Positive Energy Functional for Massless Scalars in Rotating Black Hole Backgrounds of Maximal Ungauged Supergravity

M. Cvetič,[1,2] G. W. Gibbons,[3] C. N. Pope,[4,3] and B. F. Whiting[5]

[1]Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA
[2]Center for Applied Mathematics and Theoretical Physics, University of Maribor, SI2000 Maribor, Slovenia
[3]DAMTP, Centre for Mathematical Sciences, Cambridge University, Wilberforce Road, Cambridge CB3 0WA, United Kingdom
[4]George P. & Cynthia W. Mitchell Institute for Fundamental Physics and Astronomy, Texas A&M University, College Station, Texas 77843-4242, USA
[5]Department of Physics, University of Florida, Post Office Box 118440, Gainesville, Florida 32611-8440, USA

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We outline a proof of the stability of a massless neutral scalar field ψ in the background of a wide class of four dimensional asymptotically flat rotating and "electrically charged" solutions of supergravity, and the low energy limit of string theory, known as STU metrics. Despite their complexity, we find it possible to circumvent the difficulties presented by the existence of ergo regions and the related phenomenon of superradiance in the original metrics by following a strategy due to Whiting, and passing to an auxiliary metric admitting an everywhere lightlike Killing field and constructing a scalar field ψ (related to a possible unstable mode ψ by a nonlocal transformation) which satisfies the massless wave equation with respect to the auxiliary metric. By contrast with the case for ψ, the associated energy density of ψ is not only conserved but is also non-negative.

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Recent successes of the LIGO and Virgo gravitational wave detectors [1] and of the event horizon telescope [2] have triggered renewed interest in probing the near horizon geometry of black holes and in testing the predictions of general relativity against alternative theories of gravity. It is important that such alternative theories (i) have a well-posed initial value formulation, (ii) that proposed matter fields have an energy momentum tensor satisfying sensible energy conditions and, ideally, (iii) for any black hole solutions obtained, that some minimum requirement of stability holds. An example obtained from string theory and that meets the first two of these requirements is the Kerr-Sen metric [3], which generalized the Kerr-Newman spacetime and has recently been the subject of much investigation in relation to the recent results from observations with the event horizon telescope [2]. The supergravity-inspired STU metrics we study further generalize the Kerr-Sen metric, and the stability result we obtain applies equally well to it. STU theories are particularly interesting because they are Lagrangian based [4] and their matter sector separately satisfies the weak, strong, and dominant energy conditions (see Ref. [5] for definitions) in common with all ungauged supergravity theories [6]. Thus, they have the potential to provide a genuine, physically testable, mathematically consistent, viable alternative against which to evaluate the predictions of general relativity as a theory of gravity.

The stability of black hole spacetimes within general relativity has been of long-standing interest since the work of Regge and Wheeler [7] in 1957 and Zerilli [8] and Vishveshwara [9] in 1970 on the Schwarzschild spacetime, and the initial work of Press and Teukolsky [10,11] for the Kerr black hole in 1973. These papers were all concerned with mode stability: whether small perturbations can grow exponentially. A deeper analysis is required to establish whether linear perturbations decay generically at late times [12], and whether linear perturbations can excite nonlinear growth [13]. A subtlety that arises for black holes is that even if the solution settles down to a stationary final state, it may not have the same mass and angular momentum as the solution around which one is linearizing. The final solution should, however, by the no-hair theorems, be a regular stationary black hole within the space of solutions of the theory under consideration.

When one considers the stability of the rotating Kerr solution [14], one encounters the difficulty of dealing with superradiance. Two key steps in understanding the stability of the Kerr solution were the separation of the relevant perturbation equations [15] and the construction, using properties of the resultant confluent Heun equations, of a modified wave equation admitting a conserved positive energy [16]. The latter has played an important role in recent advances in this subject [17,18].

