Novel Method to Reliably Determine the Photon Helicity in $B \to K_1\gamma$

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A sizable right-handed photon polarization in $b \to s\gamma$ is a clear signal for new physics. In this Letter, we point out that the photon helicity in $b \to s\gamma$ can be unambiguously extracted by combining the measurements in $B \to K_1\gamma$ and the Cabibbo-favored $D \to K_1 e^+\nu$ decay. We propose a ratio of up-down asymmetries in $D \to K_1 e^+\nu$ to quantify the hadronic effects. A method for measuring, in experiment, the involved partial decay widths in the ratio is discussed, and experimental facilities like BESIII, Belle-II and LHCb are likely to measure this ratio. We also give the angular distribution that is useful for extracting the photon polarization in the presence of different kaon resonances.

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Introduction.—Nowadays, searching for new physics is the primary objective in particle physics. In the standard model (SM), the photon helicity in $b \to s\gamma$ decay is predominantly left handed, and thereby, its measurement plays a unique role in probing right-handed couplings in new physics (NP) [1–3]. A representative example is the left-right symmetric model [4,5], in which the photon can acquire a significant right-handed component. However, to date, there are not many experimental results on the photon helicity.

It is noticed that the photon helicity is related to an up-down (UD) asymmetry $\mathcal{A}_{\rm UD}$ in $B \to K_1\gamma$ [6–8] and, more generally, the angular distribution in $B \to K_{\rm res}(\to K\pi\pi)\gamma$. Throughout this Letter, K_1 denotes the axial-vector meson $K_1(1270)$ and $K_1(1410)$. However, measuring the up-down asymmetry in $B \to K_1\gamma$ [9] does not directly reveal the photon helicity since the detailed knowledge of $K_1 \to K\pi\pi$ is a prerequisite. Previous theoretical analyses have adopted nonperturbative models to parametrize the $K_1 \to K\pi\pi$ amplitude, and thus, considerable hadronic uncertainties are inevitably introduced [6–8,10,11].

In this Letter, we will point out that one can combine the measurements in $B \to K_1\gamma$ and semileptonic $D \to K_1 l^+\nu (l = e, \mu)$ decays to determine the photon polarization in $b \to s\gamma$ without any theoretical ambiguity. In particular, we propose a ratio of up-down asymmetries in $D \to K_1 e^+\nu$, $\mathcal{A}'_{\rm UD}$, to quantify the hadronic effects in $K_1 \to K\pi\pi$ decay and point out that the photon helicity can be expressed in terms of $\mathcal{A}_{\rm UD}$ and $\mathcal{A}'_{\rm UD}$. Experimental facilities including BESIII, Belle-II, and LHCb are likely to measure this ratio $\mathcal{A}'_{\rm UD}$.

Photon polarization in $B \to K_1(\to K\pi\pi)\gamma$.—Let us start with the angular distribution in $B \to K_1(\to K\pi\pi)\gamma$. The effective Hamiltonian for $b \to s\gamma$ has the general form

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^*(C_{7L}\mathcal{O}_{7L} + C_{7R}\mathcal{O}_{7R}),$$
$$\mathcal{O}_{7L,R} = \frac{em_b}{16\pi^2}\bar{s}\sigma_{\mu\nu}\frac{1\pm\gamma_5}{2}bF^{\mu\nu}, \quad (1)$$

where $C_{7L,7R}$ are the Wilson coefficients for $\mathcal{O}_{7L,R}$. Because of the chiral structure of $W^\pm$ couplings to quarks in the SM, the emitted photon in $b \to s\gamma$ is mostly left handed and the right-handed configuration is suppressed by $C_{7R}^{\rm SM}/C_{7L}^{\rm SM} \approx m_s/m_b$. For $\bar{b} \to \bar{s}\gamma$, it is vice versa.

