

## Lateral Optical Force due to the Breaking of Electric-Magnetic Symmetry

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Lateral optical forces in a direction perpendicular to light propagation have attracted increasing interest in recent years. Up to now, all lateral forces can be attributed to the symmetry breaking in the lateral directions caused by either the morphology of the scatterer geometry or the optical fields impinging on the scatterer. Here we demonstrate, both numerically and analytically, that when an isotropic scatterer breaks the electric-magnetic symmetry, a new type of anomalous lateral force can be induced along the direction of translational invariance where the illumination striking the scatterer has no propagation, field gradient, or spin density vortex (Belinfante's spin momentum). Our analytical results are rigorous for an arbitrary size scatterer, ensuring the universality of our conclusion. Furthermore, the electric-magnetic symmetry-breaking-induced lateral force is comparable in magnitude to other components of the optical force and reversible in direction for different polarizations of the illuminating light, rendering it capable of practical optical manipulation as well as enriching the understanding of light-matter interaction.

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Optical forces (OFs) exerted on an object immersed in optical fields have been extensively used for intact manipulation of microparticles in recent decades [1–7]. Physically, the OFs result from the transfer of linear momentum from light to the object through scattering. Their presence, on the other hand, can be alternatively traced to the breaking of symmetry, in either the geometry of the object or the optical fields impinging on the object. The longitudinal OFs along the propagation direction of light, ranging from the one earliest realized by Kepler when explaining why the tail of a comet points away from the sun [8], to the one first applied by Ashkin to practical optical manipulation [1], and to many more recent cases [9–16], may be attributed to the breaking of the forward-backward symmetry owing to the wave propagation. The transverse OFs perpendicular to the propagation direction of light, which play an indispensable role in optical tweezers [3], are traced to the symmetry breaking (inhomogeneity) of field intensity. The lateral OFs, which act along a direction without the field inhomogeneity nor the propagation and have become a focus of study [17–31], originate indeed from the breaking of symmetry as well. The pioneering results of the lateral OFs [17,18] come from the symmetry breaking of the scatterer's morphology in a cambered rod. The theoretical proposals of inducing lateral OFs on particles near a substrate (or another particle) rely on either multiple

scattering between the chiral particle and the substrate (or the other particle) [19–21] or unidirectional surface plasmonic resonance on the substrate induced by spin-orbit coupling of light [22,23] to break the lateral symmetry of optical fields impinging on the scatterers. The lateral OFs in two-wave interference and evanescent waves [24–27] arise from the hidden lateral asymmetry of the ambient optical fields that manifests itself as the lateral component of the Belinfante's spin momentum (BSM) [24,32–34]. Other lateral OFs [28–30] depend on particle chirality, which interacts with the lateral asymmetry appearing in the form of the spin angular momentum [35,36] and the curl of the energy current density. In all the previous studies, the physical mechanism of the lateral OFs was traced in either small or large particle limits [17–30], sacrificing the universality in order to circumvent the complexity in analytical expressions of OFs exerting on a particle of generic sizes.

In this Letter we demonstrate a new type of lateral OF in an apparent laterally symmetric system. An isotropic spherical particle is subjected to an anomalous OF along the lateral direction where the impinging optical field is translationally invariant. The lateral OF does not depend on the symmetry breaking of either the morphology of the scatterer or wave propagation. With a vanishing lateral component of BSM in the incident optical fields, neither

does this lateral OF rely on the spin density vortex to break the lateral symmetry. We trace the origin of the lateral OF, both numerically and, in particular, analytically for arbitrary particle size, to the breaking of the electric-magnetic symmetry (EMS) of the system due to the presence of the scatterer. In addition, the EMS-breaking-induced lateral OF has a magnitude comparable with other components of the OF and its direction is reversible by changing the polarization of the illuminating light, enabling its application to the practical optical manipulation in a unique manner.

