Breakdown of Diffusion on Chiral Edges

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We show that dirty quantum Hall systems exhibit large hydrodynamic fluctuations at their edge that lead to anomalously damped charge excitations in the Kardar-Parisi-Zhang universality class $\omega \simeq ck - i\mathcal{D}k^{3/2}$. The dissipative optical conductivity of the edge is singular at low frequencies $\sigma(\omega) \sim 1/\omega^{1/3}$. These results are direct consequences of the charge continuity relation, the chiral anomaly, and thermalization on the edge—in particular translation invariance is not assumed. Diffusion of heat similarly breaks down, with a universality class that depends on whether the bulk thermal Hall conductivity vanishes. We further establish the theory of fluctuating hydrodynamics for surface chiral metals, where charge fluctuations give logarithmic corrections to transport.

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Introduction and results.—Quantum Hall (QH) droplets feature gapless excitations at their edge [1]. At temperatures far below the bulk gap, the bulk essentially remains nondissipative but the edge is expected to thermalize; thermalization implies that modes not protected by conservation laws should relax. In particular, the plethora of chiral Luttinger liquid channels predicted for certain QH states are damped by disorder and interactions, and only the collective excitations corresponding to charge [2,3] and heat [4] survive at late times. Early experiments in GaAs [5,6] indeed observed a single linearly dispersing collective excitation—the edge magnetoplasmon, associated with charge fluctuations—and later experiments found evidence for the neutral heat mode [7,8]. More recently, these QH edge modes were observed in graphene [9,10] and cold atoms [11,12].

Charge propagates ballistically on the edge, in the direction fixed by the sign of the filling $\nu = n/B$. The damping of this mode was first studied at zero temperature in Ref. [13]. In the hydrodynamic regime, i.e., at finite temperature T and low frequencies $\omega\tau_{\mathrm{th}} \ll 1$, the chiral ballistic front is expected to broaden diffusively [3]. The thermalization time τ_{th} may be controlled by various mechanisms depending on the microscopics of the edge [14]—the central assumption in this Letter is that it is sufficiently small so that frequencies $\omega \lesssim 1/\tau_{\mathrm{th}}$ can be probed experimentally.

Using fluctuating hydrodynamics, we will find that nonlinearities are relevant; large charge fluctuations lead to a breakdown of diffusion and drive the edge to a dissipative fixed point in the Burgers-Kardar-Parisi-Zhang (KPZ) universality class [17,18], with dynamic critical exponent $z = 3/2$ controlling the broadening of the chiral ballistic front $\omega \simeq ck - i\mathcal{D}k^{3/2}$. Breakdown of diffusion leads to a failure of the Einstein relation, and the optical conductivity is singular at low frequency $\sigma(\omega) \sim 1/\omega^{1/3}$. Singular low frequency transport is a hallmark of large hydrodynamic fluctuations [17,19]: when hydrodynamic interactions are instead irrelevant, response functions at the lowest frequencies are analytic and the interesting physics is instead hidden, e.g., in the temperature dependence of transport parameters. We stress that momentum conservation is not assumed—disorder therefore does not have to be introduced by hand, and does not regulate the singularity in $\sigma(\omega)$ which is only cut off by finite system size.

Without momentum conservation, hydrodynamic fluctuations are usually irrelevant and give small "long-time tail" corrections to diffusive transport [20,21] $\sigma(\omega) = \chi D + |\omega|^{d/2}$, where D is the diffusion constant and d the spatial dimension. The difference here stems from the fact that the $U(1)$ symmetry has a chiral anomaly. The interplay of anomalies and hydrodynamics has been appreciated since the work of Son and Surowka [22]. Although anomalies often only lead to subtle effects on transport, we show that the $(1+1)$D chiral anomaly has dramatic consequences, with ballistic propagation and large hydrodynamic fluctuations.

The connection between momentum-conserving hydrodynamics and the KPZ universality class has been long known [17,19,23]. More recently it was shown that the hydrodynamics of a nonintegrable spin chain, despite the lack momentum conservation, shows KPZ scaling at intermediate energies [24]. We show here that systems with a chiral anomaly display KPZ scaling all the way down to arbitrarily low energies even without momentum conservation.