The differential decay rate for $B \to K_1(\to K\pi\pi)\gamma$ can be expressed as a sum of contributions from left- and right-polarized photons [6,7,11]

$$\frac{d\Gamma_{K_1\gamma}}{d\cos\theta_K} = \frac{|A|^2|\vec{J}|^2}{4} \times \left[1 + \cos^2\theta_K + 2\lambda_\gamma\cos\theta_K\frac{{\rm Im}[\vec{n}\cdot(\vec{J}\times\vec{J}^*)]}{|\vec{J}|^2}\right]. \quad (2)$$

Hereafter, θ_K is chosen as the relative angle between the normal direction $\vec{n}$ of the K_1 decay plane and the opposite flight direction of the photon in the K_1 rest frame. The coefficient A is a nonperturbative amplitude. The $\vec{J}$ characterizes the $K_1 \to K\pi\pi$ decay amplitude with $\mathcal{A}(K_1 \to K\pi\pi) = \vec{\epsilon}_{K_1}\cdot\vec{J}$. The $\cos\theta_K$ is a parity-odd quantity, and the left-handed and right-handed polarizations contribute with an opposite sign. The parameter λ_γ is defined as

$$\lambda_\gamma \equiv \frac{|\mathcal{A}(B \to K_{1R}\gamma_R)|^2 - |\mathcal{A}(B \to K_{1L}\gamma_L)|^2}{|\mathcal{A}(B \to K_{1R}\gamma_R)|^2 + |\mathcal{A}(B \to K_{1L}\gamma_L)|^2}, \quad (3)$$

with $\lambda_\gamma \simeq -1$ for $b \to s\gamma$ and $\lambda_\gamma \simeq +1$ for $\bar{b} \to \bar{s}\gamma$ in the SM.

Compared to the angular distribution in the above equation, an integrated up-down asymmetry is more convenient on the experimental side [6]

$$\mathcal{A}_{\mathrm{UD}} \equiv \frac{\Gamma_{K_1\gamma}[\cos\theta_K > 0] - \Gamma_{K_1\gamma}[\cos\theta_K < 0]}{\Gamma_{K_1\gamma}[\cos\theta_K > 0] + \Gamma_{K_1\gamma}[\cos\theta_K < 0]} = \lambda_\gamma \frac{3}{4} \frac{\mathrm{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)]}{|\vec{J}|^2}. \quad (4)$$

The LHCb collaboration has measured the up-down asymmetry in $B^+ \to K^+\pi^-\pi^+\gamma$ [9] with $\mathcal{A}_{\mathrm{UD}} = (6.9 \pm 1.7) \times 10^{-2}$ in the range of $m_{K\pi\pi} = [1.1, 1.3]$ GeV. In this kinematics region, it is expected that the asymmetry is dominated by $K_1(1270)$, but other contributions might also be important. Nevertheless, it can be seen that, even assuming the dominance of $K_1(1270)$, it is also essential to fathom the hadronic factor $\mathrm{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)]/|\vec{J}|^2$. Many estimations on this input factor have been made in either model-dependent or phenomenological approaches [6–8,10–12]. Unfortunately, the current understanding of $K_1 \to K\pi\pi$ is very limited, due to the complicated intermediate states of $K^*\pi$, $K\rho$, $(K\pi)_{S\text{-wave}}\pi$ and $K(\pi\pi)_{S\text{-wave}}$, and their phases for interferences. Thus, considerable hadronic uncertainties are inevitably introduced and beyond control. Therefore, the accurate result of λ_γ has never been achieved so far, even though the up-down asymmetry has been well measured.

Determination of photon helicity by combining $B \to K_1\gamma$ and $D \to K_1 e^+\nu_e$ decays.—Now, we proceed with the angular distribution for $D \to K_1(\to K\pi\pi)e^+\nu$ and demonstrate that combining the measurements in $B \to K_1\gamma$ and $D \to K_1 e^+\nu_e$ can determine the photon helicity in $b \to s\gamma$ in a model-independent way.

With the kinematics shown in Fig. 1, one can derive the angular distribution for $D \to K_1(\to K\pi\pi)e^+\nu_e$ as

$$\begin{aligned}\frac{d\Gamma_{K_1 e\nu_e}}{d\cos\theta_K d\cos\theta_l} = & d_1[1 + \cos^2\theta_K \cos^2\theta_l] \\ & + d_2[1 + \cos^2\theta_K]\cos\theta_l \\ & + d_3\cos\theta_K[1 + \cos^2\theta_l] \\ & + d_4\cos\theta_K\cos\theta_l \\ & + d_5[\cos^2\theta_K + \cos^2\theta_l]. \quad (5)\end{aligned}$$