As an illustration of such a lateral OF, consider an isotropic spherical particle immersed in an  $n_p$ -beam optical lattice (or quasilattice), which is modeled by the interference field composed of  $n_p$  plane waves, as schematically plotted in Fig. 1(a) for  $n_p = 5$ . The  $n_p$  plane waves have the same amplitude  $E_0$  and the same wavelength  $\lambda$ . All their wave vectors lie in the  $xoy$  plane. The electric field is thus given by

$$\mathbf{E} = \sum_{j=1}^{n_p} \mathbf{E}_j, \quad \mathbf{E}_j = E_0 \mathcal{E}_j e^{ik\hat{\mathbf{k}}_j \cdot \mathbf{r}}, \quad (1)$$

where  $k = 2\pi/\lambda$  is the wave number in the background medium. The direction of the  $j$ th wave vector is denoted by  $\hat{\mathbf{k}}_j = \cos \phi_j \hat{\mathbf{x}} + \sin \phi_j \hat{\mathbf{y}}$ . The position vector is  $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$ . The vector  $\mathcal{E}_j$  characterizes the polarization by

$$\mathcal{E}_j = -q \sin \phi_j \hat{\mathbf{x}} + q \cos \phi_j \hat{\mathbf{y}} - p \hat{\mathbf{z}}, \quad (2)$$

where  $p$  and  $q$  are two complex numbers satisfying the normalization  $|p|^2 + |q|^2 = 1$ , and making up the two-dimensional polarization vector  $(p, q)$  of the incident plane waves. We assume that all the  $n_p$  plane waves have the same polarization vector. The time dependence  $e^{-i\omega t}$  is assumed.

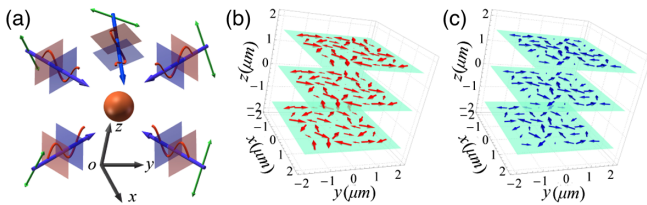


FIG. 1. (a): Schematic illustration of the system that a spherical particle is immersed in an interference field composed of  $n_p$  plane waves with all the wave vectors lying in the  $xoy$  plane and sharing the same polarization vector  $(p, q)$ . Shown in panel (a) is an example for  $n_p = 5$ . (b) and (c): The time-averaged Poynting vector (b) and Belinfante's spin momentum vector (c) at three parallel planes with different values of  $z$ . Both the vectors show the vanishing  $z$  components and the translational invariance along  $z$  direction. An anomalous lateral optical force in the  $z$  direction is induced on an isotropic spherical particle.

The  $n_p$ -beam optical lattices governed by Eqs. (1) and (2) are translationally invariant along the  $z$  direction, since the electric and magnetic fields are independent of the  $z$  coordinate. In Fig. 1(b) we display the Poynting vector for an example with  $n_p = 5$ ,  $p = q = 1/\sqrt{2}$ , and  $\phi_j = 0^\circ, 50^\circ, 110^\circ, 200^\circ, 300^\circ$  for  $j = 1, 2, \dots, 5$ , respectively. The Poynting vectors all lie in the  $xoy$  plane and stay unchanged at different values of  $z$ , indicating neither propagation nor a field gradient exists in the  $z$  direction. In addition, one can establish that, when each plane wave is linearly polarized, with  $p^*q$  being a real number, the optical field does not carry the  $z$  component of the BSM [24,25,32,33],  $\mathbf{P}_S = \frac{1}{2} \nabla \times \mathbf{S}$ , where  $\mathbf{S}$  denotes the time-averaged spin angular momentum density [35,36]. In Fig. 1(c) we exhibit the BSM vectors, again for  $p = q = 1/\sqrt{2}$ , at planes with different values of  $z$ , visualizing the vanishing of their  $z$  components besides the invariance along the  $z$  direction. This shows the lateral OF does not stem from the symmetry breaking of the illumination hidden behind the finite lateral BSM [21,24–27].

