

## Topological Nernst Effect of the Two-Dimensional Skyrmion Lattice

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The topological Hall effect (THE) and its thermoelectric counterpart, the topological Nernst effect (TNE), are hallmarks of the skyrmion lattice phase (SkL). We observed the giant TNE of the SkL in centrosymmetric Gd<sub>2</sub>PdSi<sub>3</sub>, comparable in magnitude to the largest anomalous Nernst signals in ferromagnets. Significant enhancement (suppression) of the THE occurs when doping electrons (holes) to Gd<sub>2</sub>PdSi<sub>3</sub>. On the electron-doped side, the topological Hall conductivity approaches the characteristic threshold  $\sim 1000$  ( $\Omega \text{ cm}$ )<sup>-1</sup> for the intrinsic regime. We use the filling-controlled samples to confirm Mott's relation between TNE and THE and discuss the importance of Gd-5d orbitals for transport in this compound.

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A skyrmion spin-vortex [1,2] represents a quantized unit of scalar spin chirality  $\chi_{\alpha\beta\gamma} = \mathbf{S}_\alpha \cdot (\mathbf{S}_\beta \times \mathbf{S}_\gamma)$ , defined for three neighboring magnetic moments on lattice sites  $\alpha$ ,  $\beta$ , and  $\gamma$ . It was realized early on that spin-winding results in an emergent gauge field acting on moving particles, leading to anomalies such as the topological (or geometrical) Hall effect (THE) and its sibling, the topological Nernst effect (TNE) [3–9]. The relative magnitude of the carrier mean-free path  $l_{\text{mfp}}$  as compared to the size of the skyrmion  $\lambda_{\text{sk}}$ , i.e., to the size of the magnetic unit cell, governs the appropriate starting point for theoretical modeling [10]. Well-known cases of noncentrosymmetric materials with a skyrmion lattice (SkL) phase, such as MnSi [2,11–13], fall into the regime  $l_{\text{mfp}} \ll \lambda_{\text{sk}}$ , and it is understood that the (weak) THE relates to a Berry-phase induced deflection of wave packets moving through the twisted spin texture in real space [6,13–15]. Meanwhile, the as-yet unexplored “intrinsic” (momentum-space) limit  $l_{\text{mfp}} \geq \lambda_{\text{sk}}$  necessitates a modification of the electronic wave functions themselves by the presence of magnetic order, which is predicted to yield a large THE and TNE due to Berry curvature in reciprocal space [5,16–20].

To describe the connection between TNE and THE, we write the electric currents  $\mathbf{J}$  emanating from an applied electric field  $\mathbf{E}$  or an applied temperature gradient ( $-\nabla T$ ) as  $J_i = \sigma_{ij} E_j$  and  $J_i = \alpha_{ij} (-\nabla_j T)$ , respectively. The TNE provides insight into the effect of a variation of the

chemical potential  $\zeta$ , as expressed by the Mott relation [21,22]

$$\alpha_{ij}/T = -(\pi^2/3)(k_B^2/e)(\partial\sigma_{ij}/\partial\epsilon)_{\epsilon=\zeta}, \quad (1)$$

where  $k_B$ ,  $e$  ( $>0$ ), and  $\epsilon$  represent the Boltzmann constant, the fundamental charge, and the band filling energy, respectively. Due to experimental constraints—such as relatively weak spin polarization and low skyrmion density in the ambient pressure, equilibrium SkL phases of chiral B20 compounds (skyrmion-skyrmion distance  $\sim 20$ – $200$  nanometers)—the TNE of a SkL has never hitherto been observed experimentally.

In this Letter, we report the large TNE of the SkL in centrosymmetric Gd<sub>2</sub>PdSi<sub>3</sub>, compare its magnitude to recently observed giant anomalous Nernst responses in ferromagnets, and show how the TNE is related to the enhancement (decrease) of the THE under electron (hole) doping through the Mott relation. We demonstrate that on the electron-doped side, the intrinsic limit of the THE may be within reach, and that the large transport responses in Gd<sub>2</sub>PdSi<sub>3</sub> are likely related to the prevalence of Gd-5d conducting orbitals in close proximity to the Fermi energy.

