higher than the typical gain obtained from parametric amplification by $\chi^{(2)}$ or $\chi^{(3)}$ processes in crystals and chips.

The combination of the high gain amplifier with a cavity could lead to interesting dynamics provided by the low threshold power that could be obtained or the study of extreme regimes of open cavities or strong pump operation. However, the best gain is reached with a noncollinear coupling between the pump and Stokes (anti-Stokes) probe field in order to satisfy the optimal phase matching condition. Turnbull et al. studied a range of angles for a proper phase matching condition to obtain high gain on a typical 4WM process [\[12\]](#page-4-9). The quantum correlations in a 4WM are drastically reduced for smaller angles between the probe and conjugate beams, and it is shown that, below 2 mrads, the intensity difference squeezing is lost [\[13\].](#page-4-10)

OPOs using 4WM in atoms have been recently reported [\[14](#page-4-11)–16] that employ a vapor cell with natural abundance within a cavity to run an OPO above threshold with twin beams separated by 6.1 GHz (for $85Rb$) and 13.6 GHz (for ⁸⁷Rb). The first evidence of quantum correlated fields in this type of setup was obtained in a double-pump scheme [\[17\]](#page-4-12) that generated weak fields slightly above the oscillation threshold [\[18\]](#page-4-13). For the continuous variables of intense fields, twin beams were observed for a seeded single resonant OPO with an open cavity for the conjugate mode [\[19\]](#page-4-14). Nevertheless, there have not been demonstrations of quantum correlations in self-oscillating cavities. In fact, Refs. [\[14,16\]](#page-4-11) reported measurements of intensity noise correlations, but they were not strong enough to reach the quantum limit.

Our purpose is to demonstrate the generation of quantum correlated beams from a doubly resonant OPO above the oscillation threshold using a $\chi^{(3)}$ interaction with a nondegenerate 4WM process that employs a hot vapor cell of alkali atoms within a cavity.

FIG. 1. (a) Sketch of the experimental setup. PBS indicates polarizing beam splitter cube, HWP half wave plate, UMZ unbalanced Mach-Zehnder interferometer, M mirror, FP confocal Fabry-Perot, FP flip-mirror, and SA spectrum analyzer. (b) Interferometer output analyzed by an FP cavity with R, S, and I as reference, signal, and idler, respectively. Inset: spectral separation of the signal and idler beams.

The experimental setup is shown in Fig. [1,](#page-1-0) and a detailed description is presented in the Supplemental Material [\[20\]](#page-4-15). We employ a bow-tie cavity with high reflectivity mirrors and a free spectral range (FSR) of 404.7(3) MHz, which is an odd integer fraction of the frequency separation between the twin beams. The vapor cell has antireflection coated windows and is kept at 90 °C for high optical density. The pump beam is collinear with the cavity mode and is injected by polarizing beam splitter $PBS₁$ and removed by $PBS₂$. After $PBS₂$, we use a half wave plate (HWP) and a third PBS (PBS_3) in order to control the output coupling of the cavity. By changing the orientation of the wave plate, the cavity finesse ranges from 5 to 30 for a field far from the atomic resonance.

The pump beam is generated by a Ti:sapphire laser tuned to the D1 line of Rb at 795 nm and with maximum power of 800 mW. The laser beam is locked to the blue of the $5^{2}S_{1/2}F = 2 \rightarrow 5^{2}P_{1/2}F = 3$ transition with an adjustable detuning.

Since the twin beams have the same polarization, we use an unbalanced Mach-Zehnder interferometer [\[21\]](#page-4-16). The interferometer visibility gives us 98% separation efficiency for the two modes from the OPO. This can be verified by using a confocal Fabry-Perot cavity (FP) with a 1.5 GHz FSR [see inset Fig. [1\(b\)](#page-1-0)]. Taking an injected reference field (R) from the pump beam, we can adjust the length of the long path for a constructive interference of the signal (S) or the idler field (I), while verifying that their frequency

FIG. 2. Output power from the OPO for a cavity length scan, with T = 91 °C, $\Delta = 1$ GHz, $\sigma = 1.8$ for a finesse of (a) $\mathcal{F} = 14$ and (b) $\mathcal{F} = 30$.

shift corresponds to the hyperfine splitting $(\pm 3.035 \text{ GHz})$. We have an overall detection efficiency of 83% for the whole system, accounting for optical losses and photodetector quantum efficiency, and calibrate the shot-noise level using the pump laser [\[22\].](#page-4-17)

Once the cavity is aligned, oscillation can be observed for sufficient pump power. During the scanning of the cavity length, whenever a doubly resonant condition for signal and idler beam is achieved, there is an intense output on $PBS₃$ (Fig. [2](#page-1-1)). The sudden transition of the intensity while scanning the cavity length shows an abrupt threshold for the oscillation. Figure $2(a)$ and $2(b)$ show, respectively, the recorded output for two different cavity finesses: $\mathcal{F} =$ 14 and $\mathcal{F} = 30$ at the same normalized pump power $\sigma = P/P_{th}$ (where P_{th} is the oscillation threshold pump power at exact resonance).