Based on the generalized Lorenz-Mie theory [37,38] and the Maxwell stress tensor method [39,40], we calculate the lateral force  $F_z$  on a dielectric spherical particle of radius  $r = 0.5 \mu\text{m}$  in vacuum under the illumination of a five-beam optical lattice given by Eqs. (1) and (2), with  $p = q = 1/\sqrt{2}$ . The distribution of the incident angles are chosen to be nonregular, with  $\phi_j = 0^\circ, 50^\circ, 110^\circ, 200^\circ, 300^\circ$ , for  $j = 1, 2, \dots, 5$ , and regular, with  $\phi_j = (j-1) \times 72^\circ$ . Figures 2(a) and 2(b) show the profiles of the lateral OF  $F_z$  acting on the particle versus its position in the  $xoy$  plane. The profiles exhibit a nonregular landscape and a regular landscape. For the regular case as displayed in Fig. 2(b), the lateral OF exposes additional odd symmetry upon tenfold rotation,  $\hat{\mathcal{R}}(2\pi/10)F_z(\phi) = -F_z(\phi + 2\pi/10)$ , a feature of the nonconservative (solenoidal) force field in the regular optical lattices [41]. Here  $\hat{\mathcal{R}}(\theta)$  denotes the rotational operator that rotates around the  $z$  axis by an angle  $\theta$ . The odd symmetry is not shared by the illumination, which has only fivefold discrete rotational symmetry,  $\hat{\mathcal{R}}(2\pi/5)\mathbf{E}(\phi) = \mathbf{E}(\phi + 2\pi/5)$ . This is similar to the case of the in-plane OF in optical lattices after it is decomposed into the gradient and scattering forces [41–46]. In addition, the lateral OFs in Figs. 2(a) and 2(b) are comparable in magnitude to the other components  $F_x$  and  $F_y$  of OFs, as shown in the Supplemental Material [47]. This differs significantly from the lateral OF induced by the BSM [24–26], which is substantially smaller than the other components of OFs, making it unfavorable for application to the lateral optical manipulations. The dependence of the lateral force on the particle size is examined in Fig. 2(c), where the red and blue lines denote the cases corresponding to panels (a) and (b). The lateral forces exhibit multiple sign reversals, since more higher

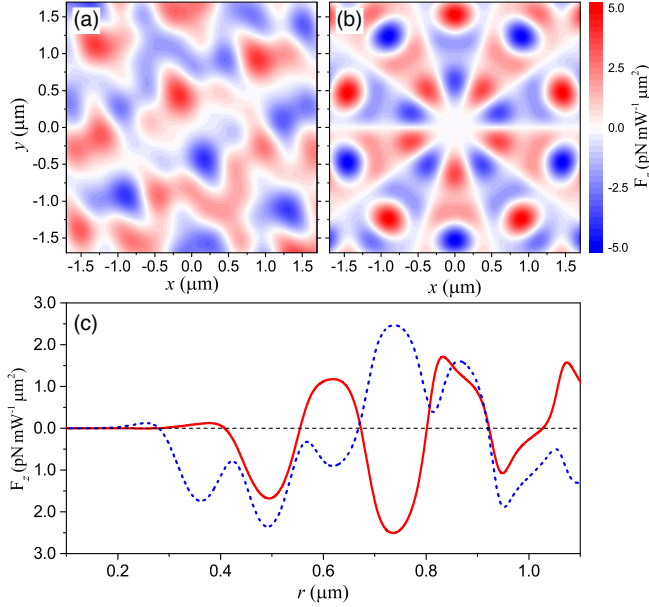


FIG. 2. The spatial profiles of the lateral force  $F_z$  acting on a spherical particle residing in vacuum and subject to the illumination of the interference fields Eqs. (1) and (2) composed of five plane waves with a nonregular distribution of the incident angles  $\phi_j = 0^\circ, 50^\circ, 110^\circ, 200^\circ, 300^\circ$  (a), and with a regular distribution of incident angles  $\phi_j = (j-1) \times 72^\circ$  (b). The particle has the radius  $r = 0.5 \mu\text{m}$ , the permittivity  $\epsilon_s = 2.5$ , and the permeability  $\mu_s = 1$ . The incident wavelength in vacuum is  $\lambda_0 = 1.064 \mu\text{m}$ . (c) The lateral force  $F_z$  versus the particle radius  $r$ . The red and blue lines correspond to the cases of panels (a) and (b). The particle is located at  $(0, 0.8, 0) \mu\text{m}$  in the Cartesian coordinate system.

order multipoles are excited on the particle as its size increases, thus leading to the interference among multipoles [48]. One therefore concludes that the emergence of

the lateral OF in a translationally invariant direction, where the illuminating field has neither propagation nor BSM [24–26] to break the lateral symmetry, is universal for generic particle sizes.