Rare-earth intermetallics with SkL phase, such as Gd<sub>2</sub>PdSi<sub>3</sub> [23], Gd<sub>3</sub>Ru<sub>4</sub>Al<sub>12</sub> [24], and GdRu<sub>2</sub>Si<sub>2</sub> [25] are highly suitable for investigating the transport response

from the emergent gauge field: a tiny vortex-vortex distance  $\lambda \sim 2\text{--}3$  nm leads to giant responses, strongly modifying the trajectory of moving charge carriers. We focus here on centrosymmetric, hexagonal  $\text{Gd}_2\text{PdSi}_3$  [Fig. 1(a)], where magnetic long-range order onsets at  $T_N \sim 20$  K. The dominant magnetic ion is  $\text{Gd}^{3+}$  in the triangular lattice plane. Dzyaloshinskii-Moriya interactions, which are intrinsic to noncentrosymmetric material platforms and which favor helical order and skyrmion spin textures, are expected to be globally absent in this centrosymmetric bulk crystal [23]. Instead, skyrmion formation is driven by frustrated interactions mediated through the conduction electrons [26,27] and remarkably, skyrmions were found to exist (for the magnetic field  $\mathbf{B} \parallel c$  axis) even at the lowest  $T = 2$  K, i.e., at  $T/T_N \sim 0.1$  [23]. Located between two phases with zero net scalar spin chirality [spiral-like IC-1 (possibly multi- $\mathbf{q}$ ) and the fanlike IC-2, Fig. 1(b)], the topologically stable SkL is bounded by sharp, first-order phase transitions as observed in the strongly hysteretic field-derivative of the magnetization [DC susceptibility  $\chi_{\text{DC}}$ , Fig. 1(d) see also Refs. [23,28]]. This phase alone was found to host enormous THE and TNE responses in our high-resolution transport experiments [Figs. 1(c), 1(e)–1(g), see Supplemental Material for technical details [29]], the Hall signal being in good agreement with previous work [23,28].

We consider cases where Nernst signals arising from noncoplanar spin arrangements have previously been reported: (i) Pyrochlore  $\text{Nd}_2\text{Mo}_2\text{O}_7$ , a canted ferromagnet where the signal is roughly proportional to the net magnetization at all  $T$  studied [62]. (ii) B-20 type, helimagnetic  $\text{MnGe}$ , where the THE and TNE probe the imbalance between positive and negative contributions to the gauge field. These originate from magnetic monopoles and antimonopoles, respectively [7,63]. (iii) Thin films of the Heusler alloy  $\text{Mn}_{1.8}\text{PtSn}$ , the magnetic structure of which was not verified independently [64]. In contrast to these cases, the Nernst effect from the SkL laid out here represents a minimal, textbooklike example of the thermoelectric response emerging from a spin texture with integer winding per magnetic unit cell.

The Nernst conductivity (sometimes referred to as transverse thermoelectric conductivity or transverse Peltier conductivity [29]) is calculated via  $\alpha_{xy} = S_{xy}\sigma_{xx} + S_{xx}\sigma_{xy}$ , requiring input from the longitudinal thermopower  $S_{xx}$  [Fig. 1(e)].  $S_{xx}$  is of comparable magnitude for the two low-field phases IC-1 and SkL; the strong difference of THE and TNE in the respective phases, as well as sharp maxima in  $\chi_{\text{DC}}$  [Fig. 1(d)], clearly distinguish these two states. Figures 1(f) and 1(g) show the experimentally obtained Nernst effect  $S_{xy}/T$  and Nernst conductivity  $\alpha_{xy}/T$  (entropy factors removed). Note that  $\alpha_{xy}/T$