The asymmetry on the peak is a consequence of the selfphase modulation associated with the $\chi^{(3)}$ nonlinearity. Close to cavity resonance, the intensity leads to a change of the field phase, thus either increasing or decreasing the absolute value of the cavity detuning. For a sufficiently high finesse (or for enough intracavity power), a bistable operation is observed as a consequence of the atom-cavity cooperativity. For a two-level system, this cooperativity is given by $C = g^2 N / \gamma_{\text{cav}} \Gamma$ with g as the atom-light coupling coefficient, N the number of atoms, γ_{cav} the cavity loss rate, and Γ the atomic spontaneous emission [\[23\]](#page-4-18). Notice that the increase of finesse, which reduces γ_{cav} , enhances the atomcavity cooperativity, leading to the steep bistability of Fig. [2\(b\)](#page-1-1) with respect to Fig. [2\(a\)](#page-1-1). We work out of the bistability regime by carefully choosing the value of \mathcal{F} .

The high gain of the medium allows the oscillation with relatively large intracavity losses (on the order of 30%) and a controllable tuning of the optical coupling, which allows us to have a fine control of the OPO threshold. Figure [3](#page-2-0) shows the total output power as a function of the input pump power for three different values of finesse. One can observe that the threshold power increases from 109 to 221 mW as the finesse of the cavity is reduced from $\mathcal{F} = 19$ to $\mathcal{F} = 10$. That is significantly smaller than the typical operational condition of OPOs using $\chi^{(2)}$ media [\[3\]](#page-4-3).

FIG. 3. Total output power as a function of the input pump power for three different values of finesse \mathcal{F} . T = 91 °C, $\Delta = 0.82$ GHz. The linear fit is just a guide to the eye.

This condition could be reached due to the high gain of the medium, typically from 100 to 500% gain, going up to 2000% in some cases [\[11\].](#page-4-8) In the oscillation regime, we should reach the saturation of the gain, matching the cavity losses [\[24\]](#page-4-19). The high gain allows the operation with extremely lossy cavities. The nearly linear dependence agrees with the model developed in [\[24\]](#page-4-19).

Avoiding the strong bistable condition, we could lock the cavity to the resonance peak, obtaining stable operation of the OPO output with power ranges from 1 to 40 mW, maintaining a single spatial mode. This is at least 10 times higher than the output power from free-space 4WM parametric amplification, which typically runs at ≤ 1 mW [\[25\]](#page-4-20). The output power can be increased up to 100 mW or more by increasing the pump power; nevertheless, this leads to the excitation of higher transverse electromagnetic modes, whose study is not within the scope of this Letter.

As we generate pairs of beams with nearly equal average intensities, we look for the twin beam generation by looking at the noise spectrum of the intensity difference. Figure [4](#page-2-1) shows the normalized intensity noise spectra for the output beams of the OPO after subtraction of electronic noise. Curve 1 shows the noise of the intensity difference

FIG. 4. Intensity noise spectra of the output beams of the OPO, normalized to the shot-noise level, for the subtraction of the photocurrents (1), signal (2), and idler (3) beams. $T = 91 \degree C$, $\Delta = 0.82$ GHz, $\mathcal{F} = 15$, $\sigma = 1.8$, $P_{th} = 159$ mW.

between the probe and the conjugate beams, showing a maximum two-mode intensity difference squeezing of $-2.7(1)$ dB with respect to the shot-noise level at 2 MHz for a pair of output fields with 12 mW of power for each beam. Experimental data can be compared to the simplest model for twin photon production inside a leaky cavity [\[26\]](#page-4-21) that is consistent with the model presented in [\[24\]](#page-4-19), corresponding to a Lorentzian profile as

$$
S(f) = 1 - \eta \frac{1}{1 + (f/BW)^2},\tag{1}
$$

where BW stands for the cavity bandwidth and η is the efficiency of the escape ratio of the photon through the output coupler $\eta = L_c/(L_c + L_i)$, where L_c is the output coupler loss and L_i corresponds to intrinsic cavity losses (adding up to 17% in the present case). The resulting curve gives a maximum noise compression of $-2.84(16)$ dB with a bandwidth of 16.1 (1) MHz, significantly smaller than the cavity bandwidth of 26 MHz.