To understand the physical mechanism, we next derive an analytical expression of the lateral OF on a spherical particle located in an  $n_p$ -beam lattice. The time-averaged OF is generally written as [34,42–45]

$$\langle \mathbf{F} \rangle = \sum_{l=1}^{\infty} \langle \mathbf{F}_{\text{int}}^{(l)} \rangle + \sum_{l=1}^{\infty} \langle \mathbf{F}_{\text{rec}}^{(l)} \rangle, \quad (3)$$

where  $l$  denotes the orders of multipoles, usually referred to as  $2^l$ -pole.  $\langle \mathbf{F}_{\text{int}}^{(l)} \rangle$  is the interception force on the  $2^l$ -pole, originating from the physical process that light is intercepted by a particle residing in the optical fields. The recoil force  $\langle \mathbf{F}_{\text{rec}}^{(l)} \rangle$  comes from the physical process that light is reemitted by the oscillating multipoles.

Substituting the electric field of the  $n_p$ -beam optical lattice, Eqs. (1) and (2), into Eq. (3), after considerable algebra (detailed in the Supplemental Material [47]), we finally arrive at the expression of the lateral OF on a spherical particle. When the polarization vector  $(p, q)$  satisfies  $p^*q = pq^*$ , namely, when the constituent plane waves are linearly polarized, the lateral OF reduces to

$$F_z = p^*q \sum_{l=1}^{\infty} F_z^{(l)} \quad (4)$$

with the  $(p, q)$ -independent  $F_z^{(l)}$  given by

$$F_z^{(l)} = \frac{2(2l+1)\pi}{l^2(l+1)^2} \text{Im}[a_l b_l^*] \sum_{i,j} [R_l^{(6)}(x_{ij}) - R_l^{(7)}(x_{ij})] (\nabla \times \text{ReS}_{\text{em},ij}^{(1)})_z - \frac{2\pi}{(l+1)^2} \text{Im}[a_{l+1} a_l^* - b_{l+1} b_l^*] \sum_{i,j} [R_l^{(3)}(x_{ij}) + R_l^{(6)}(x_{ij})] (\nabla \times \text{ReS}_{\text{em},ij}^{(1)})_z, \quad (5)$$

where  $a_l$  and  $b_l$  are the Mie coefficients [49], whereas  $R_l^{(3)}(x_{ij})$ ,  $R_l^{(6)}(x_{ij})$ , and  $R_l^{(7)}(x_{ij})$  are linear combinations of Legendre polynomials given in the Supplemental Material [47], with the argument  $x_{ij} = \hat{\mathbf{k}}_i \cdot \hat{\mathbf{k}}_j$ . Here  $p^*q$  is of odd parity while  $F_z^{(l)}$  is of even parity, leading to a parity odd force  $F_z$  in Eq. (4). The vector  $\text{ReS}_{\text{em},ij}^{(1)}$  represents the real part of the dimensionless vector  $\mathbf{S}_{\text{em},ij}^{(1)}$  defined by

$$\mathbf{S}_{\text{em},ij}^{(1)} = (\mathcal{E}_i \times \mathcal{B}_j^*) e^{ik(\hat{\mathbf{k}}_i - \hat{\mathbf{k}}_j) \cdot \mathbf{r}}, \quad (6)$$

which depends solely on the illuminating optical field, with  $\mathcal{B}_j = \hat{\mathbf{k}}_j \times \mathcal{E}_j$ . The lateral force given by Eq. (4) is therefore dimensionless as well. It actually gives the numerical value of the OF in units of  $\epsilon_b E_0^2 / k^2$ , with  $\epsilon_b$  being the background permittivity. Equation (5) contains only quadratic terms of the Mie coefficients, revealing such a lateral OF comes solely from the recoil force, a remarkable feature for OF in a direction where the illumination shining on the particle has translational invariance.