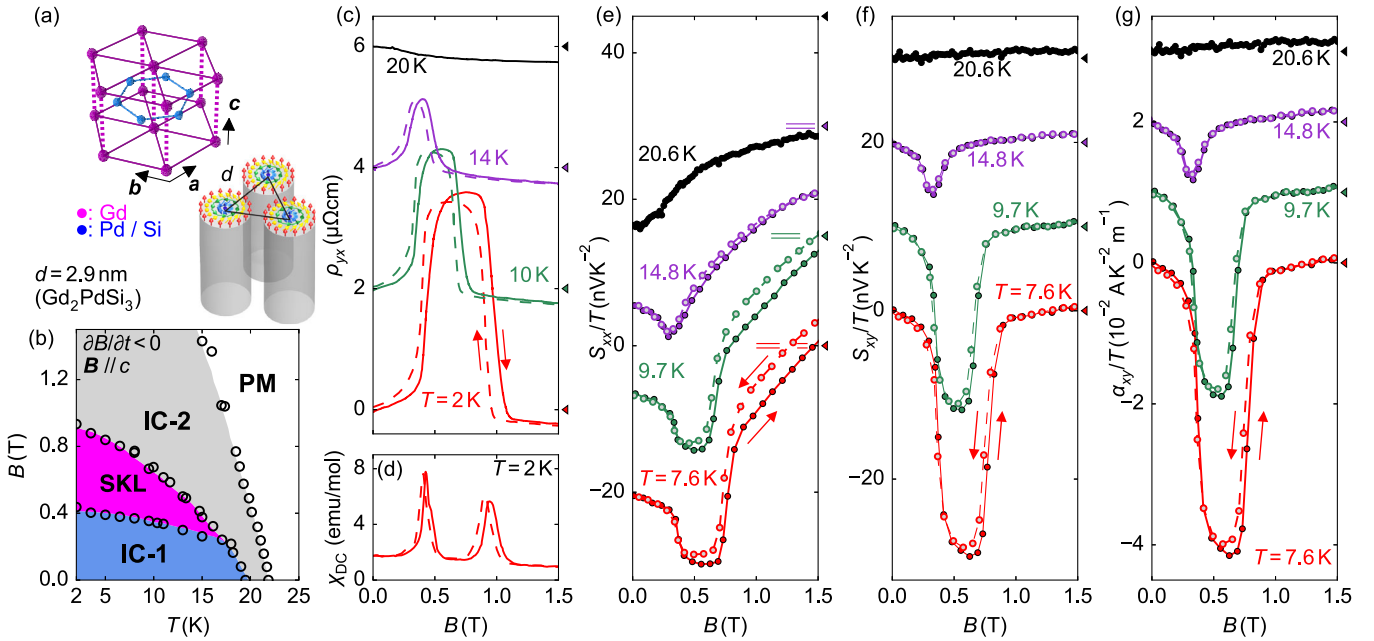


FIG. 1. (a) Hexagonal structure (basic  $\text{AIB}_2$ -type) of  $\text{Gd}_2\text{PdSi}_3$ . Inset: high-density SkL state with nanometer-sized interskyrmion distance  $d$ . (b) Magnetic phase diagram for  $\mathbf{B} \parallel c$  and decreasing field  $\partial B/\partial t < 0$ , as adapted from Ref. [61]. Labels indicate the IC-1 ground state, the skyrmion lattice phase, the IC-2 fanlike state, and the paramagnetic regime (PM). (c) Hall resistivity, (e) thermopower, (f) Nernst effect, and (g) Nernst conductivity. For the latter three, entropy factor  $\sim T$  was removed. (d) Magnetic susceptibility  $\chi_{\text{DC}}(B)$  at the lowest  $T = 2$  K. Curves in panels (c),(e)–(g) were shifted by vertical offsets of  $2 \mu\Omega \text{ cm}$ ,  $15 \text{ nV K}^{-2}$ ,  $10 \text{ nV K}^{-2}$ , and  $0.01 \text{ AK}^{-2} \text{ m}^{-1}$ , respectively. The zero level for each curve is indicated by a colored triangle at the right side of the panel. Solid arrows mark the direction of the field ramp. In (e), red, green, and purple twinned lines also indicate zero levels for  $T = 7.6, 9.7,$  and  $14.8$  K.

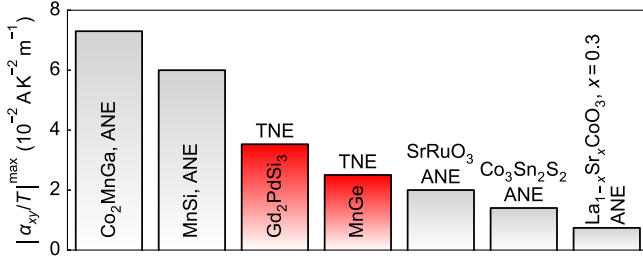


FIG. 2. Comparison of large Nernst conductivity  $\alpha_{xy}/T$  driven by magnetic order for various materials reported in the literature. Grey shading indicates anomalous Nernst response (ANE, proportional to the net magnetization  $M$ ), while red shading is reserved for the topological Nernst effect (TNE, proportional to the scalar spin chirality).

represents the intrinsic transverse thermoelectric response [21], while the derived observable  $S_{xy}$  depends strongly on  $T$  (via the entropy factor) and the resistivity, i.e., the scattering properties, of the material [65].

