Laser Control of Resonance Tunneling via an Exceptional Point

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According to the familiar Breit-Wigner formula, tunneling through a potential barrier is strongly enhanced when the energy of the projectile is equal to the resonance energy. Here we show how a weak continuous wave laser can qualitatively change the character of resonance tunneling, and enforce a sudden and total suppression of the transmission by inducing an exceptional point (EP, special non-Hermitian degeneracy). Our findings are relevant not only for laser control of transmission in the resonance tunneling diodes, but also in the context of electron scattering through any type of metastable (e.g., autoionization, Auger, intermolecular Coulombic decay) atomic or molecular states, and even in the case of transmission of light or sound waves in active systems with gain and loss.

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The phenomenon of resonance tunneling was first encountered by Breit and Wigner in their theory of slow neutron capture [1]. The ubiquitous Breit-Wigner formula states that tunneling through a potential barrier is strongly enhanced when the energy of the projectile is equal to the resonance energy. The simplest and most important example of resonant tunneling is a single particle transmission through a double barrier potential (see, e.g., Ref. [2]). Dramatic enhancement of conductivity by resonance tunneling has been exploited in a large variety of contexts in physics and engineering, e.g., for producing a new class of functional materials designed for specific device applications (see Refs. [3–6]). For resonance tunneling in quantum wells and quantum wires, diodes, and in molecular electronics see, for example, Refs. [7–13].

Here we show that coherent destruction of resonance tunneling can be obtained by applying an external field with a certain specific fine-tuned intensity and frequency. Coherent destruction of tunneling by an external field was already discussed in the context of the double well potential and its generalizations [14–16]. However, the just mentioned works dealt with bound states, while in our present Letter we consider resonance tunneling in a double barrier potential, i.e., a process which involves solely the continuum. Moreover, in Refs. [14–16] the destruction of tunneling arises due to localized eigenstates, while our present work focuses on controlling the single particle transmission through a double barrier potential using light induced exceptional points (EPs).

The EPs, special non-Hermitian degeneracies (convertible to a Jordan block degeneracy by similarity transformation [17]), are intensively studied theoretically and experimentally in various fields of physics, e.g., in optics and laser physics [18–28], in acoustic systems [29,30], and in electronic systems [31,32]. For a review on the physics of EPs, see Ref. [33].

Experiments showing a dramatic effect of the EPs on physical properties require typically very accurate measurements of an exponentially decaying signal [34]. In our present Letter, we predict theoretically a new dynamical effect of the EP that does not require measurements of very weak signals. In fact, the strength of the relevant signal is as high as in standard measurements of resonance tunneling, performed, e.g., in diodes. More specifically, we will show here that, by applying a weak intensity laser field (either forming an EP or bringing the system close to an EP), the transmission peak in the resonance tunneling probability is suppressed and a dip rather than a peak is obtained. Our finding applies generally to any situation involving resonance tunneling in atoms, molecules, quantum dots, quantum wells, and diodes.

The EPs of quantum mechanics are well defined only within the non-Hermitian formulation of the theory [35]. Correspondingly, our subsequent treatment of the tunneling problem will be based upon the non-Hermitian (complex scaled) Lippmann-Schwinger equation (LSE) [36], which takes the form

$$T_{f\leftarrow i}(E) = \langle \phi_f(E)|\phi_i(E)\rangle - 2i\pi \langle \phi_f(E)|\hat{V}|\phi_i(E)\rangle - 2i\pi \sum_\alpha \frac{(\phi_f^\theta(E)|\hat{V}^\theta|\psi_\alpha^\theta)_{\mathbf{r}_\theta}(\psi_\alpha^\theta|\hat{V}^\theta|\phi_i^\theta(E))_{\mathbf{r}_\theta}}{E - E_\alpha^\theta}. \quad (1)$$