The angular coefficients are given as

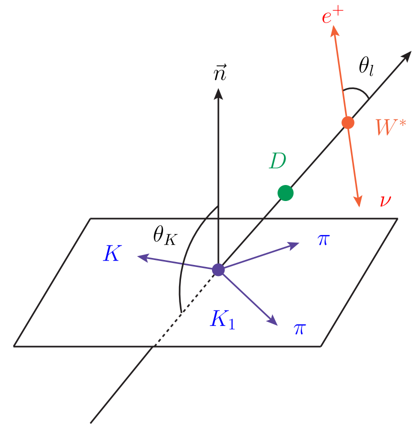

FIG. 1. Kinematics for $D \to K_{\mathrm{res}}(\to K\pi\pi)e^+\nu$. The relative angle between the normal direction of the K_{res} decay plane and the opposite of D flight direction in the K_{res} rest frame is denoted as θ_K, while θ_l is introduced as the relative angle between the flight directions of e^+ in the $e^+\nu$ rest frame and the $e^+\nu$ in the D rest frame.

$$\begin{aligned} d_1 &= \frac{1}{2}|\vec{J}|^2(4|c_0|^2 + |c_-|^2 + |c_+|^2), \\ d_2 &= -|\vec{J}|^2(|c_-|^2 - |c_+|^2), \\ d_3 &= -\mathrm{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)](|c_-|^2 - |c_+|^2), \\ d_4 &= 2\mathrm{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)](|c_-|^2 + |c_+|^2), \\ d_5 &= -\frac{1}{2}|\vec{J}|^2(4|c_0|^2 - |c_-|^2 - |c_+|^2). \quad (6)\end{aligned}$$

Above, we have neglected the lepton mass, and the $c_{0,+,-}$ corresponds to the nonperturbative amplitudes for D decays into K_1 with different polarizations.

Compared to the angular distribution for $B \to K_1\gamma$, the one for $D \to K_1 e^+\nu$ is different in three aspects. In $B \to K_1\gamma$, the emitted photon is on shell, and thus, only transverse polarizations are allowed, but the longitudinal polarization also exists in $D \to K_1 e^+\nu$ contributing with the amplitude c_0. Second, while only one angle θ_K is constructed for $B \to K_1\gamma$, two angles θ_K and θ_l are involved in $D \to K_1 e^+\nu$. Third, in $B \to K_1\gamma$, the $\cos\theta_K$ itself is a parity-odd quantity. In Eq. (2), the parity-even term $(1 + \cos^2\theta_K)$ is accompanied with $|\vec{J}|^2$, and the parity-odd term contains the decay factor $\mathrm{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)]$. In $D \to K_1 e^+\nu$, the $\cos\theta_K$ dependence is similar, but the lepton pair is produced through the $V - A$ current. This interaction also gives the parity-even term and the parity-odd term in $\cos\theta_l$. We have picked up the two parity-odd terms, formed by the $\cos\theta_K(1 + \cos^2\theta_l)$ that is proportional to $\mathrm{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)]$ and the $\cos\theta_l(1 + \cos^2\theta_K)$ that is proportional to $|\vec{J}|^2$. The ratio of the coefficients of these two terms, namely d_3 and d_2, give the required hadronic factor $\mathrm{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)]/|\vec{J}|^2$.

To pick up the d_2 and d_3 in a simpler way, we also propose to explore a ratio of up-down asymmetries (or forward-backward asymmetries)

$$\mathcal{A}'_{\rm UD} \equiv \frac{\Gamma_{K_1 e\nu_e}[\cos\theta_K > 0] - \Gamma_{K_1 e\nu_e}[\cos\theta_K < 0]}{\Gamma_{K_1 e\nu_e}[\cos\theta_l > 0] - \Gamma_{K_1 e\nu_e}[\cos\theta_l < 0]}. \quad (7)$$

It is straightforward to find

$$\mathcal{A}'_{\rm UD} = \frac{{\rm Im}[\vec{n}\cdot(\vec{J}\times\vec{J}^*)]}{|\vec{J}|^2}. \quad (8)$$

Apparently, quantifying the $\mathcal{A}_{\rm UD}$ in $B\to K_1\gamma$ and $\mathcal{A}'_{\rm UD}$ in $D\to K_1 e^+\nu$ in experiment will help to extract the photon helicity in $b\to s\gamma$

$$\lambda_\gamma = \frac{4}{3}\frac{\mathcal{A}_{\rm UD}}{\mathcal{A}'_{\rm UD}}. \quad (9)$$

