From Stochastic Spin Chains to Quantum Kardar-Parisi-Zhang Dynamics

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We introduce the asymmetric extension of the quantum symmetric simple exclusion process which is a stochastic model of fermions on a lattice hopping with random amplitudes. In this setting, we analytically show that the time-integrated current of fermions defines a height field that exhibits quantum nonlinear stochastic Kardar-Parisi-Zhang dynamics. Similarly to classical simple exclusion processes, we further introduce the discrete Cole-Hopf (or Gärtner) transform of the height field that satisfies a quantum version of the stochastic heat equation. Finally, we investigate the limit of the height field theory in the continuum under the celebrated Kardar-Parisi-Zhang scaling and the regime of almost-commuting quantum noise.

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Random unitary dynamics arise in quantum mechanics as an efficient way of describing the evolution of systems interacting with environments or external fields. The original idea was introduced by Caldeira and Leggett to study the effective dynamics of collections of spins interacting with bosonic baths [1]. The properties of such systems are expected to notably differ from their isolated counterparts due to fluctuation and dissipation arising from interactions with unknown degrees of freedom. Random unitary dynamics are also useful to study typical and universal behaviors of quantum chaotic systems. As a consequence, their studies has been recently revitalized, notably in the context of random unitary circuits [2–9], or many body systems [10–16]. By adding stochasticity, these systems ought to lose their fine properties pertaining to particularities, such as conservation laws, thus allowing the emergence of generic properties. These include the production of entanglement [2,4,17–24], the scrambling of information [3,6,25,26], or the spreading of operators [5,7,8] in systems converging to thermal or out-of-equilibrium steady states. In particular, in some quantum stochastic models [4,14,15,19], it has been argued that the growth and fluctuations of the entanglement entropies are governed by the Kardar-Parisi-Zhang (KPZ) equation [27–33]. Large deviation fluctuations for the growth of entanglement in stochastic conformal field theories have been shown to belong to the KPZ class [34]. Some scaling features of the KPZ equation have also recently been found in super-diffusive nonstochastic spin chain models [35–38] for the long-time decay of the spin-spin correlation functions. The extent by which the KPZ-like behavior is universal in quantum many-body systems is an open question.

A model describing stochastic hoppings of fermions on a lattice, which can be seen as a continuous-time version of the random unitary circuit models, has recently been introduced [12,39,40]. The average dynamics of such quantum model reduces to the classical symmetric simple exclusion process (SSEP) [40], a well-known model of out-of-equilibrium classical statistical physics. One remarkable result of [39,40] consisted in showing that, beyond the decoherence effects at work in the mean dynamics, quantum coherences and hence entanglement patterns have a rich structure in such models. In this Letter, we extend these results to asymmetric processes by introducing an asymmetric version of the quantum SSEP model and deciphering its connection with both the classical asymmetric simple exclusion process (ASEP) and the classical or quantum KPZ equation. This hence provides one theoretical example of direct appearance of the KPZ physics in quantum spin chains. Since, classically, symmetric and asymmetric models do not belong to the same universality class, we expect that, in the quantum world, a similar statement will hold.

The connection between the quantum ASEP defined below, Eqs. (1) and (2), and the KPZ dynamics will be developed along different angles. First, we will show that the average dynamics of the model is described by the classical ASEP. Then, we will show how the time-integrated current of fermions follows a quantum stochastic dynamics, akin to the classical KPZ one. This connection is realized in three different manners: by mapping the spin chain dynamics to a quantum discrete version of the KPZ equation, Eqs. (5), or to a quantum analog of the stochastic discrete heat equation (SHE), Eqs. (8), or to a quantum version of the stochastic Burgers equation, Eq. (10). In all these instances, the noise keeps its quantum character and depends on off-diagonal quantum coherences. Since the classical ASEP and its scaling limit are at the root of the macroscopic fluctuation theory (MFT) [41], which is an effective theory coding for large deviation fluctuations in diffusive out-of-equilibrium classical systems, the quantum ASEP [(1), (2)] and its mapping to the quantum KPZ dynamics open the route towards the extension of the MFT to quantum coherence and entanglement fluctuations in quantum many body systems.