Here, $T_{f\leftarrow i}(E)$ stands for the transmission coefficient associated with the transition from an appropriate initial quantum state, $\phi_i(E)$, to a given final state, $\phi_f(E)$, of the same energy E. Symbol $\hat{V}$ represents the associated interaction potential invoking the transitions. The last term of Eq. (1) is constructed by means of the complex coordinate method [35]. Meaning that the relevant scattering coordinate(s) are rotated into the complex plane by an angle θ, i.e., $\mathbf{r} \mapsto \mathbf{r}_\theta = \mathbf{r}e^{i\theta}$. The initial and the final states $\phi_{i,f}^\theta(E)$ are identified here with the energy normalized continuum eigenfunctions of the asymptotic (interaction free) Hamiltonian $\hat{H}_0^\theta = \hat{H}^\theta - \hat{V}^\theta$. On the other hand, ψ_α^θ stands for a normalized eigenfunction of the full Hamiltonian $\hat{H}^\theta$ corresponding to an eigenvalue E_α^θ. Because $\hat{H}^\theta$ is a non-Hermitian operator, the c product $(\cdots|\cdots)$ is used rather than the standard scalar product $\langle\cdots|\cdots\rangle$, see Chaps. 8 and 9 of Ref. [35] for details. Further explanation of Eq. (1) and its derivation is given in the Supplemental Material [37].

The transmission probability for a particle of an impact energy E to pass through the interaction region of nonzero $\hat{V}$ is given as $P_{f\leftarrow i}(E) = |T_{f\leftarrow i}(E)|^2$. Figure 1 presents an illustrative example, in which $P_{f\leftarrow i}(E)$ describes tunneling through an one-dimensional (1D) symmetric double well potential. The calculation is based upon Eq. (1). As we recall, the spatial coordinate x has been rotated here into the complex plane by angle θ.

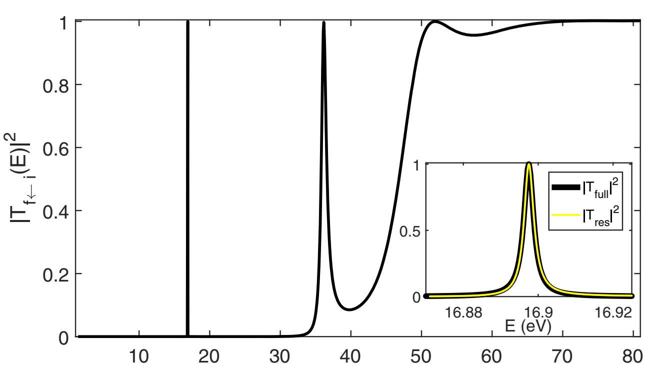

FIG. 1. The probability of transmission through an 1D double barrier potential, which takes the form $V(x) = (0.5x^2 - 0.8)e^{-0.1x^2}$ in atomic units. The calculation is based upon evaluating the right-hand side of Eq. (1) in a numerically exact manner. The spatial coordinate x has been rotated here into the complex plane by angle $\theta = 0.2$. For comparison, the yellow line displays results obtained by using Eq. (2), which assumes that a single isolated resonance controls the tunneling. One can see that Eqs. (1) and (2) are in agreement in the vicinity of the resonance peak.

For the impact energies E located in the vicinity of a single isolated narrow resonance (RES), the LSE of Eq. (1) can be dramatically simplified [36,38]. Namely, one may pick up only a single term $\alpha = \text{RES}$ from the second line of Eq. (1), and write just

$$\frac{T_{f\leftarrow i}(E)}{-2\pi i} \doteq \frac{(\phi_f^\theta(E)|V^\theta|\psi_{\text{RES}}^\theta)(\psi_{\text{RES}}^\theta|V^\theta|\phi_i^\theta(E))}{E - \mathcal{E}^{\text{RES}} + i\Gamma^{\text{RES}}/2}; \quad (2)$$

valid for $E \approx \mathcal{E}^{\text{RES}}$. Note that ψ_{RES}^θ is associated here with the complex eigenvalue $E^{\text{RES}} = \mathcal{E}^{\text{RES}} - i\Gamma^{\text{RES}}/2$, with $\mathcal{E}^{\text{RES}}$ being the real valued resonance energy and Γ^{RES} the pertinent resonance width. An approximative formula (2) is tested in the inset of Fig. 1 against the numerically exact results coming from Eq. (1). One can see that both formulas (1) and (2) provide practically identical results for $E \approx \mathcal{E}^{\text{RES}}$. Moreover, $|T_{f\leftarrow i}(E = \mathcal{E}^{\text{RES}})|^2 \doteq 1$. It has been shown [38] that an additional approximation converts $T_{f\leftarrow i}(E) = (2)$ into an even simpler appearance