The $D\to K_1(1270)e^+\nu$ channel is a Cabibbo-favored decay process, and thus, its decay branching fraction is expected to be sizable. On the experimental side, an earlier evidence for $D^0\to K_1^-(1270)e^+\nu$ has been found by CLEO in Ref. [13]. Quite recently, using the 2.93 fb^{-1} data sample of the e^+e^- collision at the center of mass energy of 3.773 GeV, the BESIII collaboration has observed $D^+\to \bar{K}_1^0(1270)e^+\nu$ for the first time with a statistical significance greater than 10σ, and the measured branching fraction is [14]

$$\mathcal{B}(D^+\to\bar{K}_1^0 e^+\nu) = (2.3\pm 0.26^{+0.18}_{-0.21}\pm 0.25)\times 10^{-3}. \quad (10)$$

From this measured branching fraction, one may infer that a direct measurement of the $\mathcal{A}'_{\rm UD}$ is feasible with more statistics in the near future. We also expect that a high precision could be achieved taking into account the fact that much more data will be accumulated at BESIII, Belle-II, and LHCb, leaving aside the Super Tau Charm Factory in the future.

Experimental implementation.—The analysis in the previous section is focused on only the K_1 contribution. Though the LHCb measurement of the spectrum of $K\pi\pi$ in $B\to K\pi\pi\gamma$ [9] and an earlier measurement by Belle [15] indicate that the $K_1(1270)$ contributions dominate in the range of $m_{K\pi\pi} = [1.1, 1.3]$ GeV, other contributions are likely non-negligible. In the next section, we will derive the angular distribution with different kaon resonances, and here, we will briefly discuss the implementation in experiment.

As shown in Eqs. (4) and (7), the up-down asymmetries are constructed using decay widths into K_1 with $\cos\theta_K > 0$ and $\cos\theta_K < 0$. In experiment, it is possible to divide the $B\to K\pi\pi\gamma$ and $D\to K\pi\pi e^+\nu$ decay widths into two regions, with $\cos\theta_K > 0$ and $\cos\theta_K < 0$, respectively. In both regions, one can make a fit of the $K\pi\pi$ spectrum by including different kaon resonances and then the involved decay widths for K_1 with $\cos\theta_K > 0$ and $\cos\theta_K < 0$ can be obtained. An analysis of the total decay width of $B\to K\pi\pi\gamma$ has been conducted by the Belle collaboration [15], deriving the branching ratio $\mathcal{B}[B^+\to K_1^+(1270)\gamma] = (4.3\pm 0.9\pm 0.9)\times 10^{-5}$. Based on the large amount of data accumulated by the LHCb collaboration and the future Belle-II experiment, it is expected that the involved decay widths for decays into K_1 with $\cos\theta_K > 0$ and $\cos\theta_K < 0$ can be obtained by such an analysis. With these partial widths, the up-down asymmetries can be obtained, and accordingly, the photon polarization can be used to probe or constrain new physics models.

Photon polarization in $B\to K\pi\pi\gamma$ and $D\to K\pi\pi e^+\nu$.—It is also meaningful to include contributions from more K_J resonances, and in particular, the $K_1(1400)$, $K_2(1430)$ contributions will interfere with that from the $K_1(1270)$. A vector $K^*(1410)$ resonance will not contribute to the photon helicity measurement, and thus, its contribution is not shown in the following.

With the K_1, K_2 resonating contributions, the $B\to K\pi\pi\gamma$ angular distribution now becomes

$$\begin{aligned}\frac{d\Gamma(B\to K\pi\pi\gamma)}{d\cos\theta_K} &= \frac{d\Gamma_{K_1\gamma}}{d\cos\theta_K} + \frac{1}{4}|B|^2|\vec{K}|^2\bigg\{(\cos^2\theta_K + \cos^2 2\theta_K) \\ &+ \lambda_\gamma \frac{2{\rm Im}[\vec{n}\cdot(\vec{K}\times\vec{K}^*)]}{|\vec{K}|^2}\cos\theta_K\cos 2\theta_K\bigg\} \\ &+ {\rm Im}[AB^*\vec{n}\cdot(\vec{J}\times\vec{K}^*)]\bigg\{\frac{1}{2}(3\cos^2\theta_K - 1) \\ &+ \lambda_\gamma\frac{{\rm Re}[AB^*(\vec{J}\cdot\vec{K}^*)]}{{\rm Im}[AB^*\vec{n}\cdot(\vec{J}\times\vec{K}^*)]}\cos^3\theta_K\bigg\}. \end{aligned} \quad (11)$$