For perspective we compare, side-by-side in Fig. 2, representative magnetic materials with anomalous (ANE, i.e., proportional to the net magnetization  $M$ ) or topological (TNE, proportional to the scalar spin chirality)  $\alpha_{xy}/T$ . Although the TNE of  $\text{Gd}_2\text{PdSi}_3$  cannot match the ultralarge ANE exhibited by the ferromagnetic Weyl semimetal  $\text{Co}_2\text{MnGa}$  [66], it easily outperforms the ANE of more metallic ferromagnets, e.g.,  $\text{SrRuO}_3$  [67] or elemental Fe and Co, for which only sparse data [68] and calculated values [69] are available at present [29].

The large TNE signal with negative sign indicates, from Eq. (1), that significant enhancement of the THE should be observed when moving the chemical potential  $\zeta$  of  $\text{Gd}_2\text{PdSi}_3$  upwards. Knowing that Pd-4d orbitals are located about 4 eV below  $\zeta$  [70], a gentle shift of  $\zeta$  may be achieved using a series of slightly carrier-doped crystals  $\text{Gd}_2(\text{Pd}_{1-x}\text{M}_x)\text{Si}_3$  with  $M = \text{Rh}$  (hole doping) or Ag (electron doping), as shown in Fig. 3. We define a generalized nominal dopant concentration  $z = -x$  on the hole-doped and  $z = x$  on the electron-doped side. The single crystals were characterized thoroughly using solid state techniques [29]. Figures 3(a) and 3(b) report the ordering temperature  $T_N$  from magnetic susceptibility as well as the modulation vector  $\mathbf{q} = (q, 0, 0)$  of the ground state from resonant elastic x-ray scattering (REXS) at the Gd-L<sub>2</sub> absorption edge in reflection geometry [sketch in Fig. 3(a)]. We determined  $q$  from scans of scattering intensity along high-symmetry lines in momentum space [Fig. 3(b), inset]. These data show that the magnetic properties are but weakly affected by chemical substitution on the Pd site.

In electrical transport measurements, the topological Hall resistivity  $\rho_{yx}^T$  changes significantly with  $z$  [Figs. 3(c) and 3(d), respectively]. We calculate the topological Hall conductivity  $\sigma_{xy}^T = \rho_{yx}^T / (\rho_{xx}^2 + \rho_{yy}^2)$ , shown in Fig. 3(e) for selected samples. The peak value

$\max[\sigma_{xy}^T(B, T = 2 \text{ K})]$  is plotted in Fig. 3(f). The green line in this panel has a slope determined directly from the magnitude of the topological Nernst conductivity  $\alpha_{xy}^T/T$  using Eq. (1) [29]. A necessary ingredient of the