In the remainder of this Letter, we establish the results discussed above for charge fluctuations. Energy fluctuations are then studied, and diffusion of heat on the edge is similarly shown to break down. Finally, we analyze fluctuations in higher dimensions. The upper critical dimension for these systems is $d_c = 2$, where fluctuations are marginally irrelevant, and the theory describes the hydrodynamics of surface chiral metals.

Charge fluctuations on the edge.—We study systems in one spatial dimension with a single $U(1)$ symmetry, that is anomalous,

$$\partial_\mu j^\mu = \frac{\nu}{4\pi}\epsilon^{\mu\rho}F_{\mu\rho}. \tag{1}$$

We are working in units where $e^2/\hbar = 1$. Such systems can be thought of as living on the boundary of a gapped bulk. When $\nu \in \mathbb{Z}$, the topological order in the bulk is trivial and the anomaly can be canceled by a properly quantized Chern-Simons term $(\nu/4\pi)AdA$ for the background field. When $\nu \notin \mathbb{Z}$ as in fractional QH states, the bulk has nontrivial topological order. We make no additional symmetry assumptions—in particular, momentum is not approximately conserved in any limit.

We are interested in the finite temperature properties of the system on the edge, at temperatures small compared to the bulk gap $k_B T \ll \Delta$. We will assume that the edge thermalizes—this implies that physics at the lowest frequencies is governed by hydrodynamics; namely, the dynamics of conserved densities: charge $n = j^0$, and heat (or energy). We postpone the treatment of heat to the next section; as we will see the dynamics of charge density alone is already surprisingly rich.

Dissipation in a theory with a nonanomalous $U(1)$ symmetry is described by simple diffusion $\omega \sim -iDk^2$. The goal of this section is to determine how the anomaly $\nu \neq 0$ changes this picture. The hydrodynamic treatment proceeds as follows [25,26]: at late times, all operators are controlled by their overlaps with hydrodynamic densities, here n. This holds in particular for the current, which can be written in terms of n—or its associated potential μ—in a gradient expansion

$$j_x = \frac{\nu}{2\pi}\mu - \chi D\partial_x\mu + \cdots, \tag{2}$$

where the charge susceptibility $\chi = \partial n/\partial\mu$ and diffusivity D are unknown functions of n (or μ), and $\cdots$ denotes higher gradient terms $O(\partial_x^2\mu)$. The anomaly fixes the leading term in the constitutive relation [27]. Combining Eqs. (1) and (2), one finds the following equation of motion for the charge density:

$$0 = \dot{n} + c\partial_x n - \partial_x(D\partial_x n) + \cdots, \tag{3}$$

with velocity $c = \nu/(2\pi\chi)$. Linearizing in the fluctuations $n = \bar{n} + \delta n$, the standard hydrodynamic approach [25] yields the retarded Green's function

$$G^R_{nn}(\omega, k) = \chi\frac{ick + Dk^2}{-i(\omega - ck) + Dk^2} + \cdots, \tag{4}$$

where the corrections … are less singular as $\omega, k \to 0$. Here and in the following, functions of n such as c, D, χ are evaluated on the background density $\bar{n}$. In the absence of an anomaly, the velocity c vanishes and one obtains a diffusive Green's function as expected. The linear analysis suggests that the anomaly $\nu \neq 0$ leads to a right-moving ballistic front at velocity $c = \nu/(2\pi\chi)$, with diffusive spreading around the front [2,3]. We will see that this conclusion is incorrect. The chiral ballistic front is tied to the bulk Hall conductivity

$$\sigma^{\text{bulk}}_{xy} = \lim_{\omega\to 0} G^R_{j_x n}(\omega, 0) = \chi c = \frac{\nu}{2\pi}, \tag{5}$$

and is a robust consequence of the anomaly. However, dissipation does not lead to diffusive spreading around the chiral front, because of a breakdown of the perturbative expansion in dissipative hydrodynamics. This can be seen by expanding the equation of motion (3) beyond leading order in δn (which we denote as n in the following for simplicity): writing $c(n) \simeq c + c'n$ with $c' \equiv \partial c/\partial n = -(\nu/2\pi)(\chi'/\chi^2)$ and $\chi' \equiv \partial\chi/\partial n$, one finds

$$\partial_x\eta_x = \dot{n} + c\partial_x n + \frac{1}{2}c'\partial_x n^2 - D\partial_x^2 n + \cdots. \tag{6}$$