The extension to the rotating electrically charged Kerr-Newman solution has been impeded by the lack of separation results analogous to those of Ref. [15]. For the special case of an electrically neutral massless scalar wave equation on a fixed Kerr-Newman background, see

Refs. [19,20]. Very recently, the Teukolsky equations for Kerr-Newman spacetime have been obtained [21] as a first, very preliminary, step towards being able to discuss the stability question in this difficult case.

The main purpose of the present Letter is to report on the mode stability of the massless wave equation on the fixed background of a class of four-dimensional four-charged rotating black hole solutions [4,22] of ungauged supergravity theory known as "STU black holes," which are generalizations of the Kerr-Newman black hole, admitting a separation of variables [23]. STU supergravity is a consistent truncation of maximal four-dimensional supergravity, having four Abelian gauge fields, and the four "charges" of the black holes we consider here are carried by these fields. If the charges are set equal, the solution reduces to the Kerr-Newman black hole [24]. The four-dimensional supergravity can be embedded within the low-energy limit of string theory, by means of a dimensional reduction on a 6-torus. For an account of the existence of Killing Staeckel and Killing-Yano tensors extending known results in the Kerr case, see Refs. [24,25] and [26].

In previous work [27,28], three of us have advocated the use of these metrics as a foil to assess the accuracy with which astronomical observations of black holes can confirm that the metric is given by that of Kerr [14]. Quite apart from the STU black holes being solutions within supergravity or string theory, they have the merit that one can view them as a parameterized family of generalizations of the Kerr black hole for which, ungauged, their accompanying matter sector obeys the dominant, strong and weak energy conditions of general relativity. A further distinct advantage they have over many rival versions of modified gravity is that STU metrics can be obtained from a Lagrangian and so have a Hamiltonian and an initial-value formulation. To be effective as a foil, it is vital to understand whether STU black holes are stable, failing which they could not become the end points of gravitational wave emission arising from the inspiral and merger of massive black holes, as have recently been observed by the LIGO and Virgo Collaborations, starting with GW150914 in 2015 [1]. A similar reliance on stability applies for the use of STU black holes (in particular the special case of the Sen [3] black hole) in relation to observations made by the event horizon telescope [2].

An outgoing mode solution is one that has no support on the past horizon or at past null infinity. With the separation of variables given in Eq. (6), an outgoing mode with complex ω would grow exponentially in time if $\mathrm{Im}\omega > 0$, and would therefore be considered "unstable." Outgoing modes where ω is real and not equal to 0, if they exist, would also be considered unstable, since they would present an obstacle to proving decay at late times ($\omega = 0$ is discussed further below). Outgoing modes with $\mathrm{Im}\omega < 0$ decay exponentially in time, and are "quasinormal modes." We shall follow the strategy employed for the Kerr metric in Ref. [16]. We start with the separated solutions of the massless wave equation $\Box_g \psi = 0$, where $\Box_g$ is the covariant d'Alembertian of the stationary STU black hole metric g, and then construct a new function Ψ satisfying $\Box_{\hat{g}} \Psi = 0$, where $\Box_{\hat{g}}$ is the covariant d'Alembertian of an auxiliary metric $\hat{g}$ with a null Killing vector field K^μ.

By contrast with ψ and the physical metric $g_{\mu\nu}$, the conserved energy current $J^\mu = -K^\nu T^\mu{}_\nu$ has a positive energy density, $J^0 \geq 0$, where $T_{\mu\nu} = \partial_\mu \Psi \partial_\nu \Psi - \frac{1}{2} \hat{g}_{\mu\nu} \hat{g}^{\rho\sigma} \partial_\rho \Psi \partial_\sigma \Psi$ is the energy-momentum tensor with respect to the auxiliary metric $\hat{g}_{\mu\nu}$. In Ref. [16] it was shown that establishing positivity of an energy functional, albeit with respect to the auxiliary metric $\hat{g}_{\mu\nu}$, of the auxiliary field Ψ suffices to show mode stability of the original massless scalar field ψ. The relationship between Ψ and ψ is nonlocal, being effected by means of a pair of certain integral and differential transforms, chosen so that an unstable mode for ψ maps to an unstable mode for Ψ.