Equations (4) and (5) represent the first rigorous analytical expressions for the lateral OF acting on a spherical



particle of arbitrary size immersed in optical lattices, thus ensuring the universality of any conclusion thereby drawn. They show many salient features of the lateral OF in the optical lattices. First, the curl operator  $\nabla \times$  in front of  $\text{Re}S_{\text{em},ij}^{(l)}$  indicates that the lateral OF is cast into a sum of purely solenoidal terms, thus it is a nonconservative scattering force [50,51], consistent with the translational invariance of illuminating fields in the  $z$  direction and, also, explaining the additional odd symmetry upon tenfold rotation of its profile in a regular optical lattice, as shown in Fig. 2(b). Second, most importantly, the lateral OF vanishes when the Mie coefficients satisfy

$$\text{Im}(a_{l+1}a_l^* - b_{l+1}b_l^*) = 0 \quad \text{and} \quad \text{Im}(a_l b_l^*) = 0. \quad (7)$$

Equations (7) can be fulfilled when  $a_l = b_l$  for all multipole orders  $l$ . In other words, whenever the presence of a scatterer in optical fields does not violate the EMS of the system in the sense that it displays symmetric electric and magnetic response, the lateral OF vanishes identically, regardless of the size and composition of the particle. It is therefore concluded that the lateral OF in our system is induced by the breaking of the EMS due to the presence of the particle. This differs fundamentally from the lateral OFs reported previously, where the emergence of the lateral OFs relies on either the asymmetry due to the morphology of the scatterer [17,18], the asymmetry arising from the illuminating optical fields mediated by nearby scatterer [19–23], or the asymmetry of the illumination hidden behind the BSM [24–27]. Third, Eqs. (4) and (5) suggest that besides the breaking of the EMS, the emergence of the lateral OF requires also optical vorticity that manifests itself in Eq. (5) as curls of some field-dependent quantities. For a simple regular optical lattice with, e.g.,  $n_p = 3$  and  $\phi_j = 2(j-1)\pi/3$  for  $j = 1, 2, 3$ , Eq. (5) reduces to

$$F_z^{(l)} = \frac{4(2l+1)\pi}{l^2(l+1)^2} \text{Im}[a_l b_l^*] C_l (\nabla \times \mathbf{P})_z - \frac{4\pi}{(l+1)^2} \text{Im}[a_{l+1} a_l^* - b_{l+1} b_l^*] D_l (\nabla \times \mathbf{P})_z, \quad (8)$$

where  $\mathbf{P} = \frac{1}{2} \text{Re}(\mathbf{E} \times \mathbf{H}^*)$  is the energy flow density while  $C_l$  and  $D_l$  are two constants dependent on  $E_0$ . So although the illumination shining on the isotropic particle is translationally invariant and exhibits no propagation in the  $z$  direction, it still possesses the asymmetry hidden in the curl of  $\mathbf{P}$ . It is with this asymmetry that the breaking of the EMS brought about by the particle couples to induce an anomalous lateral OF. The demand for a finite circulation, as shown in Eqs. (5) and (8), requires  $n_p \geq 2$  to generate such a lateral OF, manifesting the role of the optical lattice. Also the optical lattice is designed to avoid the lateral Poynting vector and BSM that could complicate the tracing of the physical origin of such a lateral force. Last but not least, the appearance of  $p^*q$  in Eq. (4) indicates that the