In the absence of additional symmetries, there is no reason for c' to vanish and nonlinearities are generically expected, see, e.g., Ref. [29]. Less relevant nonlinear terms coming from the n dependence of the diffusivity D are omitted. We included a noise term η_x in the constitutive relation, whose symmetric Green's function is constrained by the fluctuation-dissipation theorem at leading order in gradients to be $\langle\eta_x(t, x)\eta_x\rangle = 2D\chi T\delta(x)\delta(t) + \cdots$. To establish the leading correction to ballistic propagation, it is convenient to work in the frame of the chiral front $x' = x - ct$, $t' = t$ (or, equivalently, $\omega' = \omega + ck$, $k' = k$). In these coordinates the equation of motion

$$\partial_x\eta_x = \partial_{t'}n + \frac{1}{2}c'\partial_x n^2 - D\partial_x^2 n + \cdots, \tag{7}$$

leads to a scaling $\omega' \sim k^2$, so that $\eta \sim k^{3/2}$ and $n \sim k^{1/2}$. One then finds that the interaction term c' is *relevant*, and drives the system to a new dissipative fixed point that is not described by diffusive spreading around the chiral front (4). In terms of the original coordinates, we expect $\omega - ck \sim k^z$, with $z < 2$ at the stable fixed point. In fact, Eq. (7) is nothing but the KPZ equation, with charge mapping to the slope of the interface $n = \partial_x h$, and the system is described by Burgers-KPZ [17,18] universality with $z = 3/2$. The symmetric Green's function is given by

$$G_{nn}(\omega,k)=\frac{\chi T}{\mathcal{D}k^z}g_{\mathrm{KPZ}}\left(\frac{\omega-ck}{\mathcal{D}k^z}\right)+\cdots, \tag{8}$$

where $\cdots$ are terms that are subleading in the scaling $(\omega-ck/k^z)\sim 1$, and g_{KPZ} is the KPZ scaling function which is known numerically to high precision [30,31]. $G_{nn}(\omega,k)$ is sharply peaked around $\omega=ck$ with a width of order $\mathcal{D}k^z$; charge fluctuations therefore obey the dispersion relation

$$\omega=ck-i\mathcal{D}k^z+\cdots. \tag{9}$$

KPZ scaling ties dissipation to thermodynamics: the dimensionful constant $\mathcal{D}\sim\mathrm{length}^z/\mathrm{time}$ is fixed in terms of parameters in the equation of motion by dimensional analysis

$$\mathcal{D}=\sqrt{T\chi}|c'|=\sqrt{\frac{T}{\chi^3}}\frac{|\nu|}{2\pi}|\chi'|. \tag{10}$$

This expression makes manifest three crucial ingredients that led to KPZ universality around the chiral front: finite temperature T, the anomaly ν, and thermodynamic nonlinearities through $\chi'=\partial\chi/\partial n$.

Equation (8) leads to a universal prediction for transport on the edge: the symmetric Green's function controls the dissipative optical conductivity at low frequencies $\omega\tau_{\mathrm{th}}\ll 1$ through the fluctuation-dissipation theorem and a Ward identity—one finds

$$\sigma(\omega)\simeq\lim_{k\to 0}\frac{\chi\omega^2}{2\mathcal{D}k^{7/2}}g_{\mathrm{KPZ}}\left(\frac{\omega}{\mathcal{D}k^{3/2}}\right)=a\frac{\chi\mathcal{D}^{4/3}}{\omega^{1/3}}, \tag{11}$$

with $a\approx 0.417816$ (see Supplemental Material [31]). While singular conductivites are common in one-dimensional momentum conserving systems [19,41,42], momentum conservation was not assumed here. This singularity as $\omega\to 0$ will be regulated in a system of finite length L, see Ref. [41] for a discussion on subtleties with the Kubo formula in this situation. Although $\lim_{\omega\to 0}\sigma(\omega,k)$ vanishes for $k\neq 0$, the relevant observable may be $\sigma(\omega,k)$ at $\omega\sim ck\sim c/L$ [19,41], in which case one finds $\sigma_{\mathrm{dc}}\sim\chi\mathcal{D}^{4/3}(L/c)^{1/3}$. This also leads to a thermal contribution to the current noise $S_{\mathrm{th}}=\sigma_{\mathrm{dc}}/L\sim L^{-2/3}$, which vanishes more slowly than the standard thermal contribution $S_{\mathrm{th}}\sim L^{-1}$. It would be interesting to explore the relevance of this correction in shot noise measurements in QH systems [43–45].