$$\frac{T_{f\leftarrow i}(E)}{-2\pi i} = \frac{\gamma_i^{\text{RES}}\gamma_f^{\text{RES}}}{E - \mathcal{E}^{\text{RES}} + i\Gamma^{\text{RES}}/2}. \quad (3)$$

Here $\gamma_{i,f}^{\text{RES}}$ are constants equal to the square roots of the associated partial widths times extra phase factors (see Sec. VIII and Exercise 8.1 in Ref. [35]). When the two partial widths are identical [as in the case of any symmetric potential $V(x)$], one has $\gamma_i^{\text{RES}} = \gamma_f^{\text{RES}}$. Therefore, $\gamma_i^{\text{RES}}\gamma_f^{\text{RES}} = e^{i\phi}\Gamma^{\text{RES}}/2$, and Eq. (3) reduces to the familiar Breit-Wigner profile mentioned at the beginning of this article. In particular one finds that $|T_{f\leftarrow i}(E = \mathcal{E}^{\text{RES}})|^2 = 1$.

Before proceeding further, let us briefly mention three physically realistic situations which are, to a good approximation, described by a 1D double barrier potential. The first situation corresponds to scattering of electrons on an atom. Here the Feshbach autoionizing resonances (such as an $4d \to 5p$ excitation of In^+ [39]) can be mapped into shape type resonances of a 1D potential (like, e.g., the Gaussian potential of Ref. [40]). The second situation is associated with an experimental setup of a two-dimensional quantum dot [9,41]. An adiabatic elimination of one spatial coordinate results here again in a 1D double barrier effective potential. The third situation concerns resonance tunneling in semiconductor quantum heterostructures (Esaki's Nobel Prize of 1973 [42,43]).

Let us move now towards discussing the main subject of this Letter, namely, to the resonance tunneling for a potential supporting an EP. Slightly more generally, we shall assume that our potential supports two resonances (RES_1 and RES_2) that are almost degenerate such that the system is close to an EP. In the just mentioned (near) EP situation, the single term approximation (2) of Eq. (1) is not applicable. Instead, one needs to use an analogous two term formula

$$\frac{T_{f\leftarrow i}(E)}{-2\pi i} \doteq \sum_{n=1,2} \frac{(\phi_f^\theta(E)|V^\theta|\psi^\theta_{\mathrm{RES}_n})(\psi^\theta_{\mathrm{RES}_n}|V^\theta|\phi_i^\theta(E))}{E - \mathcal{E}^{\mathrm{RES}_n} + i\Gamma^{\mathrm{RES}_n}/2}; \quad (4)$$

valid again for $E \approx \mathcal{E}^{\mathrm{RES}_{1,2}}$.

It was shown in Refs. [44,45] that, for specific toy models fine-tuned as to match the EP condition (defined by degeneracy of eigenvectors), an EP induces a dip (zero) in the profile of $|T_{f\leftarrow i}(E)|^2$ for $E = \mathcal{E}^{\mathrm{EP}}$. Let us emphasize in this context that, unlike Refs. [44,45], our study describes the most general setting which includes also a *dynamical* effect of the EP on $|T_{f\leftarrow i}(E)|^2$. The term dynamical highlights here the fact that proximity of our system to the EP can be controlled during a single transmission experiment through varying the intensity and frequency of the used laser light. Bringing our system towards an EP does not require changing the parameters of the static potential, as opposed to Refs. [44,45]. Furthermore, the latter possibility of using lasers is highly relevant from an experimental point of view.