For the auxiliary metric $d\hat{s}^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu$, we have

$$d\hat{s}^2 = \Omega^2 \left(\frac{dr^2}{\Delta} + d\theta^2 \right) + \frac{(P + a^2 \cos^2\theta) \Delta \sin^2\theta}{a^2 \Omega^2} d\phi^2 - \frac{2\Delta^{1/2} \sin\theta}{a} dt d\phi, \tag{1}$$

where Δ is given in Eq. (4), and P is given in Eq. (17), and Ω is given in Eq. (23). The function P is manifestly positive for all $r \geq r_+$ (the radius of the outer horizon). This implies that with respect to the auxiliary metric $\hat{g}_{\mu\nu}$ the azimuthal Killing vector ∂_ϕ is everywhere spacelike, and the Killing vector ∂_t is everywhere lightlike. By contrast with respect to the physical STU metric $g_{\mu\nu}$ the Killing vector ∂_t is timelike outside the ergosphere upon which it becomes null and spacelike within the ergosphere.

The fact that ∂_t is lightlike with respect to $\hat{g}_{\mu\nu}$ and the fact that $T_{\mu\nu}$ satisfies the dominant energy condition, implies the positivity of the energy density J^0. Specifically, from $J^\mu = -K^\nu T^\mu{}_\nu$ and $K = \partial_t$, the total energy $\int \sqrt{-\hat{g}} d^3 x J^0$ is

$$\frac{1}{2a} \int \sin\theta dr d\theta d\phi [(P + a^2 \cos^2\theta)(\partial_t \Psi)^2 + \Delta (\partial_r \Psi)^2 + (\partial_\theta \Psi)^2], \tag{2}$$

which is manifestly positive, reducing to that for the Kerr metric in Ref. [16] if the "electric charges" are set to zero.

We shall now present a brief summary of the key steps in the derivation of the auxiliary metric (1). Further details will be given in a longer paper to follow [29].

The four-charge rotating STU black hole solution was obtained in Ref. [22]; a convenient form for the metric is [4]:

$$ds^2 = -\frac{\rho^2 - 2\mu r}{W}(dt + \bar{\omega} d\phi)^2 + W\left(\frac{dr^2}{\Delta} + d\theta^2 + \frac{\Delta \sin^2\theta d\phi^2}{\rho^2 - 2\,\mu r}\right), \quad (3)$$

where

$$\begin{aligned} W^2 &= r_1 r_2 r_3 r_4 + a^4 \cos^4\theta + [2r^2 + 2\,\mu r \Sigma_i^2 + 8\,\mu r \Pi_c \Pi_s \\ &\quad - 4\mu^2(\Sigma_{ijk}^2 + 2\Pi_s^2)] a^2 \cos^2\theta, \\ r_i &= r + 2\mu s_i^2, \qquad \rho^2 = r^2 + a^2\cos^2\theta, \\ \Delta &= r^2 - 2\,\mu r + a^2 = (r - r_-)(r - r_+), \\ \bar{\omega} &= \frac{2\,\mu a [r \Pi_c - (r - 2\mu)\Pi_s] \sin^2\theta}{\rho^2 - 2\,\mu r}, \\ c_i &= \cosh \delta_i, \qquad s_i = \sinh \delta_i, \\ \Pi_c &= c_1 c_2 c_3 c_4, \qquad \Pi_s = s_1 s_2 s_3 s_4, \\ \Sigma_i^2 &= s_1^2 + s_2^2 + s_3^2 + s_4^2, \qquad \Sigma_{ijk}^2 = \sum_{i<j<k} s_i^2 s_j^2 s_k^2. \end{aligned} \quad (4)$$