Nondissipative response such as the bulk Hall conductivity $\sigma_{xy}^{\mathrm{bulk}}$ is controlled instead by the real part of the retarded Green's function $\mathrm{Re}G_{nn}^R$, which can be obtained from $\mathrm{Im}G_{nn}^R$ by analyticity. This is done in the Supplemental Material [31], where we show that the quantized bulk Hall conductivity (5) is unchanged. Finally, long-range Coulomb interactions can be taken into account as usual in the random phase approximation by resumming a geometric series of diagrams involving photons—this does not qualitatively change the dispersion relation, which simply receives logarithmic corrections; see, e.g., Ref. [13].

The fate of heat.—We now extend the discussion to include the other hydrodynamic mode: heat, or energy. Heat has less privileged a status than charge, since it can leak out of the edge through phonons and will therefore only be approximately conserved. However, the timescale for heat loss may be parametrically longer than τ_{th} as it is controlled by different physics—a possibility affirmed by the experimental observation of the collective heat mode [7]. Neglecting first thermoelectric effects, charge and heat decouple and can be treated separately. A nonzero bulk thermal Hall conductivity κ_{xy} then gives heat a finite chiral speed of sound [4] $c_{\mathrm{heat}}=(\kappa_{xy}/c_V)$, where c_V is the specific heat, and the analysis in the previous section holds with heat replacing charge. This result is largely unaffected by coupling between charge n and energy ε—expanding the continuity relations as in Eq. (6) now yields a system of KPZ equations,

$$\partial_t n_a+C_{ab}\nabla n_b+D_{ab}\nabla^2 n_b+\lambda_{abc}n_b\nabla n_c+\cdots=0, \tag{12}$$

with $n_1=n$ and $n_2=\varepsilon$. As long as the velocity eigenvalues are distinct, going into the rest frame of any eigenmode one finds that interactions with the other eigenmodes are kinematically disfavored. One therefore expects two independent KPZ modes around each chiral ballistic front—this is indeed what is observed numerically [46,48].

One important exception is when the thermal Hall conductivity vanishes $\kappa_{xy}=0$, so that the heat mode does not propagate ballistically [4] (this happens, e.g., for $\nu=2/3$). Although a linearized analysis would suggest that heat then diffuses, its nonlinear coupling to the fluctuating charge mode also leads to a breakdown of diffusion in this case. This nonlinear coupling comes from the fact that in a background field $F_{0x}=E_x$, the energy continuity relation is changed to $\dot{\varepsilon}+\partial_x j_x^\varepsilon=E_x j_x$, which, using Eq. (2), fixes the leading term in the constitutive relation for the energy current [49] $j_x^\varepsilon=(\nu/4\pi)\mu^2+\cdots$. The two modes that diagonalize the C matrix in Eq. (12) are now $\delta\mu=\chi_{nn}^{-1}\delta n+\chi_{n\varepsilon}^{-1}\delta\varepsilon$ and $\delta s=(1/T)(\delta\varepsilon-\mu\delta n)$. The former is still described by KPZ universality, with a correlator of the form (8). Instead when $\kappa_{xy}=0$, entropy fluctuations have a vanishing speed and self-coupling $\lambda_{sss}=0$. However coupling to the KPZ mode $\lambda_{s\mu\mu}\neq 0$ leads to superdiffusion $\omega_{\mathrm{heat}}\sim -ik^{z_{\mathrm{heat}}}$. Although the exponent is not known analytically, a "mode coupling" approximation gives $z_{\mathrm{heat}}=5/3$ and seems consistent with numerics (see Refs. [23,41] for reviews). This approximation yields again $\bar{\kappa}(\omega)\sim 1/\omega^{1/3}$; however, soft heat modes now lead to a more singular charge conductivity

$\sigma(\omega) \sim 1/\omega^{2/5}$. It is interesting that soft fluctuations are further enhanced in the $\nu = 2/3$ state, where experiments have suggested higher sensitivity to finite system size [53].