The match of this simple model to the current result is quite surprising if we consider that the noise spectra of the twin beams generated from parametric amplification on atomic vapor can have rich spectra [\[27\]](#page-4-22), with twin beam correlation shifting from squeezing to excess noise in the frequency range that is of the order of the atomic spectral linewidth of 6 MHz for the atom [\[10\],](#page-4-7) depending on power broadening as well. The resulting profile is a consequence of the interplay of an amplifier with a variable gain bandwidth with a fixed cavity bandwidth, which cannot be fully accounted for by simplified models where a broadband amplifier is considered [\[24\]](#page-4-19).

Curves 2 and 3 in Fig. [4](#page-2-1) show the normalized intensity noise for each beam. As expected, the noise for each beam presents excess of noise and should be subjected to a more detailed treatment. Nevertheless, some distinctive features appear in this situation. The noise rapidly diverges for small analysis frequency, consistent with the cavity dynamics of an OPO, as a consequence of the phase diffusion between the converted fields [\[24\]](#page-4-19). This feature is absent in the case of the injected optical parametric amplifier (OPA) [\[10\]](#page-4-7), where the excess noise is not so sharp. That is also a consequence of the possibility of perfect squeezing of twin beams for lossless cavities leading to strong excess noise in each field that is different from the finite squeezing level that is expected from single pass in a parametric amplifier. Oddly enough, the noise of each beam goes down as the analysis frequency grows. In fact, it is expected that it could eventually evolve to noise compression for appropriate pump power [\[24\]](#page-4-19).

The present configuration provides a versatile tool to study the squeezing generation for different cavity couplings adjusted by the intracavity wave plate. It changes both the loss ratio of the photon pair and the cavity bandwidth. Amplifying gain could be changed as well

FIG. 5. Noise intensity difference as a function of the input pump power normalized by the threshold power σ for three different cavity finesses and an analysis frequency of 7 MHz. (a)–(c) correspond to three different pump detunings relative to the atomic line.

by the control of the pump detuning. Figure $5(a)$ –(c) show the squeezing level for pump detuning frequencies of $\Delta = 1.00$, 0.82, and 0.64 GHz. We perform this characterization for three different finesse values and as a function of the pump power above threshold. We notice that, consistent with the expected results from either a simplified [\[26\]](#page-4-21) or detailed model [\[24\]](#page-4-19), the squeezing level of the twin beams is insensitive to the pump power. This demonstrates also the robustness of the noise compression under depletion of the pump field, even in the doubly resonant condition. Other expected behavior is the reduction of the squeezing with reduced cavity coupling, which is consistent with the simple model shown in Eq. (1) .

A curious feature comes from the role of the amplifier gain. When the pump field is tuned closer to the atomic resonance at 0.64 GHz, the squeezing level is reduced to −1 dB due to the increased absorption in the atomic medium. On the other hand, since we have a higher gain, we could have lower threshold powers, and we demonstrate squeezing in values of σ as high as 4.5 times above the threshold. As for the squeezing level, the best result is obtained for a lower gain, obtained for a detuning of 1 GHz, with an open cavity, ranging from −2 to −3 dB. We expect that for reduced cavity losses the present configuration could provide a squeezing level at least as good as those obtained from the direct parametric amplification [\[11\]](#page-4-8).

FIG. 6. Noise spectra for different temperatures of the vapor cell. T = 91 °C (Curve 1), 96 °C (C2), 101 °C (C3), 108 °C (C4) with $\sigma = 1.8$, $\Delta = 0.82$ GHz.

While finesse and detuning with respect to the atomic transition seem to keep the quantumness of the outcoming fields within the range of parameters we have studied, the atomic density is a determinant factor for twin beam generation. Figure [6](#page-3-1) shows the OPO noise spectra for different temperatures of the vapor cell for the same normalized pump power σ . Notice that Curve 1 for 91 °C shows the maximum level of squeezing, and as the temperature is increased up to 108 °C, the quantum correlations are lost and eventually the intensity noise difference is above the shot noise. In other words, the increase of atomic density will increase the gain but should lead to an increase of the losses as well, which deteriorate the quantum correlations of the twin beams. This could be the parameter that inhibited the observation of quantum correlations in previous realizations [\[14,16\]](#page-4-11) since the temperature typically used is around 105 °C, which corresponds to a noise spectrum in between Curves 3 and 4 in Fig. [6](#page-3-1).