Changing variables from r and θ to

$$x = \frac{r - r_-}{r_+ - r_-}, \qquad y = \frac{1}{2}(\cos\theta + 1), \quad (5)$$

the massless scalar wave equation $\Box \psi = 0$ can be separated by writing

$$\psi = e^{-i\omega t} e^{im\phi} \frac{X(x)}{\sqrt{x(x-1)}} \frac{Y(y)}{\sqrt{y(y-1)}}, \quad (6)$$

with $X(x)$ and $Y(y)$ satisfying

$$\frac{X''(x)}{X(x)} + V_x = 0, \qquad \frac{Y''(y)}{Y(y)} + V_y = 0, \quad (7)$$

where a prime denotes a derivative with respect to the argument, and

$$V_x = -\tilde{\alpha}^2 + \frac{\tilde{\alpha}\tilde{\kappa} + \tilde{\lambda} + \frac{1}{2}\tilde{\kappa}^2}{x} + \frac{\frac{1}{4} - \tilde{\beta}^2}{x^2} + \frac{\tilde{\alpha}\tilde{\kappa} - \tilde{\lambda} - \frac{1}{2}\tilde{\kappa}^2}{x-1} + \frac{\frac{1}{4} - \tilde{\gamma}^2}{(x-1)^2}, \quad (8)$$

$$V_y = -\alpha^2 + \frac{\alpha\kappa + \lambda + \frac{1}{2}\kappa^2}{y} + \frac{\frac{1}{4} - \beta^2}{y^2} + \frac{\alpha\kappa - \lambda - \frac{1}{2}\kappa^2}{y-1} + \frac{\frac{1}{4} - \gamma^2}{(y-1)^2}. \quad (9)$$

The constants in the potential V_x are given by

$$\begin{aligned} \tilde{\alpha} &= i\omega(r_+ - r_-), \qquad \tilde{\kappa} = -\frac{i}{4}\omega(r_+ + r_-)\sum_i \cosh 2\delta_i, \\ \tilde{\beta} &= \frac{i[am - \omega(r_+ + r_-)(\Pi_c r_- + \Pi_s r_+)]}{r_+ - r_-}, \\ \tilde{\gamma} &= -\frac{i[am - \omega(r_+ + r_-)(\Pi_c r_+ + \Pi_s r_-)]}{r_+ - r_-}, \\ \tilde{\lambda} &= \frac{1}{2} + \tilde{\alpha}(\tilde{\gamma} - \tilde{\beta}) - \frac{1}{2}(\tilde{\gamma} - \tilde{\beta})^2 + \lambda_T + \nu, \end{aligned} \quad (10)$$

(from which $\omega_+ = a/[(r_+ + r_-)(\Pi_c r_+ + \Pi_s r_-)]$) with

$$\nu = \frac{\omega^2(r_+ + r_-)^2}{32}\left[\sum_i \tilde{c}_i^2 - 6\sum_{i<j}\tilde{c}_i\tilde{c}_j + 16(\Pi_c - \Pi_s - 1)^2 + 32(2\Pi_c - 1)\right], \quad (11)$$

where $\tilde{c}_i = \cosh 2\delta_i$; and in V_y we have

$$\begin{aligned} \alpha &= 2a\omega, \qquad \kappa = 0, \qquad \beta = -\frac{m}{2}, \qquad \gamma = \frac{m}{2}, \\ \lambda &= \frac{1}{2} + \alpha(\gamma - \beta) - \frac{1}{2}(\gamma - \beta)^2 + \lambda_T. \end{aligned} \quad (12)$$

In these expressions, λ_T is the analogue of the separation constant employed in Ref. [15] in the case of the Kerr metric. Note that $\omega = m\omega_+$ is the superradiant threshold, where $\tilde{\gamma}$ in Eq. (10) changes sign.