In fact, an EP induced occurrence of a dip can be anticipated from Eq. (4). Namely, close to an EP degeneracy one has $\mathcal{E}^{\mathrm{RES}_1} - i\Gamma^{\mathrm{RES}_1}/2 \doteq \mathcal{E}^{\mathrm{RES}_2} - i\Gamma^{\mathrm{RES}_2}/2 \doteq \mathcal{E}^{\mathrm{EP}} - i\Gamma^{\mathrm{EP}}/2$ and $|\psi^\theta_{\mathrm{RES}_1})(\psi^\theta_{\mathrm{RES}_1}| \doteq -|\psi^\theta_{\mathrm{RES}_2})(\psi^\theta_{\mathrm{RES}_2}|$, with the latter property coming from the c-orthonormality condition $(\psi^\theta_{\mathrm{RES}_n}|\psi^\theta_{\mathrm{RES}_{n'}}) = \delta_{nn'}$ (see the Supplemental Material [37]and Sec. IX.E in Ref. [35] for details). The right-hand side of Eq. (4) then approximately vanishes for $E \doteq \mathcal{E}^{\mathrm{EP}}$. Therefore, instead of a peak of the resonance transmission probability one expects a dip (i.e., total reflection).

Interestingly, the use of Siegert state formalism [46–48] (where only the resonance and antiresonance poles of the scattering matrix are taken into consideration) enables us to get a simple closed form expression for $|T_{f\leftarrow i}(E \approx \mathcal{E}^{\mathrm{EP}})|^2$. One has

$$P_{f\leftarrow i}(E = \mathcal{E}^{\mathrm{EP}} + \Delta E) \simeq \left| \frac{\Gamma^{\mathrm{EP}}/2}{\Delta E - i\Gamma^{\mathrm{EP}}/2} + \frac{\Gamma^{\mathrm{EP}}/2}{\Delta E + i\Gamma^{\mathrm{EP}}/2} \right|^2. \quad (5)$$

Showing that $P_{f\leftarrow i}(E = \mathcal{E}^{\mathrm{EP}} + \Delta E)$ consists basically of two mutually complex conjugated Breit-Wigner contributions which are coherently added together. The transmission profile (5) possesses a zero at $E = \mathcal{E}^{\mathrm{EP}}$ embedded between two maxima that are shifted by $\Delta E = \pm\Gamma^{\mathrm{EP}}/2$. This analytical result is confirmed by our numerical calculation (see Fig. 2 plotted and discussed below).

Having analyzed the impact of an EP on resonance transmission in a rather general setting, let us move on now towards a slightly more specific case of EPs induced and controlled by laser light. Consider again a particle of an effective mass m moving in 1D along coordinate x under an influence of a symmetric short ranged potential $V(x)$. Assume that this particle also interacts with a weak linearly polarized laser pulse of a given frequency ω and an adiabatic envelope $\epsilon_0(t)$. The corresponding complex scaled Hamiltonian is for our present purposes most conveniently written in the acceleration gauge [49], and possesses the form