Higher dimension: Surface chiral metals and chiral magnetic effect.—Two reasons drive us to generalize the theory above to higher dimensions: first, we will find that the KPZ fixed point can be accessed perturbatively from the upper critical dimension $d_c = 2$; second, chiral systems with diffusive broadening naturally occur in higher dimensions as well. In $d = 2$ the theory we consider furnishes the low-energy description of surface chiral metals [54,55]. These are boundaries of three-dimensional materials made from layered QH systems—they exhibit propagation of a chiral diffusive front in the direction of the layer and regular diffusion in the transverse direction, which was shown to be stable against localization [54]. In $d = 3$, the theory describes the hydrodynamics of a charge current subject to the chiral magnetic effect in the presence of a background magnetic field (decoupling momentum and energy fluctuations). The chiral magnetic effect [56] corresponds to a nonvanishing equilibrium value of the charge current in the presence of a magnetic field, and is due to the chiral anomaly. This effect arises in condensed matter systems such as Weyl semimetals [57,58], in heavy ion physics [59] and astrophysics [60].

The common feature to all such systems is the presence of a chiral front with diffusive broadening along a given direction, which we label with x, and of ordinary diffusion in the orthogonal directions, which we label with y^A, where $A = 2, \ldots, d$. Up to first order in gradients the constitutive relations for the current are

$$j_x = \frac{\nu}{2\pi}\mu - \chi D_x \partial_x \mu, \qquad j_A = -\chi D_\perp \partial_A \mu, \tag{13}$$

where the chemical potential μ is an arbitrary function of the charge density n. Working again in the frame of the chiral front $x' = x - ct$, $y' = y$, $t' = t$, the conservation equation for Eq. (13) reads

$$\partial_{t'} n + \frac{1}{2} c' \partial_x n^2 - D_{ij} \partial_i \partial_j n = \partial_i \eta^i + \cdots, \tag{14}$$

where $D_{ij} = \mathrm{diag}(D_x, D_\perp, \ldots, D_\perp)$. The correlator of the noise current $\eta^i = (\eta^x, \eta^A)$ is again fixed by thermal equilibrium: $\langle \eta_i(t,x,y)\eta_j \rangle = 2D_{ij}\chi T \delta(t)\delta(x)\delta^{(d-1)}(y)$. In this frame, $\omega \sim k^2$, implying that $\eta^i \sim k^{(d/2)+1}$, $n \sim k^{(d/2)}$, and thus c' scales as $k^{(2-d)/2}$, i.e., the interaction becomes marginal in $d = 2$ and irrelevant in $d > 2$. This stochastic system was first studied in Ref. [61] in the context of driven diffusive systems, where the drive plays a crucial role in enhancing hydrodynamic fluctuations. We emphasize that our system is not driven: instead the anomaly enhances fluctuations. To study the effects of fluctuations and determine the RG fate in $d = 2$ we implement the effective field theory approach to hydrodynamics [40,62], reviewed in the Supplemental Material [31]. This framework allows us to perform a dynamical RG analysis keeping all the symmetries manifest, and appropriately capturing contact terms in correlation functions. The central object is the path integral

$$Z = \int DnD\varphi_a e^{iS[n,\varphi_a]}, \tag{15}$$

where $\varphi_a(t, x, y^A)$ is an auxiliary field, and can be related to the noise currents in Eq. (14) following the Martin-Siggia-Rose (MSR) formalism [63]. The action associated to the stochastic equation (14) is given in the Supplemental Material [31]. Renormalization can be studied as usual by integrating out modes in a momentum shell $M \leq |\vec{k}| \leq \Lambda$. We find that $\chi, c', D_\perp$ do not renormalize. It is illuminating to express the renormalization of D_x in terms of a rescaled coupling. To this aim, we rescale $\partial_x \to \partial_x/\sqrt{D_x}$, $\partial_A \to \partial_A/\sqrt{D_\perp}$, $\varphi_a \to \varphi_a/\sqrt{T\chi}$, $n \to n\sqrt{T\chi}$ to canonically normalize the fields. Then the cubic coupling becomes $\lambda = c'\sqrt{(\chi T/D_x)}$. The β function for λ is

$$\beta_\lambda = -\frac{\varepsilon}{2}\lambda + \frac{\lambda^4}{32\pi c' \sqrt{D_\perp \chi T}}, \tag{16}$$

which shows that the coupling is marginally irrelevant in $d = 2$. For $d = 2 - \varepsilon$, the diffusive fixed point with $\lambda = 0$ is unstable, and the stable fixed point can be accessed perturbatively: $\lambda^{*3} = 16\pi\varepsilon c' \sqrt{D_\perp \chi T}$. As $d \to 1$, the fixed point will approach the KPZ universality class. For $d \geq 2$, $\lambda = 0$ becomes stable and is the only fixed point [64]. This is summarized in Fig. 1. It is interesting that the generalization to higher dimensions (14) is distinct from the one natural for the KPZ equation, where interactions are instead marginally relevant in $d = 2$.