An unstable mode would have

$$X(x) \sim \begin{cases} e^{\tilde{\alpha}x} x^{-\tilde{\kappa}} & \text{as } x \to \infty, \\ (x-1)^{-\tilde{\gamma}} & \text{as } x \to 1. \end{cases} \quad (13)$$

Following the strategy in Ref. [16], we define a new radial function $\tilde{h}(x)$ by means of the integral transform ($\mathrm{Im}\omega > 0$),

$$\tilde{h}(x) = e^{\hat{\alpha}x} x^{\hat{\beta}} (x-1)^{\hat{\gamma}} \times \int_1^\infty dz e^{2\tilde{\alpha}xz} e^{-\tilde{\alpha}z} z^{-\frac{1}{2}-\tilde{\beta}} (z-1)^{-\frac{1}{2}-\tilde{\gamma}} X(z), \quad (14)$$

where

$$\begin{aligned} \hat{\alpha} &= -\tilde{\alpha}, \qquad \hat{\kappa} = -\tilde{\beta} + \tilde{\gamma}, \qquad \hat{\beta} = -\frac{1}{2}(\tilde{\beta} + \tilde{\gamma} + \tilde{\kappa}), \\ \hat{\gamma} &= -\frac{1}{2}(\tilde{\beta} + \tilde{\gamma} - \tilde{\kappa}), \qquad \hat{\lambda} = \tilde{\lambda}. \end{aligned} \quad (15)$$

The function $\tilde{X}(x) = \sqrt{x(x-1)}\tilde{h}(x)$ satisfies the same equation as does $X(x)$ in Eq. (7), except that V_x in Eq. (9) is now given by replacing the tilded constants by the hatted ones given in Eq. (15) [16], and has been chosen so that an unstable mode for $X(x)$ maps to an unstable mode for $\tilde{X}(x)$. With these transformations, the radial equation becomes

$$x(x-1)\tilde{h}'' + (2x-1)\tilde{h}' + [\omega^2 P(x) + \omega^2 a^2 - 4am\omega x - \lambda_T]\tilde{h} = 0, \quad (16)$$

where

$$P(x) = (r_+ + r_-)^2 \left[\frac{(\sum_i \tilde{c}_i - 4\Pi_c + 4\Pi_s)^2 (x-1)}{64x} + \frac{(\sum_i \tilde{c}_i + 4\Pi_c - 4\Pi_s)^2 x}{64(x-1)} + 2(\Pi_c + \Pi_s)x \right] + (r_+ - r_-)^2 x(x-1) - a^2, \quad (17)$$

and the function $P(x)$ is manifestly positive for all $x \geq 1$.

In a similar vein, one may introduce an angular function $v(y)$, this time related to $Y(y)$ via a differential transform, with $\tilde{Y}(y) = \sqrt{y(y-1)}v(y)$ satisfying the same equation as does $Y(y)$ in Eq. (7), except that V_y in Eq. (9) is now given by replacing the constants $\alpha, \beta, \gamma, \kappa$ by barred ones, given by [16] (Note: λ is unaffected)

$$\bar{\alpha} = \alpha, \quad \bar{\kappa} = -\beta + \gamma, \quad \bar{\beta} = \frac{1}{2}n + \beta, \quad \bar{\gamma} = \frac{1}{2}n - \gamma, \quad (18)$$

where the integer n is equal to $|m|$. Taking m, without loss of generality, to be non-negative, v satisfies

$$y(y-1)v'' + (2y-1)v' + [4a^2\omega^2 y(1-y) + 4am\omega(y-1) - \lambda_T]v = 0. \quad (19)$$