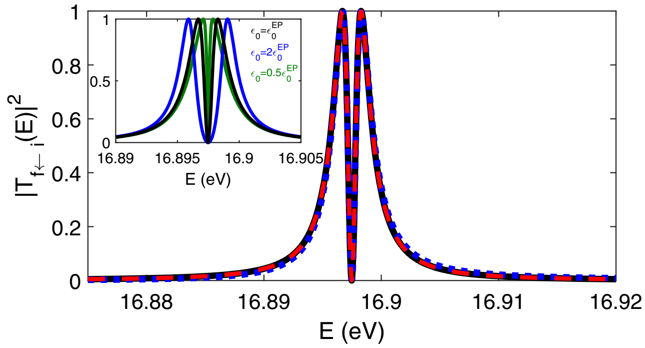

FIG. 2. The probability of transmission associated with scattering at the 1D double barrier potential of Fig. 1 dressed by a fine-tuned cw laser. The fine-tuned laser parameters generating an EP are equal to $\omega^{\mathrm{EP}} = 0.918\,930\,602\,139\,6$ a.u. and $\epsilon_0^{\mathrm{EP}} = 8.625\,863\,370\,106\,809 \times 10^{-5}$ a.u. Note the dip that is surrounded by two maxima which are separated by Γ^{EP}, in agreement with Eq. (5). One has $\Gamma^{\mathrm{EP}} = 0.001\,585\,516\,26$ eV. Note also that predictions of the LSE (1) (solid black line) agree with (4) (dashed red line) and (5) (dashed blue line) to all significant digits in the energy window plotted here. The results presented in the inset were obtained for laser frequency that equals to $\omega = \omega^{\mathrm{EP}}$, whereas the laser field strength ϵ_0 differs from ϵ_0^{EP} as indicated in the figure. Note that the dip survives even for $\epsilon_0 \neq \epsilon_0^{\mathrm{EP}}$ and its energy location remains unaffected.

$$\hat{H}^\theta(t) = -\frac{\hbar^2}{2m} e^{-2i\theta}\partial_{xx} + V(xe^{i\theta} + \alpha_0(t)\cos\omega t); \quad (6)$$

where $\alpha_0(t) = \epsilon_0(t)/(m\omega^2)$. Note that the particle becomes asymptotically free for $x \to \pm\infty$, this introduces naturally the concept of scattering by a time dependent target $V(xe^{i\theta} + \alpha_0(t)\cos\omega t)$. Such a scattering theory is rather straightforward to formulate when using continuous wave (cw) lasers, for which the envelope $\epsilon_0(t)$ is a constant, and $\hat{H}^\theta(t)$ is time periodic with the period $T = 2\pi/\omega$ of one optical cycle. Under such circumstances one may take advantage of Floquet theory [50], and move straightforwardly into studying an equivalent time independent coupled channel scattering problem. Moreover, in a single photon approximation (which is justified for weak enough laser intensities), one may deal with only two coupled scattering Floquet channels. The resulting effective two channel Floquet Hamiltonian takes then the following appearance:

$$\hat{H}^{\theta} = \begin{pmatrix} -\frac{\hbar^2}{2m}e^{-2i\theta}\partial_{xx} + V(xe^{i\theta}) & \frac{\alpha_0 V'(xe^{i\theta})}{2} \\ \frac{\alpha_0 V'(xe^{i\theta})}{2} & -\frac{\hbar^2}{2m}e^{-2i\theta}\partial_{xx} + V(xe^{i\theta}) + \hbar\omega \end{pmatrix}. \quad (7)$$

Here $V'(x) = \partial_x V(x)$. Scattering theory defined by this Hamiltonian can be formulated and resolved by taking advantage of the LSE (1) and by following all the subsequent considerations leading ultimately to Eq. (5). A self contained derivation of the Hamiltonian $\hat{H}^{\theta} = (7)$ is given in the Supplemental Material [37], together with a detailed description of the pertinent initial or final states $\phi^{\theta}_{i,f}(E)$, etc. In passing, we note that the concept of an EP directly induced by time periodic modulation has recently been reported in classical optics, both theoretically [51] and experimentally [52].

We proceed further with taking the same 1D test problem as in Fig. 1 and exposing it to a weak cw laser light as explained in the previous paragraph. The laser parameters (ω, ϵ_0) are deliberately fine-tuned in such a way as to induce an EP, via arranging for coalescence between the field free bound state (dressed by one photon) and the lowest field free shape resonance of $V(x)$. Subsequently, the corresponding two channel transmission probabilities are calculated, both in a numerically exact fashion based upon the LSE (1), using the associated two term approximation (4), and using the simple dip formula (5). All the underlying technical details are relegated to the Supplemental Material [37].

Physical significance of the obtained results is neatly documented by Fig. 2, which depicts the overall transmission profile for the impact energies E in the vicinity of $\mathcal{E}^{\mathrm{EP}}$ [recall that the meaning of $\mathcal{E}^{\mathrm{EP}}$ is clarified in Eq. (5)]. Most importantly, the Lorentzian peak observed for resonant scattering in the absence of laser (inset of Fig. 1) is split by the used weak laser (which generates the EP) into two peaks separated by an energy gap equal to Γ^{EP}. One also observes the dip (zero) of transmission, located between the just mentioned two peaks and anticipated in our theoretical discussion above. In passing, we note that a similar split-peak profile has been recently reported in optics [27].