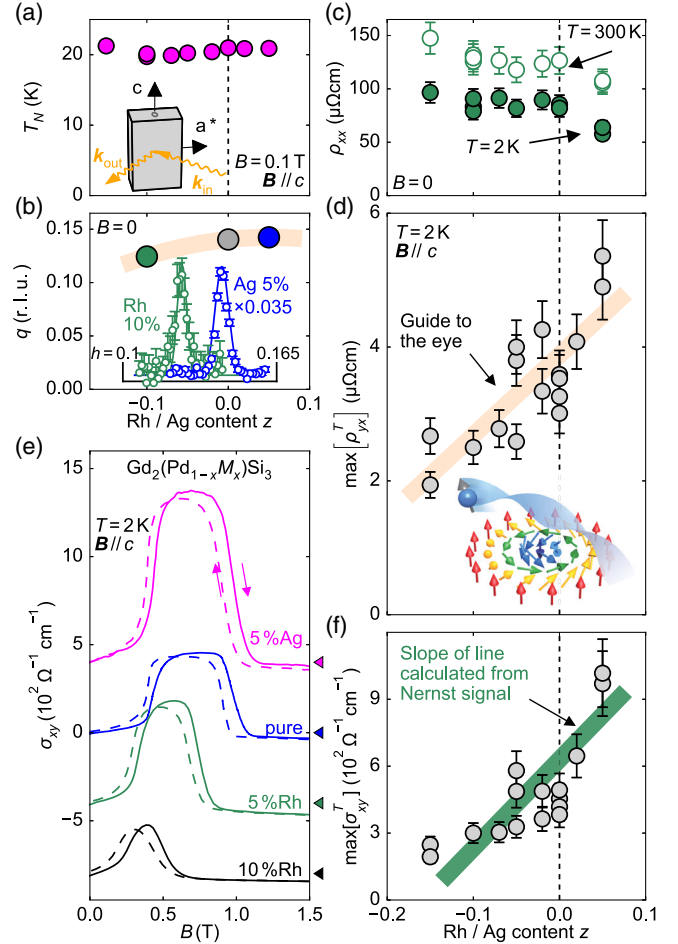


FIG. 3. (a) Néel temperature  $T_N$  of  $\text{Gd}_2(\text{Pd}_{1-x}\text{M}_x)\text{Si}_3$  from magnetic susceptibility.  $M = \text{Rh}, \text{Ag}$  corresponds to  $z < 0$  ( $z > 0$ ), respectively (see text). Inset: sample geometry for resonant elastic x-ray diffraction (REXS) at the Gd-L<sub>2</sub> edge in reflection geometry. (b) Magnetic wave number  $q$ , determined from REXS. The data point for pure  $\text{Gd}_2\text{PdSi}_3$  (grey disk) is reproduced from Ref. [23]. Inset: line scans of REXS intensity along  $(2-h, 1, 0)$  in reciprocal space for two samples, after the subtraction of a constant background. The beam energy was set to  $E = 7.935 \text{ keV}$  and the sample temperature was  $T = 4 \text{ K}$  ( $5 \text{ K}$ ) for the Ag-doped (Rh-doped) crystal. Scale of y axis (not shown) is intensity normalized by monitor counts (arbitrary units). Statistical errors of detector counts are indicated. (c) Longitudinal resistivity as well as extremal (d) topological Hall resistivity and (f) topological Hall conductivity. Systematic (sample shape) errors are indicated. In (e), raw data of Hall conductivity as a function of magnetic field. Inset of (d), illustration of the real-space Berry-phase mechanism for the THE and TNE [6,12]. Red lines in (b),(d) are guides to the eye, while the green line in (e) was calculated from the Nernst conductivity (see text). Vertical dashed lines mark  $z = 0$ .

calculation, the inverse density of states  $\partial\varepsilon/\partial z$ , was estimated from the specific heat of isoelectronic  $\text{Y}_2\text{PdSi}_3$  [29,71]. Good agreement of the doping study with the Nernst signal—without any adjustable parameters—amounts to a direct experimental confirmation of Eq. (1) for the THE and TNE.

The combined band filling and thermoelectric experiment establishes  $\text{Gd}_2\text{PdSi}_3$  as a model material for the quantitative exploration of transport responses in the presence of rather mild changes to the magnetic properties. Note that the Mott relation for the TNE has, to the best of our knowledge, never before been demonstrated in the literature. Even for the ANE, the only test without adjustable parameters was carried out using thin films of  $\text{Cr}_x(\text{Sb}_{1-y}\text{Bi}_y)_{2-x}\text{Te}_3$ , with ferromagnetism induced by dilute Cr spins ( $x = 0.15$ , Refs. [29,72]). We emphasize that not only can the sign of the filling-induced change to  $\sigma_{xy}^T$  be predicted from  $\alpha_{xy}^T$  but even its magnitude to within several tens of percents.

We further scrutinize underlying assumptions made in comparing the TNE with the THE for the doped crystals. First, Eq. (1) describes the regime of linear response. Hence, we note that despite being moderately large ( $-\partial_x T \sim 0.3$  K/mm), the longitudinal  $T$  gradient in our Nernst experiment on  $\text{Gd}_2\text{PdSi}_3$  is too small to unpin the SkL, and to result in a flux-flow type Nernst effect [29]. Second, Smrčka and Středa derived Eq. (1) by changing  $\zeta$  while leaving all other parameters unaffected—such as character of the magnetic ordering, nature of the electronic bands, and scattering processes [22]. We have demonstrated the stability of the magnetic order in Fig. 3 and, because even stoichiometric  $\text{Gd}_2\text{PdSi}_3$  already has significant residual resistivity, the introduction of dopants does not excessively affect the scattering properties [29]. Third, the electronic spectrum of  $\text{Gd}_2\text{PdSi}_3$  is sufficiently broadened by disorder to justify the linear approximation [green line in Fig. 3(f)] [29].