An interesting effect shown in Figs. [4](#page-2-1) and [6](#page-3-1) is a small peak that appears in the noise of each individual beam and is not suppressed by the subtraction. We performed a characterization of its frequency, and we have shown [\[20\]](#page-4-15) that it is proportional to the pump power. Although we lack a proper model for its origin, this dependence leads to an association with the AC Stark shift. Since it is narrow enough, its presence does not affect the overall noise profile of the OPO output.

The OPO based on 4WM with hot atomic vapor has shown a significant level of squeezing in the twin beams $(-2.7(1)$ dB, $-3.7(1)$ dB after correction for quantum efficiency) that could be immediately optimized with the design of a dedicated cavity. This design is simplified by the possibility of using a relatively high transmittance for the output coupler ($\approx 30\%$) while dramatically reducing the intracavity loss. Moreover, since we are free from thermal effects and defective absorption of typical nonlinear crystals, an extended operational range of pump power could be studied, reaching more that 4 times above threshold before transverse multimode operation shows up.

That allows the production of quantum correlation of very intense fields with more than 20 mW each, a situation very distinct from the usual OPA using this medium. Measurements above this power were limited by saturation on the photodetectors, so we do not rule out that squeezing should remain for even higher intensities. In the case of transverse multimode operation, an aperture in the smaller waist can be used to either select or manipulate those modes, leading to the production of fields featuring interesting quantum images.

The presented results agree with the model of a gain medium in an open cavity beyond the limit of weak coupling [\[24\]](#page-4-19). In this case, pump depletion could be fully considered, and it was shown that, while pump depletion does not affect the robust twin beam generation, it leads to interesting dynamics of the noise compression of each beam, including here the depleted pump. Some interesting features, such as the precise interplay between the bandwidths of the atomic amplifier and the cavity or the origin of the narrow peak whose frequency closely follows the proportionality to the pump, remain the subject of future studies.We may conclude that the success in the observation of quantum correlations in the present implementation came from the control of the atomic density, from operating at lower temperatures, and from the use of a ring cavity in order to avoid the effect of multiple coupling of propagating and counterpropagating modes in the atomic media. This system should provide a useful tool for the production of quantum correlated states close to atomic resonances, eventually leading to the observation of entanglement in such fields.