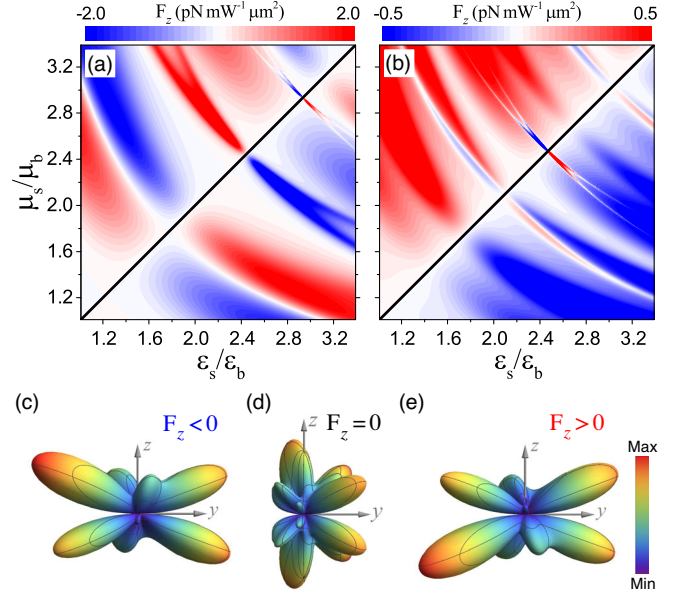


FIG. 3. (a) and (b): The lateral optical force  $F_z$  versus particle permittivity  $\epsilon_s/\epsilon_b$  and permeability  $\mu_s/\mu_b$  when a particle is immersed in vacuum (a) and water (with  $\epsilon_b = 1.33^2$ ) (b). The black solid lines denote the vanishing of the lateral force. (c)–(e): The spherical plots of the  $z$  component of the scattered irradiance by a particle in vacuum with  $(\epsilon_s, \mu_s) = (2.5, 1.0)$  (c),  $(2.5, 2.5)$  (d), and  $(1.0, 2.5)$  (e). The up-down ( $\pm z$ ) asymmetry in the scattered irradiance in panels (c) and (e) signifies a lateral force arising from the breaking of the electric-magnetic symmetry brought about by the particle. All other parameters are the same as those in Fig. 2(a), and the particle is located at  $(0, 0.8, 0) \mu\text{m}$ .

lateral OF reverses its direction if either  $p$  or  $q$  changes its sign, offering an extra possibility to tailor the OF uniquely by tuning the illuminating polarization.

In Fig. 3, the EMS-breaking-induced lateral OF is plotted versus the particle permittivity  $\epsilon_s$  and permeability  $\mu_s$ , while other parameters correspond to those in Fig. 2(a), with the particle positioned at  $(0, 0.8, 0) \mu\text{m}$ . The black solid lines correspond to the zeros of the lateral OF, which mark the positions where the EMS is fulfilled. As shown in Fig. 3(a) for the particle in vacuum, the lateral OF is induced for  $\epsilon_s \neq \mu_s$ , manifesting the fact that EMS has been broken. The lateral OF also shows an antisymmetric profile with respect to the black solid line,  $F_z(\epsilon_s, \mu_s) = -F_z(\mu_s, \epsilon_s)$ , in agreement with Eqs. (4) and (5). Figure 3(b) shows the lateral OF on a particle immersed in water (with the refractive index  $n_w = 1.33$ ), where the black line shows up at  $\epsilon_s/\epsilon_b = \mu_s/\mu_b$ , with  $\epsilon_b = n_w^2$  and the background permeability  $\mu_b = 1$ . The black line marking the vanishing lateral OF remains diagonal when plotted against relative permittivity and relative permeability, corroborating the requirement of EMS, Eq. (7). It is the breaking of the EMS brought about by the isotropic particle,  $\epsilon_s/\epsilon_b \neq \mu_s/\mu_b$ , that gives rise to the lateral OF, irrespective of the particle size and composition.

The emergence of the EMS-breaking-induced lateral OF can be vividly visualized by the angular dependence of the scattered irradiance, defined as the energy scattered into a unit solid angle per unit time [11,16,48,49], owing to the fact that the lateral force is purely a recoil force, as indicated by Eq. (5). Figures 3(c)–3(e) are typical spherical plots of the  $z$  component scattered irradiance for a particle with  $(\epsilon_s, \mu_s) = (2.5, 1.0)$ ,  $(2.5, 2.5)$ , and  $(1.0, 2.5)$ , respectively. Both Figs. 3(c) and 3(e) clearly show asymmetry in the scattered irradiance pattern with respect to the  $z = 0$  plane, clearly signifying a bias of the OF in the direction opposite to the scattered irradiance, according to the conservation of momentum. In fact, the integration of the scattered irradiance as shown in Figs. 3(c)–3(e) over a solid angle yields the negative of the lateral force  $F_z$  [34]. In Fig. 3(d), on the other hand, an up-down symmetry in the scattered irradiance pattern is observed as a result of the EMS, leading to the vanishing of a net lateral OF.

Finally, we have also used Gaussian beams with optical axes lying in the  $z = 0$  plane instead of plane waves. For the EMS-breaking case, the lateral force profile for the beam waist  $w_0 = 10\lambda_0$  remains similar to that of plane waves and gets distorted as  $w_0$  decreases (see Supplemental Material [47]). For the EMS preserving case, the calculated lateral force on the particle in the  $z = 0$  plane is of the same order as the numerical noise, even when the beams are tightly focused with  $w_0 \sim \lambda_0$ . This signifies that our observation is valid for realistic beams.