The model.—We consider an asymmetric extension of the quantum SSEP [39]. It describes fermions on a lattice undergoing hopping between nearest-neighbor with random amplitudes. In the Heisenberg picture, the infinitesimal stochastic Hamiltonian generating the flow on observables $\hat{O}_t \mapsto \hat{O}_{t+dt} = e^{idH_t}\hat{O}_t e^{-idH_t}$ is given by

$$dH_t = \sum_{j=-\infty}^{+\infty} [c^\dagger_{j+1} c_j dW^j_t + c^\dagger_j c_{j+1} d\bar{W}^j_t], \qquad (1)$$

where $(c^\dagger_j, c_j)$ are fermionic creation and annihilation operators acting at site j with anticommutation relations $\{c^\dagger_j, c_i\} = \delta_{ij}$ and $(dW^j_t, d\bar{W}^j_t)$ are *quantum noises*, attached to the edges, see Fig. 1. Quantum noises are operators acting on the Hilbert space of the reservoir, with canonical equal-time commutation relations $[dW^j_t, d\bar{W}^k_t] = \delta_{jk} dt$. Physically, they represent operators in the interaction picture creating and annihilating excitations in a bosonic reservoir. We define the stochastic average $\mathbb{E}[\cdot]$ as the trace over the degrees of freedom of the bath. Within stochastic averages, the noise satisfies the so-called quantum Itô rules $d\bar{W}^j_t dW^k_t = \delta_{jk}\alpha dt$ and $dW^k_t d\bar{W}^j_t = \delta_{jk}(1+\alpha)dt$ where α is the average number of excitations in the bath, and $\mathbb{E}[dW^j_t] = \mathbb{E}[d\bar{W}^j_t] = 0$. Our model further assumes that the noise is Markovian in the sense that there is no memory effect due to the large number of degrees of freedom of the bath: this implies $d\bar{W}^j_t dW^k_{t'} = dW^k_{t'} d\bar{W}^j_t = 0$ for $t \neq t'$. We refer the reader to [42–45] for more details on quantum noise.

The equation of motion of an operator $\hat{O}$ follows from expanding the flow (1) of $\hat{O}_t$ [46],

$$d\hat{O}_t = [\mathcal{L}^*_{\rm TASEP}(\hat{O}_t) + \alpha\mathcal{L}^*_{\rm SSEP}(\hat{O}_t)]dt + i\sum_{j=-\infty}^{+\infty} [c^\dagger_{j+1} c_j dW^j_t + c^\dagger_j c_{j+1} d\bar{W}^j_t, \hat{O}_t]. \qquad (2)$$

The superoperators are here given by $\mathcal{L}^*_{\rm TASEP}(\star) = \sum_{j=-\infty}^{+\infty}[c^\dagger_{j+1}c_j \star c^\dagger_j c_{j+1} - \frac{1}{2}\{(1-\hat{n}_j)\hat{n}_{j+1}, \star\}]$ and $\mathcal{L}^*_{\rm SSEP}(\star) = \sum_{j=-\infty}^{+\infty}[c^\dagger_j c_{j+1} \star c^\dagger_{j+1} c_j + c^\dagger_{j+1} c_j \star c^\dagger_j c_{j+1} - \frac{1}{2}\{\hat{n}_j(1-\hat{n}_{j+1}) + (1-\hat{n}_j)\hat{n}_{j+1}, \star\}]$. The notation $\{,\}$ stands for the anticommutator and $\hat{n}_j$ is the local density $c^\dagger_j c_j$. The superscript * denotes the fact that we are dealing with the dual of a Lindbladian on the operator space and we will explain the meaning of the subscript in the following. As an example, the dual Linbladians evaluated on the number operator $\hat{n}_k$ yield

$$\mathcal{L}^*_{\rm TASEP}(\hat{n}_k) = \hat{n}_{k+1}(1-\hat{n}_k) - \hat{n}_k(1-\hat{n}_{k-1}),$$
$$\mathcal{L}^*_{\rm SSEP}(\hat{n}_k) = (\hat{n}_{k+1} - \hat{n}_k) - (\hat{n}_k - \hat{n}_{k-1}). \qquad (3)$$