It has been shown in Ref. [53] that when a particle interacts with an oscillating laser field, a field free bound state is transformed into a resonance, which in the context of scattering and tunneling plays the same mediating role as, e.g., the barrier resonances in an absence of laser. Namely, the transmission can get arbitrarily close to unity. The light induced EP presented above is created by the coalescence of the same type of resonance state as studied in Ref. [53] with a barrier resonance of our symmetric 1D potential.

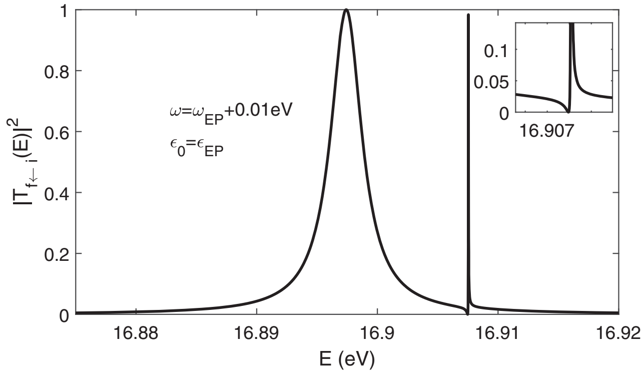

FIG. 3. The probability of transmission associated with scattering at the 1D double barrier potential of Fig. 1 dressed by a cw laser detuned from the EP condition. The detuned laser frequency equals to $\omega = \omega^{\mathrm{EP}} + 0.01$ eV, the laser field strength $\epsilon_0 = \epsilon_0^{\mathrm{EP}}$. Note the presence of the dip (shown separately in the inset). Note also that the predictions of the LSE (1) agree with Eq. (4) to all significant digits in the energy window plotted here.

One may ask at this point how does the profile plotted in Fig. 2 change when the laser parameters (ω, ϵ_0) are detuned from the EP condition. An answer is provided by Fig. 3 and by the inset of Fig. 2. One may see that the dip feature of $|T_{f\leftarrow i}(E)|^2$ survives both when detuning the frequency (Fig. 3) and the laser field strength (inset of Fig. 2) from the EP. Additionally, Fig. 3 is clearly reminiscent of the typical Fano profile. This observation supports the suggestion of Heiss and Wunner [45] that all such Fano line shapes imply the proximity of an EP. Importantly, Fig. 3 demonstrates that an imprint of the EP can be seen even when the laser intensity is quite far from its EP value. In passing we note that the separation ΔE of the transmission peaks of Fig. 2 and the laser field strength ϵ_0 are found to obey the relationship $\Delta E = (\epsilon_0/\epsilon_0^{\mathrm{EP}})\Gamma^{\mathrm{EP}}$.

In summary, the qualitative difference between the behavior of transmission in Fig. 1 and Figs. 2 and 3 represents the main message of the present Letter. We may conclude that the laser induced EP enables us to control totally the character of scattering through a double barrier potential. Importantly, the EP induced effects are visible in the transmission profile even when the laser parameters (ω, ϵ_0) are not exactly fine-tuned to $(\omega^{\mathrm{EP}}, \epsilon_0^{\mathrm{EP}})$. Note that, up to now, the only EP effects that have been discussed in the literature so far for the case of $(\omega, \epsilon_0) \neq (\omega^{\mathrm{EP}}, \epsilon_0^{\mathrm{EP}})$ include the so-called asymmetric switch [22,23], the survival probability [54,55], and scattering in optical microspirals [56]. Moreover, the dynamical effect presented here does not require any modification in the fabrication of the systems, e.g., the diodes, and can be observed in any processes which involve tunneling, by applying laser with the corresponding frequency and intensity.

Our findings open the door to a plethora of new ideas, such as control of resonance tunneling in quantum cascade [57] by a light induced EP. Similar approaches may apply for atomic, molecular, and mesoscopic systems, or for the case of active systems with gain and loss. Furthermore, the labile nature of resonance tunneling leads to a new regime where electrical switches can be controlled through electromagnetic radiation.

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