In conclusion, we have demonstrated numerically and analytically a new type of lateral OF on an isotropic spherical particle immersed in optical lattices that are modeled by an interference field composed of an arbitrary number of plane waves with the same polarization vector and amplitude. The lateral OF is induced in a direction along which the light shining on the particle has neither field gradient nor propagation. Neither does the illumination carry Belinfante's spin momentum that breaks its lateral symmetry. By deriving an analytical expression for such a lateral OF for a particle of generic size, the origin of the lateral OF is traced, unambiguously and without loss of universality regarding particle size, to the breaking of EMS brought about by the particle. The lateral OF explored here is comparable in magnitude to other components of the OFs and reversible in direction through tweaking the illuminating polarization, thus offering opportunity for unique practical applications in optical manipulation, as well as adding to our physical understanding of light-matter interaction.

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- [1] A. Ashkin, *Phys. Rev. Lett.* **24**, 156 (1970).
- [2] D. G. Grier, *Nature (London)* **424**, 810 (2003).
- [3] A. Ashkin, *Optical Trapping and Manipulation of Neutral Particles Using Lasers* (World Scientific, Hackensack, 2006).
- [4] K. Dholakia and T. Čižmár, *Nat. Photonics* **5**, 335 (2011).
- [5] M. Padgett and R. Bowman, *Nat. Photonics* **5**, 343 (2011).
- [6] M. L. Juan, M. Righini, and R. Quidant, *Nat. Photonics* **5**, 349 (2011).
- [7] R. W. Bowman and M. J. Padgett, *Rep. Prog. Phys.* **76**, 026401 (2013).
- [8] J. Kepler, *De Cometis Libelli Tres* (Augsburg, Augustae Vindelicorum, 1619).
- [9] J. Baumgartl, M. Mazilu, and K. Dholakia, *Nat. Photonics* **2**, 675 (2008).
- [10] C. Maher-McWilliams, P. Douglas, and P. F. Barker, *Nat. Photonics* **6**, 386 (2012).
- [11] J. Chen, J. Ng, Z. F. Lin, and C. T. Chan, *Nat. Photonics* **5**, 531 (2011).
- [12] S. Sukhov and A. Dogariu, *Phys. Rev. Lett.* **107**, 203602 (2011).
- [13] D. B. Ruffner and D. G. Grier, *Phys. Rev. Lett.* **109**, 163903 (2012).
- [14] A. Dogariu, S. Sukhov, and J. J. Sáenz, *Nat. Photonics* **7**, 24 (2013).
- [15] O. Brzobohatý, V. Karásek, M. Šiler, L. Chvátal, T. Čižmár, and P. Zemánek, *Nat. Photonics* **7**, 123 (2013).
- [16] H. J. Chen, S. Y. Liu, J. Zi, and Z. F. Lin, *ACS Nano* **9**, 1926 (2015).
- [17] G. A. Swartzlander, Jr., T. J. Peterson, A. B. Artusio-Glimpse, and A. D. Raisanen, *Nat. Photonics* **5**, 48 (2011).
- [18] S. H. Simpson, S. Hanna, T. J. Peterson, and G. A. Swartzlander, Jr., *Opt. Lett.* **37**, 4038 (2012).
- [19] S. B. Wang and C. T. Chan, *Nat. Commun.* **5**, 3307 (2014).
- [20] H. J. Chen, Y. K. Jiang, N. Wang, W. L. Lu, S. Y. Liu, and Z. F. Lin, *Opt. Lett.* **40**, 5530 (2015).
- [21] H. Magallanes and E. Brasselet, *Nat. Photonics* **12**, 461 (2018).
- [22] F. J. Rodríguez-Fortuño, N. Engheta, A. Martínez, and A. V. Zayats, *Nat. Commun.* **6**, 8799 (2015).
- [23] S. Sukhov, V. Kajorndejnukul, R. R. Naraghi, and A. Dogariu, *Nat. Photonics* **9**, 809 (2015).
- [24] K. Y. Bliokh, A. Y. Bekshaev, and F. Nori, *Nat. Commun.* **5**, 3300 (2014).
- [25] A. Y. Bekshaev, K. Y. Bliokh, and F. Nori, *Phys. Rev. X* **5**, 011039 (2015).
- [26] M. Antognozzi, C. R. Bermingham, R. L. Harniman, S. Simpson, J. Senior, R. Hayward, H. Hoerber, M. R. Dennis,