It is worth emphasizing that the validity of Mott's relation is in itself experimental evidence for the suitability of the rigid band scenario in describing mild changes of composition in  $\text{Gd}_2(\text{Pd}_{1-x}\text{M}_x)\text{Si}_3$  ( $M = \text{Rh}, \text{Ag}$ ). However, the notion of rigid bands warrants some more careful examination. As a first step, we have calculated the partial density of states (P-DOS)  $g_P(\varepsilon)$  of pure  $\text{Gd}_2\text{PdSi}_3$  in the framework of density functional theory (DFT) [Fig. 4(a)], using the  $\text{Ce}_2\text{CoSi}_3$  structure type. The calculations were carried out in the fully spin-polarized state (see the Supplemental Material for technical details [29]). Figures 4(b) and 4(c) further illustrate the effect of doping. The size of the unit cell for the doped cases was doubled along  $c$ , and one Pd atom was replaced with Rh or Ag [29]. Although this is an imperfect approximation for randomly distributed dopants in the experimental study, shifting of the total density of states (DOS) to the left (right) side for electron (hole) doping

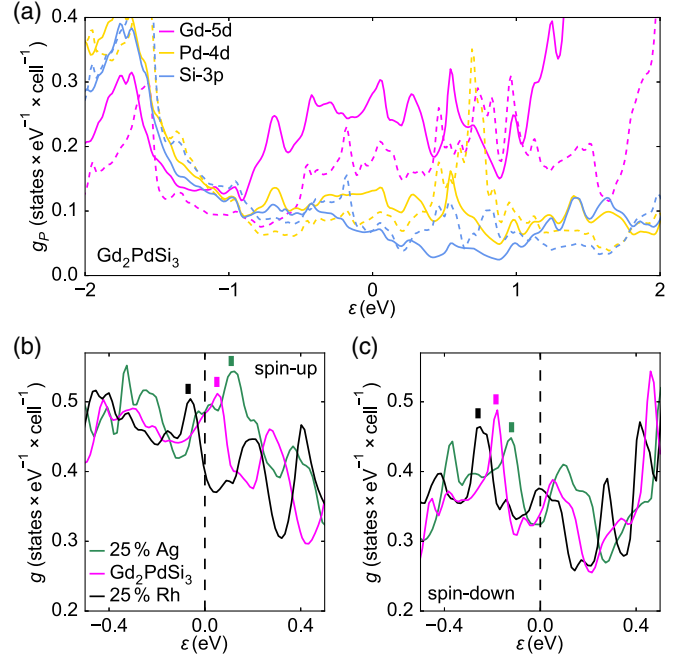


FIG. 4. Density functional theory calculations, neglecting Gd-4f, in the spin-polarized state of pure  $\text{Gd}_2\text{PdSi}_3$ . (a) Partial density of states (P-DOS)  $g_P(\varepsilon)$ ; solid and dashed lines represent spin-up and spin-down bands, respectively. (b),(c) Total DOS  $g(\varepsilon)$  for  $\text{Gd}_2\text{PdSi}_3$  and two doped derivatives. The vertical fat lines are guides to the eye, marking the shift of prominent features in  $g(\varepsilon)$  with band filling.

was observed in these calculations, consistent with the rigid band scheme.

The P-DOS in Fig. 4(a) is consistent with previous photoemission work on  $R_2\text{PdSi}_3$  ( $R = \text{Tb}, \text{La}, \text{Gd}$ ) [70] and indicates that Gd-5d orbitals dominate the total DOS at  $\zeta$ . Sizable Hund's rule coupling within the atomic shell of Gadolinium underpins the large THE and TNE in this material. Considering the giant  $\sigma_{xy}^T$  on the electron-doped side in Fig. 3, it is instructive to examine key material parameters. We draw on experimental data of the carrier mobility (from the normal Hall effect and  $\rho_{xx}$ ) to estimate the carrier mean free path in stoichiometric  $\text{Gd}_2\text{PdSi}_3$  [29]. Under the assumption of a twofold spin degenerate spherical (tubular) Fermi surface,  $l_{\text{mfp}} = 3.9$  nm ( $\approx 2.8$  nm). Despite disorder, this is comparable to or larger than the characteristic dimensions of the spin texture. Moreover, an order-of-magnitude estimate for the typical Hall conductivity in the intrinsic region yields  $\sigma_{xy}^{\text{int}} \approx 950 \Omega^{-1} \text{cm}^{-1}$ , not far from the present experimental result [29].

In conclusion, the insights presented here not only demonstrate a giant TNE from skyrmions and establish the validity of Mott's relation for THE and TNE, but also provide a guiding post for driving deeper into the intrinsic regime of THE and TNE for centrosymmetric skyrmion hosts. The present Nernst response of high-density

skyrmion textures in the centrosymmetric magnet  $\text{Gd}_2\text{PdSi}_3$  is on par with the largest anomalous Nernst signals ever observed in ferromagnets.

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