The form of these equations echoes the fermion number conservation, which leads to the local continuity equation, $d\hat{n}_k = (j_k dt + dB^k_t) - (j_{k-1}dt + dB^{k-1}_t)$, with noise

$$dB^k_t = i[c^\dagger_{k+1} c_k dW^k_t - c^\dagger_k c_{k+1} d\bar{W}^k_t], \qquad (4)$$

and current $j_k = [\hat{n}_{k+1}(1-\hat{n}_k) + \alpha(\hat{n}_{k+1} - \hat{n}_k)]$.

Correspondence to the ASEP.—The averaged dynamics of this model can be linked to classical exclusion processes. Let us define pointer states as elements of the form $\mathbb{P}_{[\epsilon]} = \prod_j^{\otimes} \mathbb{P}^{\epsilon_i}_j$ with $\epsilon \equiv \{\epsilon_j\}$, $\epsilon_j \in \{\pm\}$ and $\mathbb{P}^+_j = c^\dagger_j c_j$, $\mathbb{P}^- = (\mathbb{I} - c^\dagger_j c_j)$. To each pointer state, one associates a classical state such that $\mathbb{P}^+_j$ ($\mathbb{P}^-_j$) corresponds to an occupied state (empty state) at site j which we denote $|\bullet|$ ($|\circ|$). The operators $\mathbb{P}_{[\epsilon]}$ are mapped onto each others by the generator of the mean evolution: $\mathbb{E}[\mathcal{L}(\mathbb{P}_{[\epsilon]})] = \sum_{\epsilon'} Q[\epsilon']\mathbb{P}_{[\epsilon']}$. Starting from a diagonal density matrix $\rho_0 = \sum_\epsilon Q_0[\epsilon]\mathbb{P}_{[\epsilon]}$, the dynamics leaves the density matrix ρ_t diagonal on average and the weights $Q_t[\epsilon]$ at time t can be associated to the classical probability amplitude for the system to be in the configuration ϵ. The master equation satisfied by the weights Q_t can be found upon evaluation of the action of the Lindbladians on a pair of adjacent sites:

$$\mathcal{L}_{\rm SSEP}(|\bullet\circ|) = -|\bullet\circ| + |\circ\bullet|,$$
$$\mathcal{L}_{\rm SSEP}(|\circ\bullet|) = -|\circ\bullet| + |\bullet\circ|,$$
$$\mathcal{L}_{\rm TASEP}(|\bullet\circ|) = 0,$$
$$\mathcal{L}_{\rm TASEP}(|\circ\bullet|) = -|\circ\bullet| + |\bullet\circ|.$$

Both operators return zero upon acting on the states $|\circ\circ|$ and $|\bullet\bullet|$ since there is no particle or the exclusion freezes the dynamics. The transition rates are equal to the ones of the corresponding classical processes: $\mathcal{L}_{\rm SSEP}$ corresponds to the symmetric simple exclusion process and $\mathcal{L}_{\rm TASEP}$ to the totally asymmetric one. We emphasize that these statements

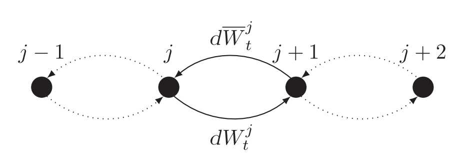

FIG. 1. Representation of the lattice and the hopping process. A fermion can jump to the right (to the left) with probability per unit time dW_t ($d\bar{W}_t$). These amplitudes correspond to an effective interaction with a bosonic bath.

are true if we restrict ourselves to the average evolution and to pointer states initial conditions. The quantum model embraces more general situations where quantum coherences play a key role [39,40]. At a large scale, the ASEP is known to converge towards the KPZ growth model under a weak asymmetry: we now show that a similar statement holds for the quantum case.