- A. Y. Bekshaev, K. Y. Bliokh, and F. Nori, *Nat. Phys.* **12**, 731 (2016).
- [27] V. Svak, O. Brzobohatý, M. Šiler, P. Ják, J. Kaňka, P. Zemánek, and S. H. Simpson, *Nat. Commun.* **9**, 5453 (2018).
- [28] A. Hayat, J. P. B. Mueller, and F. Capasso, *Proc. Natl. Acad. Sci. U.S.A.* **112**, 13190 (2015).
- [29] H. J. Chen, C. H. Liang, S. Y. Liu, and Z. F. Lin, *Phys. Rev. A* **93**, 053833 (2016).
- [30] T. Zhang, M. R. C. Mahdy, Y. Liu, J. H. Teng, C. T. Lim, Z. Wang, and C. Qiu, *ACS Nano* **11**, 4292 (2017).
- [31] B. A. van Tiggelen, A. Nussle, and G. L. J. A. Rikken, *Phys. Rev. A* **87**, 063424 (2013).
- [32] F. J. Belinfante, *Physica (Utrecht)* **7**, 449 (1940).
- [33] M. V. Berry, *J. Opt. A* **11**, 094001 (2009).
- [34] Y. K. Jiang, H. J. Chen, J. Chen, J. Ng, and Z. F. Lin, [arXiv:1511.08546](https://arxiv.org/abs/1511.08546).
- [35] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Photons and Atoms: Introduction to Quantum Electrodynamics* (John Wiley and Sons, New York, 1989).
- [36] L. Allen, S. M. Barnett, and M. J. Padgett, *Optical Angular Momentum* (Institute of Physics Pub., Bristol, 2003).
- [37] G. Gouesbet, *Opt. Commun.* **283**, 517 (2010).
- [38] G. Gouesbet and G. Grehan, *Generalized Lorenz-Mie Theories* (Springer, Berlin, 2011).
- [39] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (John Wiley and Sons, New York, 1999).
- [40] Q. Ye and H. Z. Lin, *Eur. J. Phys.* **38**, 045202 (2017).
- [41] Y. K. Jiang, H. Z. Lin, X. Li, J. Chen, J. J. Du, and J. Ng, *ACS Photonics* **6**, 2749 (2019).
- [42] Y. K. Jiang, J. Chen, J. Ng, and Z. F. Lin, [arXiv:1604.05138](https://arxiv.org/abs/1604.05138).
- [43] H. X. Zheng, X. N. Yu, W. L. Lu, J. Ng, and Z. F. Lin, *Comput. Phys. Commun.* **237**, 188 (2019).
- [44] X. N. Yu, Y. K. Jiang, H. J. Chen, S. Y. Liu, and Z. F. Lin, *Phys. Rev. A* **100**, 033821 (2019).
- [45] G. J. Ha, H. X. Zheng, X. N. Yu, and Z. F. Lin, *J. Opt. Soc. Am. B* **37**, 67 (2020).
- [46] J. J. Du, C. H. Yuen, X. Li, K. Ding, G. Q. Du, Z. F. Lin, C. T. Chan, and J. Ng, *Sci. Rep.* **7**, 18042 (2017).
- [47] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.125.073901> for numerical simulations of optical forces under the illumination of multiple plane waves or Gaussian beams and the analytical derivation of lateral optical forces.
- [48] H. J. Chen, Q. Ye, Y. W. Zhang, L. Shi, S. Y. Liu, Z. Jian, and Z. F. Lin, *Phys. Rev. A* **96**, 023809 (2017).
- [49] C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles* (John Wiley, New York, 1998).
- [50] S. Sukhov and A. Dogariu, *Rep. Prog. Phys.* **80**, 112001 (2017).
- [51] S. Albaladejo, M. I. Marqués, M. Laroche, and J. J. Sáenz, *Phys. Rev. Lett.* **102**, 113602 (2009).