The $(B, \vec{K})$ are nonperturbative coefficients relating to $K_2(1430)$, and their explicit forms can be found in Ref. [7]. As shown in the above equation, the photon polarization λ_γ can also be extracted through the K_2 contribution in the third line of the above equation or the $K_1 - K_2$ interference term in the fifth line. But again, such determinations require the knowledge of nonperturbative matrix elements, ${\rm Im}[\vec{n}\cdot(\vec{K}\times\vec{K}^*)]/|\vec{K}|^2$, and ${\rm Re}[AB^*(\vec{J}\cdot\vec{K}^*)]/{\rm Im}[AB^*\vec{n}\cdot(\vec{J}\times\vec{K}^*)]$.

Including the resonances, we also give the angular distributions for $D\to K_{\rm res}(\to K\pi\pi)e^+\nu$

$$\frac{d\Gamma(D\to K\pi\pi e\nu_e)}{d\cos\theta_K d\cos\theta_l} = \sum_{K_J = K_1, K_2, K'_{12}} \frac{d\Gamma_{K_J e\nu_e}}{d\cos\theta_K d\cos\theta_l}. \quad (12)$$

The $D \to K_2(\to K\pi\pi)e^+\nu$ contribution is

$$\begin{aligned}\frac{d\Gamma_{K_2 e\nu_e}}{d\cos\theta_K d\cos\theta_l} &= |c_0'|^2\frac{3}{2}\sin^2(2\theta_K)\sin^2\theta_l|\vec{K}|^2 + 2|c_+'|^2|\vec{K}|^2\cos^4\frac{\theta_l}{2}\bigg\{(\cos^2\theta_K + \cos^2 2\theta_K) \\ &+ 2\cos\theta_K\cos 2\theta_K\frac{\mathrm{Im}[\vec{n}\cdot(\vec{K}\times\vec{K}^*)]}{|\vec{K}|^2}\bigg\} + 2|c_-'|^2|\vec{K}|^2\sin^4\frac{\theta_l}{2}\bigg\{(\cos^2\theta_K + \cos^2 2\theta_K) \\ &- 2\cos\theta_K\cos 2\theta_K\frac{\mathrm{Im}[\vec{n}\cdot(\vec{K}\times\vec{K}^*)]}{|\vec{K}|^2}\bigg\}, \end{aligned} \tag{13}$$

The coefficients in the third and fifth terms on the right hand side mimic the required input $\mathrm{Im}[\vec{n}\cdot(\vec{K}\times\vec{K}^*)]/|\vec{K}|^2$ in the third line of Eq. (11). The $K_1 - K_2$ interference is given as

$$\begin{aligned}\frac{d\Gamma_{K_{12}^I e\nu_e}}{d\cos\theta_K d\cos\theta_l} &= -4\sqrt{3}\sin^2(\theta_K)\cos\theta_K\sin^2\theta_l\mathrm{Re}[c_0(c_0')^*\vec{J}\cdot\vec{K}^*] \\ &- 8\cos^4\frac{\theta_l}{2}\mathrm{Im}[c_+(c_+')^*\vec{n}\cdot(\vec{J}\times\vec{K}^*)]\bigg\{\frac{1}{2}(3\cos^2\theta_K - 1) + \cos^3\theta_K\frac{\mathrm{Re}[c_+(c_+')^*(\vec{J}\cdot\vec{K}^*)]}{\mathrm{Im}[c_+(c_+')^*\vec{n}\cdot(\vec{J}\times\vec{K}^*)]}\bigg\} \\ &- 8\sin^4\frac{\theta_l}{2}\mathrm{Im}[c_-(c_-')^*\vec{n}\cdot(\vec{J}\times\vec{K}^*)]\bigg\{\frac{1}{2}(1 - 3\cos^2\theta_K) + \cos^3\theta_K\frac{\mathrm{Re}[c_+(c_+')^*(\vec{J}\cdot\vec{K}^*)]}{\mathrm{Im}[c_+(c_+')^*\vec{n}\cdot(\vec{J}\times\vec{K}^*)]}\bigg\}. \end{aligned} \tag{14}$$