The Kardar-Parisi-Zhang quantum height.—Let us recall that the classical Kardar-Parisi-Zhang dynamics is described by a trio of equations: the KPZ equation, the stochastic heat equation, and the stochastic Burgers equation. Our task consists in unveiling their quantum counterpart in the context of our model. Let $\tilde{n}_j \equiv n_j - \frac{1}{2}$ be the centered local density and $\hat{h}_k \equiv \sum_{j=-\infty}^{k} \tilde{n}_j$ be the cumulated fermion number, or identically the time-integrated current of fermions [47], to which we refer as the quantum height field [48]. In classical exclusion processes, the height growth is related to the so-called corner growth process consisting in the growth of an interface from its corners, transforming local minima (maxima) onto local maxima (minima) whenever a particle passes through the corresponding site, see Fig. 2.

From now on and for convenience, we introduce the forwards gradient of the height as $\nabla_+ \hat{h}_k = \hat{h}_{k+1} - \hat{h}_k$, its backwards gradient as $\nabla_- \hat{h}_k = \hat{h}_k - \hat{h}_{k-1}$ and the discrete Laplacian as $\Delta = \nabla_+ \nabla_-$. From Eq. (2) or from the summation of the evolution of the number operators $\hat{n}_j$, we obtain the exact dynamics of the height as

$$d\hat{h}_k = \left[\left(\alpha + \frac{1}{2}\right)\Delta\hat{h}_k - \nabla_+\hat{h}_k\nabla_-\hat{h}_k + \frac{1}{4}\right]dt + dB_t^k. \quad (5)$$

Equation (5) plays the role of a discretized quantum KPZ equation. Note that the nonlinearity is a direct consequence of the noncommutativity of the noise. We emphasize that the noise term depends on off-diagonal observables with respect to the occupation number basis: therefore the fluctuations of the height, contrary to the classical case,

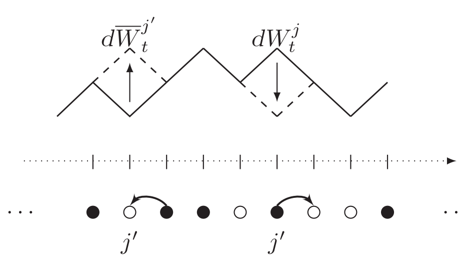

FIG. 2. Mapping from the particle picture to the height field. Bottom. Particles hop on the lattice and the occupied sites are black filled. Top. The configuration of particles is represented by an interface incremented with the presence of a particle at the corresponding site. The particle transport maps to a corner growth process: if a particle hops to the left (to the right), a corner is raised (lowered).

is strongly dependent on the existence of quantum coherences. Note also that the deterministic part of Eq. (5) comprises a nonlinear term rendering the equation intrinsically difficult to solve and the correlator of the noise depends on the height as

$$dB_t^k dB_t^{k'} = \delta_{kk'}\left[\frac{1}{2}\Delta\hat{h}_k + \left(\alpha + \frac{1}{2}\right)\left(\frac{1}{2} - 2\nabla_+\hat{h}_k\nabla_-\hat{h}_k\right)\right]dt. \quad (6)$$