For surface chiral metals in $d = 2$ where interactions are marginally irrelevant, the chiral diffusive fixed point is approached slowly and transport parameters run logarithmically. In this case one can solve the RG flow equation (16) and find the conductivity at low frequencies,

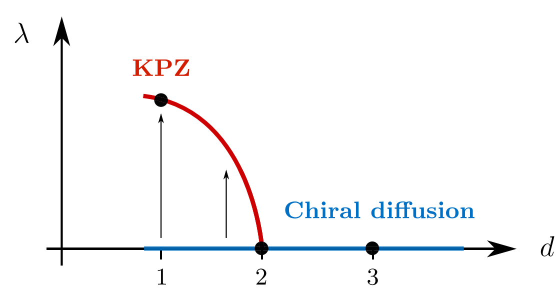

FIG. 1. Fixed points λ^* as a function of spatial dimension d.

$$\sigma_{xx}(\omega) = \chi D_x(\omega) \simeq \chi \left[\frac{3\chi T c'^2}{32\pi\sqrt{D_\perp}} \log\frac{1}{\omega}\right]^{2/3}. \quad (17)$$

In $d = 3$, pertaining to the chiral magnetic effect, the coupling λ is irrelevant. As shown in the Supplemental Material [31], one finds $\sigma_{xx}(\omega) \sim \sigma_{xx}(0) + \lambda^2\omega^{(1/2)}$. This is the same frequency power generated by fluctuation corrections in three-dimensional momentum-conserving systems.

Discussion.—We have shown that hydrodynamic fluctuations on edges realizing the chiral anomaly lead to a breakdown of diffusion, giving rise to singular low frequency transport and anomalous damping of edge modes. In particular, edge magnetoplasmons are predicted to be anomalously damped, Eqs. (9), (10). The charge susceptibility χ and $\chi' = \partial\chi/\partial n$ are nonuniversal but expected to have weak field and temperature dependence. The linear dependence of damping on filling ν for gapped bulks has been widely observed experimentally, see, e.g., Refs. [5,6]. The temperature and wave vector dependence of damping have been less systematically reported—the weak temperature dependence of damping observed in graphene in Ref. [10] is consistent with Eq. (10). To our knowledge only Ref. [6] studied wave vector dependence; the dependence they observe is between linear to quadratic, which would neatly agree with Eq. (9). Moreover, the overall size of damping observed is consistent with Eqs. (9), (10): estimating $\chi' \sim \omega_c$ to be of order the bulk gap and $\chi \sim 1/c$, one finds a quality factor $Q \sim \sqrt{\hbar\omega T}/\omega_c \sim 100$ at $\omega \sim 20$ MHz, consistent with Ref. [6]. However more thorough investigation—which is well within experimental reach—is needed to unequivocally confirm our prediction.

In the presence of edge reconstruction [65] with weak interedge interactions, our results can, in principle, apply to each edge (and perhaps most usefully to the outermost one), with the appropriate anomaly. If interedge interactions are strong enough, we instead expect only a single collective charge and heat mode to be long lived.

Nonlinear charge fluctuations have been argued to lead to a breakdown of the linearized edge picture even at $T = 0$ in the absence of dissipation, where the Burger's equation is stabilized not by the diffusive term in Eq. (3) but by a nondissipative two-derivative term (the Benjamin-Ono equation) [29], whose coefficient is related to the bulk Hall viscosity. This term leads to a nonanalytic correction to the dispersion relation $\omega - ck \sim k|k|$—we expect this nonanalyticity softens at finite temperature, and is then less relevant than the diffusive term.

Although we have focused on charge and heat modes at the edge of QH systems, the necessary ingredients—chiral edge modes protected by a continuity relation—are realized in a number of other systems, where similar conclusions will hold for transport of charge, heat, or spin on the boundary. These include gapped quantum spin liquids and topological superconductors [66], quantum anomalous Hall [67], and quantum spin Hall systems [68] (when spin conservation is a good approximation).

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