The $c'_{0,+,-}$ correspond to the nonperturbative amplitudes for D decays into the K_2. In this interference term, the relation between $\{(\mathrm{Re}[c_+(c_+')^*(\vec{J}\cdot\vec{K}^*)])/(\mathrm{Im}[c_+(c_+')^*\vec{n}\cdot(\vec{J}\times\vec{K}^*)])\}$ and $\mathrm{Re}[AB^*(\vec{J}\cdot\vec{K}^*)]/\mathrm{Im}[AB^*\vec{n}\cdot(\vec{J}\times\vec{K}^*)]$ is less obvious.

Discussion.—Although the lepton mass has been neglected, we have checked that our method is still valid, through the angular distribution analysis, when the lepton is massive. Thus, this analysis also applies to $D \to K_{\mathrm{res}}(\to K\pi\pi)\mu^+\nu_\mu$.

In the above, we have elucidated the method using the angular distribution of $D \to K_{\mathrm{res}}(\to K\pi\pi)l^+\nu$ decay, but one can also use the $B_s \to K_{\mathrm{res}}(\to K\pi\pi)l\bar{\nu}$, $D_s \to K_{\mathrm{res}}(\to K\pi\pi)l^+\nu$ decays and $\tau \to K_{\mathrm{res}}(\to K\pi\pi)\nu$. An estimate of the branching fraction of $B_s \to K_1 l\bar{\nu}$ is about $(3.65^{+2.27}_{-1.87}) \times 10^{-4}$ [16] and might be measured in the future. The $D_s \to K_{\mathrm{res}}l^+\nu$ is a $c \to d$ transition suppressed by the Cabibbo-Kobayashi-Maskawa matrix element and needs more data.

Combining the measurements in $B \to K\pi\pi\gamma$ and $D \to K\pi\pi\ell\nu$ can give the absolute value of $|C_{7R}/C_{7L}|$ via λ_γ in Eq. (3). This constraint on C_{7R}/C_{7L} is complementary to those from the time-dependent CP asymmetries in $B^0 \to f_{CP}\gamma$ (where f_{CP} is the CP eigenstate) [1,17,18] which measure $S_{f_{CP}\gamma} \propto \mathrm{Im}[e^{-i\phi}C_{7R}/C_{7L}]$ (where ϕ is the phase in the $B^0 - \bar{B}^0$ mixing), and the angular distributions in $B \to K^*(\to K\pi)\gamma(\to e^+e^-)$ [19–21] with $A_T^{(2)}(0) \propto \mathrm{Re}[C_{7R}/C_{7L}]$ and $A_T^{(im)}(0) \propto \mathrm{Im}[C_{7R}/C_{7L}]$.

Photon helicity and the right-handed couplings are similar for $b \to s\gamma$ and $b \to d\gamma$ in the SM, but might be different in NP models. Thus, the $B \to a_1(1260)\gamma \to \pi\pi\pi\gamma$ is also of great interest, and combining the measurements $B \to a_1\gamma$ and $D \to a_1 e^+\nu$ will allow us to determine the photon helicity in $b \to d\gamma$ in a model-independent way.

Conclusions.—Since photons emitted in $b \to s\gamma$ decay are predominantly left handed, measuring photon polarizations in this mode can test the standard model and probe the new physics effects with large right-handed couplings. It has been noticed that the photon polarization in $b \to s\gamma$ is related to the up-down asymmetry in $B \to K_1\gamma$. But unfortunately, this observation does not provide a full understanding of photon polarization, since this requires the knowledge on the $K_1 \to K\pi\pi$ decay, and introduces uncontrollable model dependence.

In this Letter, we have pointed out that one can combine the measurements in $B \to K_1\gamma$ and semileptonic $D \to K_1 l^+\nu (l = e, \mu)$ decays and determine the photon polarization in $b \to s\gamma$ without any theoretical ambiguity. In particular, we proposed a ratio of up-down asymmetries in $D \to K_1 e^+\nu$, $\mathcal{A}'_{\mathrm{UD}}$, to quantify the hadronic effects in $K_1 \to K\pi\pi$ decay. Experimental facilities including BESIII, Belle-II, and LHCb are likely to measure this ratio in the future.

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