We now perform an "unseparation of variables," defining a wave function $\Psi(t,x,y,\phi) = e^{-i\omega t + im\phi}\tilde{h}(x)v(y)$. Expressed in terms of the original radial and angular variables r and θ by using (5), Ψ satisfies the wave equation

$$\left[\partial_r(\Delta\partial_r) + \frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta) - (P + a^2\cos^2\theta)\partial_t^2 - 2a\left(\cos\theta + \frac{r-\mu}{\epsilon_0\mu}\right)\partial_t\partial_\phi \right]\Psi = 0, \quad (20)$$

where $\epsilon_0 = (r_+ - r_-)/(r_+ + r_-)$. (Note: λ_T cancels between the radial and angular terms in the unseparation process.) This generalizes the result given in Ref. [16] for the Kerr metric, the difference being the replacement of the function f in that paper by the function P in Eq. (20). By pursuing techniques similar to those used in Refs. [18,30,31], our results for $\mathrm{Im}\omega > 0$ extend to real $\omega \neq 0$. For $\omega = 0$, ψ in Eq. (6) becomes time independent. The radial equation then reverts to the hypergeometric equation, with three regular singular points r_-, r_+, and ∞, and requires a separate discussion. In the Kerr limit, Teukolsky [15] has shown that this equation does not have solutions which are well behaved both at the horizon and at infinity (see also Ref. [11]). By a similar method, that conclusion also holds in the present case.

One can read off, up to an arbitrary conformal scaling, an inverse metric $\hat{g}^{\mu\nu}$ such that the terms involving two derivatives of Ψ in Eq. (20) are of the form $\hat{g}^{\mu\nu}\partial_\mu\partial_\nu\Psi$. Thus, the nonzero components are of the form

$$\hat{g}^{rr} = \Delta\Omega^{-2}, \qquad \hat{g}^{\theta\theta} = \Omega^{-2},$$
$$\hat{g}^{tt} = -(P + a^2\cos^2\theta)\Omega^{-2},$$
$$\hat{g}^{t\phi} = -a\left(\cos\theta + \frac{r-\mu}{\sqrt{\mu^2 - a^2}}\right)\Omega^{-2}. \quad (21)$$

One may then look for a choice of the conformal factor such that the remaining terms in the wave equation (20) are reproduced also; that is, that a multiple of Eq. (20) may be written as the covariant d'Alembertian

$$\frac{1}{\sqrt{-\hat{g}}}\partial_\mu(\sqrt{-\hat{g}}\hat{g}^{\mu\nu}\partial_\nu\Psi) = 0. \quad (22)$$

That an Ω exists is nontrivial, since the same function Ω must produce both the $(\partial_r\Delta)\partial_r\Psi$ term and the $\cot\theta\partial_\theta\Psi$ term. In fact, an Ω does exist, and can be given by

$$\Omega^2 = \Delta^{1/2}\left(\cos\theta + \frac{r-\mu}{\sqrt{\mu^2 - a^2}}\right)\sin\theta. \quad (23)$$

The auxiliary metric itself is given in Eq. (1). An extended version of our work presented here is in preparation [29].

To appreciate more fully the complexity of the STU metrics, it is necessary to expand completely out the compact expressions presented in Eqs. (4) and (17), and used in Eqs. (3) and (20). It should be recognized that any proof of stability for STU metrics may require a process at least as complicated as that which has only been introduced this year by Ref. [21] for the Kerr-Newman problem, in a spacetime that was discovered over 50 years ago. Hopefully, results will be quicker in the STU case, with our results providing the first hint of encouragement that similar such gargantuan efforts as in Ref. [21] may, in the end, be worthwhile for STU metrics too.

In conclusion, we have provided a proof of mode stability for a massless scalar field ψ in the background of a wide class of four-dimensional asymptotically flat rotating and "electrically charged" supergravity black holes, whose energy-momentum tensor satisfies the dominant energy condition in general relativity. We handled the difficulties presented by the existence of ergo regions in the original metrics by passing to an auxiliary metric admitting an everywhere lightlike Killing field and constructing a scalar field Ψ related to ψ by a nonlocal integral transformation which satisfies the massless wave equation with respect to the auxiliary metric. We find that the associated energy density of Ψ is not only conserved but is also non-negative. Our results open the way to establishing decay estimates as obtained in Ref. [17].

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