To further study the model, let us remark that there is a miracle: defining the operator $\hat{Z}_k \equiv e^{\delta \hat{h}_k}$ with a suitable choice of parameter δ makes the dynamics of $\hat{Z}$ linear. This transformation is similar to the so-called discrete Cole-Hopf (or Gärtner) change of variable for classical models [50–52] and for this reason we will refer to $\hat{Z}$ as the quantum Cole-Hopf operator. From Eqs. (2) and (5), the evolution of the Cole-Hopf operator [53] is

$$d\hat{Z}_k = [(1+\alpha)(1-\hat{n}_k)\hat{n}_{k+1}e^{\delta/2} - \alpha\hat{n}_k(1-\hat{n}_{k+1})e^{-\delta/2}] \times 2\sinh\left(\frac{\delta}{2}\right)\hat{Z}_k dt + 2\sinh\left(\frac{\delta}{2}\right)dB_t^{k,(-)}\hat{Z}_k, \quad (7)$$

where we further introduced the decorated noise $dB_t^{k,(\pm)} = i[e^{\pm\delta/2}c_{k+1}^\dagger c_k dW_t^k - e^{\mp\delta/2}c_k^\dagger c_{k+1} d\bar{W}_t^k]$. Equation (7) remains nonlinear in the number operators for an arbitrary value of δ, but a wise choice allows us to cancel the nonlinearity: $\delta = \log[\alpha/(1+\alpha)]$. Fixing this value, the evolution of the Cole-Hopf operator is simplified to

$$d\hat{Z}_k = [\sqrt{\alpha(\alpha+1)}\Delta\hat{Z}_k - (\sqrt{\alpha+1} - \sqrt{\alpha})^2\hat{Z}_k]dt - \frac{1}{\sqrt{\alpha(\alpha+1)}}dB_t^{k,(-)}\hat{Z}_k. \quad (8)$$

The constant drift can be absorbed by redefining the Cole-Hopf operator as $\tilde{Z}_k \equiv e^{\mu t}\hat{Z}_k$ with $\mu = (\sqrt{\alpha+1} - \sqrt{\alpha})^2$. This amounts to shift the height field in another frame in translation with the original one [54]. Going from Eq. (5) to Eq. (8), we traded a nonlinear equation and an additive noise with a linear equation with a multiplicative noise. Remark the mechanisms making our Cole-Hopf transform work rely on the operator's algebra making it a nontrivial quantum feature. Since it describes a diffusion on $\mathbb{Z}$, the stochastic averaged dynamics of the Cole-Hopf operator $\tilde{Z}_k$ can be computed exactly [55],

$$\mathbb{E}[\tilde{Z}_k(t)] = \sum_{\ell\in\mathbb{Z}} I_{k-\ell}[2t\sqrt{\alpha(\alpha+1)}]e^{-2t\sqrt{\alpha(1+\alpha)}}\tilde{Z}_\ell(0), \quad (9)$$

where I is the modified Bessel function of the first kind [56]. We emphasize that Eq. (9) is operator valued since the trace over the fermionic degrees of freedom has not been taken. Finally, the third equation of the trio is obtained readily. Indeed, the backwards gradient of the quantum height field, $\tilde{n}_k = \nabla_- \hat{h}_k$ verifies a quantum discrete viscous stochastic Burgers equation

$$d\tilde{n}_k + \tilde{n}_k(\nabla_+ + \nabla_-)\tilde{n}_k dt = \left(\alpha + \frac{1}{2}\right)\Delta\tilde{n}_k dt + \nabla_- dB_t^k. \quad (10)$$

Due to the linear time growth of the height, it is classically more convenient to study the stochastic Burgers equation to obtain stationary measures in the KPZ physics. We expect the same phenomenon to remain valid in the quantum realm. Now that we have unveiled the trio of equation governing the KPZ physics, let us turn to the introduction of quantum replica and to the continuous field theory limit.