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- [1] M. A. Nielsen and I. L. Chuang, *Quantum Computation* and Quantum Information (Cambridge University Press, Cambridge, England, 2011).
- [2] Z. Y. Ou, S. F. Pereira, H. J. Kimble, and K. C. Peng, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.68.3663) 68, 3663 (1992).
- [3] A. S. Villar, L. S. Cruz, K. N. Cassemiro, M. Martinelli, and P. Nussenzveig, Phys. Rev. Lett. 95[, 243603 \(2005\).](https://doi.org/10.1103/PhysRevLett.95.243603)
- [4] A. Dutt, K. Luke, S. Manipatruni, A. L. Gaeta, P. Nussenzveig, and M. Lipson, [Phys. Rev. Applied](https://doi.org/10.1103/PhysRevApplied.3.044005) 3, [044005 \(2015\).](https://doi.org/10.1103/PhysRevApplied.3.044005)
- [5] M. Pysher, Y. Miwa, R. Shahrokhshahi, R. Bloomer, and O. Pfister, Phys. Rev. Lett. 107[, 030505 \(2011\)](https://doi.org/10.1103/PhysRevLett.107.030505).
- [6] O. Pinel, P. Jian, R. M. de Araujo, J. Feng, B. Chalopin, C. Fabre, and N. Treps, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.108.083601) 108, 083601 [\(2012\)](https://doi.org/10.1103/PhysRevLett.108.083601).
- [7] S. Yokoyama, R. Ukai, S.C. Armstrong, C. Sornphiphatphong, T. Kaji, S. Suzuki, J.-i. Yoshikawa, H. Yonezawa, N. C. Menicucci, and A. Furusawa, [Nat.](https://doi.org/10.1038/nphoton.2013.287) Photonics 7[, 982 \(2013\)](https://doi.org/10.1038/nphoton.2013.287).
- [8] M. Chen, N. C. Menicucci, and O. Pfister, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.112.120505) 112[, 120505 \(2014\).](https://doi.org/10.1103/PhysRevLett.112.120505)
- [9] A. Lvovsky, B. Sanders, and W. Tittel, [Nat. Photonics](https://doi.org/10.1038/nphoton.2009.231) 3, 706 [\(2009\).](https://doi.org/10.1038/nphoton.2009.231)
- [10] C. F. McCormick, V. Boyer, E. Arimondo, and P. D. Lett, Opt. Lett. 32[, 178 \(2007\)](https://doi.org/10.1364/OL.32.000178).
- [11] Q. Glorieux, L. Guidoni, S. Guibal et al., [Proc. SPIE](https://doi.org/10.1117/12.854953) Quantum Opt. 7727[, 772703 \(2010\).](https://doi.org/10.1117/12.854953)
- [12] M. T. Turnbull, P. G. Petrov, C. S. Embrey, A. M. Marino, and V. Boyer, Phys. Rev. A 88[, 033845 \(2013\)](https://doi.org/10.1103/PhysRevA.88.033845).
- [13] R. Ma, W. Liu, Z. Qin, X. Su, X. Jia, J. Zhang, and J. Gao, Opt. Lett. 43[, 1243 \(2018\)](https://doi.org/10.1364/OL.43.001243).
- [14] J. Okuma, N. Hayashi, A. Fujisawa, M. Mitsunaga, and K.-i. Harada, Opt. Lett. 34[, 698 \(2009\).](https://doi.org/10.1364/OL.34.000698)
- [15] X. Yu, M. Xiao, and J. Zhang, [Appl. Phys. Lett.](https://doi.org/10.1063/1.3295694) 96, 041101 [\(2010\).](https://doi.org/10.1063/1.3295694)
- [16] J. Sheng, H. Wu, X. Yang, U. Khadka, and M. Xiao, Opt. Lett. 37[, 1655 \(2012\)](https://doi.org/10.1364/OL.37.001655).
- [17] H. Wu and M. Xiao, Phys. Rev. A **80**[, 063415 \(2009\).](https://doi.org/10.1103/PhysRevA.80.063415)
- [18] The photon flux of each field, estimated as 10^7 photons/ sec, was measured with avalanche photodiodes, limiting the exploration of the quantum features to violations of Cauchy-Schwarz inequalities.
- [19] X. Pan, H. Chen, T. Wei, J. Zhang, A. M. Marino, N. Treps, R. T. Glasser, and J. Jing, [Phys. Rev. B](https://doi.org/10.1103/PhysRevB.97.161115) 97, 161115(R) [\(2018\).](https://doi.org/10.1103/PhysRevB.97.161115)
- [20] See Supplemental Material at [http://link.aps.org/](http://link.aps.org/supplemental/10.1103/PhysRevLett.125.083601) [supplemental/10.1103/PhysRevLett.125.083601](http://link.aps.org/supplemental/10.1103/PhysRevLett.125.083601) for details of the data analysis and the experimental setup.
- [21] E. H. Huntington, G. N. Milford, C. Robilliard, T. C. Ralph, O. Glöckl, U. L. Andersen, S. Lorenz, and G. Leuchs, Phys. Rev. A 71[, 041802\(R\) \(2005\).](https://doi.org/10.1103/PhysRevA.71.041802)
- [22] The shot-noise level is calibrated by using a fraction of the coherent pump field (red arrow in Fig. [1](#page-1-0)) with the same power as the twin beams. The reference beam is injected in the open port of $PBS₆$ to perform the balanced detection.
- [23] A. Lambrecht, E. Giacobino, and J.M. Courty, [Opt.](https://doi.org/10.1016/0030-4018(94)00493-E) Commun. 115[, 199 \(1995\)](https://doi.org/10.1016/0030-4018(94)00493-E).
- [24] B. A. F. Ribeiro, R. B. de Andrade, M. Martinelli, and B. Marques, companion paper, [Phys. Rev. A](https://doi.org/10.1103/PhysRevA.102.023522) 102, 023522 [\(2020\)](https://doi.org/10.1103/PhysRevA.102.023522).
- [25] A. M. Marino, V. Boyer, and P. D. Lett, *[Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.100.233601)* **100**, [233601 \(2008\).](https://doi.org/10.1103/PhysRevLett.100.233601)
- [26] S. Reynaud, [Eur. Phys. Lett.](https://doi.org/10.1209/0295-5075/4/4/008) 4, 427 (1987).
- [27] N. V. Corzo, Q. Glorieux, A. M. Marino, J. B. Clark, R. T. Glasser, and P. D. Lett, [Phys. Rev. A](https://doi.org/10.1103/PhysRevA.88.043836) 88, 043836 [\(2013\)](https://doi.org/10.1103/PhysRevA.88.043836).