Comment on "Sign-Reversing Hall Effect in Atomically Thin High-Temperature $Bi_{2.1}Sr_{1.9}CaCu_{2.0}O_{8+\delta}$ Superconductors"

Materials fabrication and measurement technologies have progressed tremendously during the past two decades. Experiments that were once very difficult in the 1990s can now be done routinely. What was reported by Zhao *et al.* [1] is such an elegant realization. An examination of their data revealed a quantitative validation to what was predicted in the 1990s [2,3], corroborated well with another recent elegant experiment [4]. Nevertheless, Zhao *et al.* were still formulating their interpretation within an independent vortex dynamical model whose theoretical foundation had been questioned. Here I wish to point out that by overlooking the vortex many-body effect their main conclusion is incorrect.

To indicate the abundance of theoretical models, they stated that, "A rich theoretical lore attributes the Hall anomalies to either vortex pinning, details of the vortex core electronic spectrum, hydrodynamic effects, superconducting fluctuations, Berry phase, and charges in the vortex core" [1]. While their brief summary did capture the mood in the community, their Ref. [5] was not on Hall anomaly. For all those independent vortex dynamical models for Hall anomaly an inadequate physics had been reasoned [6]. Instead, a different kind of Hall anomaly model was proposed [2,3] to explain such a generic effect. It was based on the vortex many-body effect in competition with pinning. A few quantitative predictions were obtained as well. It is essentially the same physics as what is known in semiconductors: A pinned vortex lattice behaves as a filled topologically trivial band, with zero Hall effect; the effective topological number carriers, either thermally activated or externally induced, are responsible for the Hall effect as well as the longitudinal resistance. It was tested qualitatively [7]. The relevancy of such a model to Hall anomaly was acknowledged experimentally [8] and supported by data from other laboratories [4,5,9–18]. The authors of Ref. [1] did not report the comparison between their data and the vortex many-body Hall anomaly model.

To advance their theory of independent vortex dynamics type, they concluded that, "However, neither the explanation nor the consensus of the Hall behavior in the entire temperature range was achieved" [1]. This may be only partially valid in regards to the consensus. Nevertheless, an existing vortex many-body model has not only explained one sign change, but also predicted that two or more sign changes are possible. In addition, the Arrhenius law behavior of the resistance is a natural outcome. Furthermore, it has two quantitative predictions that the authors may already have data to directly check against.

(1) At low enough magnetic field the sign change may disappear. It is curious that they did not present data for magnetic fields below 2 T but above the effective H_{c1}. In thin films the effective H_{c1} can be very small.

(2) The quantitative formulas for the "activation energy" appeared in the Arrhenius law was given [3], the energy scale to generate effective carriers, vortex vacancies (or interstitials) in the vortex lattice. This energy scale again may be tested directly. The contributions of vacancies and interstitials to resistances are additive, same signs to longitudinal but opposite to Hall resistance.

From a general symmetry and topology perspective it was demonstrated that the Magnus force is insensitive to nonmagnetic impurities. Such a result was regarded as one of very few results in many-body physics [19]. Direct experimental measurement of the Magnus force on moving vortices validated such a prediction [20]. Full vortex dynamics has now been obtained from microscopic BCS theory without the uncontrolled relaxation time approximation [21]. In order for their independent vortex dynamical theory to apply, their Ref. [13], the Magnus force would have to be practically reduced to zero to have a near zero Hall effect. This key ingredient in their theory was achieved by a nontopological means of relaxation time approximation, but it should be pointed out that the type of theory relied on by the authors was questioned long ago because of the invalidation in the use of relaxation time approximation to obtain vortex friction. It has been known that, similar to a moving vortex, the friction of a moving object in a Fermi sea can be obtained without such approximation [22].

In conclusion, Zhao *et al.* had interpreted their main data based on an invalid independent vortex dynamics model and have reached their main conclusion incorrectly. Instead, Hall anomaly in type-II superconductors is generally an effect of the vortex many-body effect, which was not discussed by the authors though it is consistent with their observations.

This work was supported in part by the National Natural Science Foundation of China No. 16Z103060007.

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Received 2 July 2019; accepted 22 May 2020; published 19 June 2020

DOI: 10.1103/PhysRevLett.124.249701

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