Quantum replica.—A possible direction to study the stochastic heat equation is by the means of the replica method pioneered by Kardar in [57]. We propose in this Letter to extend it to our quantum model. To this aim, we define the equal-time nth quantum replica as $u(k_1, \ldots, k_n) = \prod_{\ell=1}^n \tilde{Z}_{k_\ell}$, [58], which evolution can be derived from the Itô rules and the discrete SHE (8) as

$$\partial_t \mathbb{E}[u] = \mathbb{E}\left[\sqrt{\alpha(\alpha+1)}\sum_{\ell=1}^n \Delta_\ell u + \sum_{\ell<m} V_{k_\ell,k_m} u\right]. \quad (11)$$

The potential V is given by the correlator of the noise

$$V_{k_\ell,k_m} = \frac{1}{\alpha(\alpha+1)}\frac{dB_t^{k_\ell,(+)} dB_t^{k_m,(-)}}{dt}, \\ = \delta_{k_m k_\ell}\left[\frac{(1-\hat{n}_{k_m})\hat{n}_{k_m+1}}{1+\alpha} + \frac{\hat{n}_{k_m}(1-\hat{n}_{k_m+1})}{\alpha}\right]. \quad (12)$$

Equation (11) describes an imaginary-time Schrödinger equation on a lattice acting on a space of operators and shares some strong similarities with the δ-Bose gas or the Lieb-Liniger model [59,60]. Indeed, the potential V_{k_ℓ,k_m} is an attractive contact interaction and since all operators Z_{k_ℓ} commute, $u(k_1, \ldots, k_n)$ is invariant by permutation of the ks and therefore each replica can be interpreted as a bosonic particle. The Lieb-Liniger model is solvable using the Bethe ansatz [61], it would be of interest to develop an extension of this ansatz for Eq. (11). We leave the development of such an ansatz for a future work.

Limit to the continuum.—So far, our quantum model was considered on an infinite lattice. To investigate its large scale properties, we propose to probe its limit in the continuum in a similar fashion as for the classical exclusion processes [52]. We will show that in a regime that we call the almost-commuting quantum noise regime and in the (1:2:3) KPZ scaling limit, the continuous version of Eq. (5) is a quantum version of the continuous Kardar-Parisi-Zhang equation. The (1:2:3) KPZ scaling [62] refers to the following (height:space:time) scaling

$$\{\hat{h} \equiv \varepsilon^{-1}\tilde{h}, k \equiv \lfloor \varepsilon^{-2} x \rfloor, t \equiv \varepsilon^{-3}\tilde{t}\}. \quad (13)$$

In addition, the almost-commuting quantum noise regime is defined as the limit of an almost infinite number of excitations in the bosonic bath $\alpha \equiv \varepsilon^{-1}\alpha_0$. Both regimes are then considered in the limit $\varepsilon \to 0$. At leading order in ε, the (1:2:3) scaling leads to the following minimal replacements $\Delta h \to \varepsilon^3 \partial_{xx}\tilde{h}$, $\nabla h \to \varepsilon \partial_x \tilde{h}$, and $\delta_{kk'} \to \varepsilon^2 \delta(x - x')$. The equation of evolution of the quantum height (5) is then transformed onto

$$d\tilde{h} = \left[\alpha_0 \partial_{xx}\tilde{h} - (\partial_x \tilde{h})^2 + \frac{1}{4\varepsilon^2}\right] d\tilde{t} + dB_{\tilde{t}}^x, \quad (14)$$

where the noise in continuum is defined as $dB_{\tilde{t}}^x \equiv \lim_{\varepsilon\to 0} \varepsilon dB_t^k$. The correlator of the noise in the continuum is purely classical at leading order in ε,

$$dB_{\tilde{t}}^x dB_{\tilde{t}}^{x'} \underset{\varepsilon\to 0}{\to} \delta(x - x')\frac{\alpha_0}{2} d\tilde{t}. \quad (15)$$

Equation (14) is extremely similar to the classical KPZ equation although being operator valued. Similarly to what happens classically [52], since we expect the quantum interface described by the height $\tilde{h}$ to be *rough*, the ε^{-2} divergence in (14) is not surprising: one needs to renormalize the field by the addition of a correction term $t/4\varepsilon^2$ to obtain a well-defined theory. It is remarkable that the quantum character of the noise disappears in the continuum limit, see Eq. (15). Nonetheless, note that the nonlinearity remains relevant: since it originated from the noncommutativity of the quantum noise, this is an evidence that the model conserves some quantum properties on large scales. The discrete SHE (8) and the discrete Burgers equation (10) converge in the same fashion to the continuous SHE and stochastic Burgers equation.

There exists a number of analytic results concerning the classical KPZ equation in $1+1$ dimensions. Explicit solutions have been found for a variety of initial conditions [63–76], it would be interesting to know whether these admit an analog in the quantum case. Furthermore, the classical stochastic Burgers equation admits a Gaussian stationary measure [68,77] with the correlator given on the r.h.s. of (16). This hints at the existence of a stationary measure in the quantum case with a correlator for the quantum height gradient $\partial_x \tilde{h}$ (or equivalently the fermion density-density correlator) given by the classical correlator

$$\mathbb{E}[\langle \partial_x \tilde{h}(0,0)\partial_x \tilde{h}(x,\tilde{t})\rangle] \sim \frac{1}{\tilde{t}^{2/3}} f_{\mathrm{KPZ}}\left(\frac{x}{2\tilde{t}^{2/3}}\right), \quad (16)$$

where f_{KPZ} was calculated in [68,77]. An important question at this stage would be to investigate the stationary measure of the quantum Burgers equation. We leave this for a future work.

Discussion and outlook.—In this Letter, we have introduced a quantum asymmetric simple exclusion process and identified a quantum growing interface $\hat{h}(t)$ sharing strong similarities with a classical counterpart: the Kardar-Parisi-Zhang height. The KPZ physics is classically described by a trio of equations: the KPZ equation itself, the stochastic Burgers equation and the stochastic heat equation and we have established here the existence of their quantum counterpart both in the discrete and the continuous settings. This model provides one analytic example of the presence of the KPZ physics in a quantum model and we hope it will stimulate the community and help bridge the gap between classical and quantum out-of-equilibrium physics.

This model raises a number of questions. From a technical point of view it would be interesting to pursue the study of the moments of the Cole-Hopf operator and to investigate the full counting statistics and the entanglement entropy. The results [40] about quantum SSEP seems to point to an underlying integrable structure. Understanding better the role of the off-diagonal quantum coherences would be important. It would also be interesting to obtain the stationary measure of the theory.

This model questions the existence of a quantum KPZ-like universality class. In the classical KPZ physics, there are two major results concerning universality. The first one is the strong universality which states that for growth models with a fixed asymmetry, there exists a fixed-point at large time [78–80], which describes among other quantities the one-point fluctuations of the height with the celebrated $1/3$ exponent $h(t) \sim v_\infty t + t^{1/3}\chi$, where v_∞ is the average velocity of the interface and χ is a random variable. The second result is the weak universality [81] that asserts that the KPZ equation is the universal scaling limit of weakly asymmetric growth models under the $(1:2:3)$ scaling (13). Extending these results to the quantum realm is one challenge that could shed some new light on quantum out-of-equilibrium problems.

Finally, solving the SSEP and ASEP models have been instrumental in the formulation of the MFT [41]. We may expect that solving their quantum versions introduced in the Letter will play an analogue role in the formulation of a quantum extension of the MFT, aiming at describing quantum coherence and entanglement fluctuations in out-of-equilibrium quantum many body systems and completing the recently emerging membrane picture [18,19,22,23] for entanglement production in many-body systems.

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[47] Since the number operators verify a continuity equation $d\hat{n}_k = (j_k dt + dB^k_t) - (j_{k-1}dt + dB^{k-1}_t)$, one can integrate it with respect to the space and time variables to obtain that $\hat{h}_k = \int_0^t [j_k dt + dB^k_t]$ and therefore the height field also has the interpretation of a time-integrated current.

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[54] From the specific structure of the decorated noise, it is possible to rearrange the noise term for convenience in three different ways $\tilde{Z}^{1/2}_k dB^k_t \tilde{Z}^{1/2}_k = \tilde{Z}_k dB^{k,(+)}_t = dB^{k,(-)}_t \tilde{Z}_k$.

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