Absolute Magnetometry with ^{3}He

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We report development of a highly accurate (parts per billion) absolute magnetometer based on ^{3}He NMR. Optical pumping polarizes the spins, long coherence times provide high sensitivity, and the ^{3}He electron shell effectively isolates the nuclear spin providing accuracy limited only by corrections including materials, sample shape, and magnetization. Our magnetometer was used to confirm calibration, to 32 ppb, of the magnetic-field sensors used in recent measurements of the muon magnetic moment anomaly $(g_\mu - 2)$, which differs from the standard model by 2.4 ppm. With independent determination of the magnetic moment of ^{3}He, this work will lead the way to a new absolute magnetometry standard.

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Magnetometry, determining the intensity of a magnetic field over space and time, is crucial to many fields of applied and fundamental science including medicine, materials science, geology and geodesy, astronomy, and fundamental physics. Often-competing magnetometry requirements include sensitivity, stability, absolute accuracy, bandwidth, and spatial resolution, and many applications require accurately determining the magnetic field intensity in Tesla $(\mathrm{kg}\cdot\mathrm{s}^{-2}\cdot\mathrm{A}^{-1})$. The most sensitive magnetometers, superconducting quantum-interference devices (SQUIDs) reach sub $\mathrm{fT}/\sqrt{\mathrm{Hz}}$ sensitivity [1], but are not absolute. Optically pumped atomic magnetometers approach this sensitivity in small fields [2,3], but are generally not highly accurate or stable. Accurate absolute magnetometers include rotating coils [4] and calibrated NMR [5].

Magnetic fields are most accurately determined by measuring the *frequency* corresponding to the energy difference of two quantum states. The frequency ω and magnetic field B are related by $B = \omega(\hbar J/|\mu|) = \omega/|\gamma|$, where $\hbar J$ is the total angular momentum, μ is the magnetic moment, and $\gamma = \mu/\hbar J$. $J = 1/2$ nuclei in diamagnetic materials are the most nearly ideal two-state quantum systems, and NMR of proton rich substances, e.g., H_2O, is most commonly encountered. For pure H_2O, the shielded proton magnetic moment for a spherical sample at 25° $\mu'_p(25°)$ is known to 11.3 ppb, limited mostly by measurement of $\mu'_p/\mu_e(H)$ [6,7].

In practical magnetometry, temperature-dependent sample, shape, and external material effects lead to perturbations of the magnetic field that must be calibrated or corrected. With much smaller intrinsic corrections illustrated by the practically engineered magnetometer demonstrated in this Letter, a new more precise and more accurate standard for magnetometry can be established with ^{3}He. Unlike H_2O, for which the signal size is proportional to $1/T$, the ^{3}He is hyperpolarized by laser optical pumping techniques; thus ^{3}He magnetometry is applicable over a very broad range of magnetic fields using NMR for fields >0.1 T and atomic sensors for lower fields [8].

Quantities relevant to determining the magnetic moment of a free ^{3}He atom are provided in the Supplemental Material [9]. Diamagnetic shielding of the atomic electrons reduces the field at the ^{3}He nucleus, referred to as the helion. The shielded helion moment μ'_h has been measured to 4.3 ppb relative to the shielded proton magnetic moment in water [33] and to 4 ppb relative to protons in high-pressure H_2 [34], but the uncertainty on μ'_h, 11.4 ppb, is dominated by the uncertainty on $\mu'_p/\mu_e(H)$. To eliminate the proton from the chain, effort is underway [35,36] to directly measure the magnetic moment of the unshielded nucleus μ_h in a Penning trap using techniques similar to those used to measure the proton [37] and antiproton [38] g factors. The high-precision theoretical calculation of the diamagnetic shielding [39], which is only weakly temperature dependent, would provide the shielded magnetic moment μ'_h at the few ppb level.

The muon magnetic moment anomaly—A spin $\frac{1}{2}$-particle (lepton) with charge q_l and mass m_l has a magnetic moment $\mu_l = g_l(q_l/2m_l)\hbar/2$. For a structureless particle with no radiative corrections $g_l = 2$ [40], however, interactions with the virtual fields of the quantum vacuum lead to corrections, and $g_l = 2(1 + a_l)$ [41–43]. The largest correction, ≈0.1%, is due to QED with the strong and weak interactions entering at 3 and 6 orders of magnitude less, respectively. For the muon, the most recent standard model determinations a_μ^{SM} are summarized in [44,45]. The muon magnetic-moment

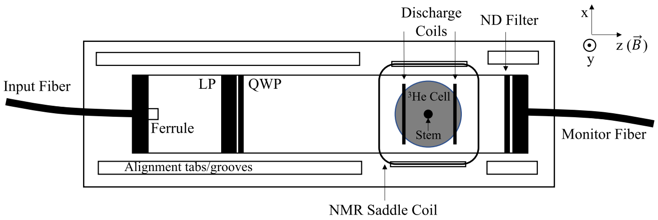

FIG. 1. The ^{3}He magnetometer assembly described in the text. The assembly is 5 cm square in cross section and 15 cm long. The distance from the input-fiber ferrule to the cell is approximately 8 cm. LP and QWP indicate the linear polarizer and quarter wave plate.

anomaly has been measured by producing muons in accelerators at CERN [46] and Brookhaven [47]. The most recent measurements $a_\mu^{\rm exp}$ from Brookhaven (E821) for μ^+ [48] and μ^- [49] reveal $a_\mu^{\rm exp} - a_\mu^{\rm SM} = (28.0 \pm 7.4) \times 10^{-10}$ or $(2.40) \pm (0.63)$ ppm, using $a_\mu^{\rm SM}$ from [45]. A new measurement with higher statistical precision and smaller systematic errors is underway at Fermilab and is expected to improve the uncertainty by a factor of 4 [50].

Experimentally, muons are confined in a 7.1 m radius magnetic storage ring with weak vertical focusing by electric quadrupole fields. The anomaly frequency, determined from the variation of the rate of μ decays into positrons or electrons that exceed a threshold energy, is $\omega_{a_\mu} \approx a_\mu (e/m_\mu)\tilde{B}$ for muon momentum near 3.094 GeV/c, where the electric field effects are effectively canceled. The field averaged over the detected muon trajectories and time is $\tilde{B} \approx 1.45$ T. A chain of measurements using proton-NMR magnetometers (probes) calibrated with high-purity H_2O yields the frequency $\tilde{\omega}'_p = (2|\mu'_p|/\hbar)\tilde{B}$. In terms of ω_{a_μ} and $\tilde{\omega}'_p$,

$$a_\mu = \frac{g_e}{2} \frac{\omega_{a_\mu}}{\tilde{\omega}'_p} \left| \frac{\mu'_p}{\mu_e} \right| \frac{m_\mu}{m_e}, \tag{1}$$

where μ'_p/μ_e is determined to 10.5 ppb [9], g_e has been determined to 0.28 ppt [51], and m_μ/m_e is determined to 22 ppb from the muonium hyperfine splitting [52] and QED [7]. In this work, the ^{3}He magnetometer independently determined and confirmed the systematic corrections to the standard H_2O reference probes used in E821.

The ^{3}He *Magnetometer*—The magnetometer illustrated in Fig. 1 is a compact device designed to operate at 1.45 T providing polarization and NMR magnetometry *in situ*. The required elements are the ^{3}He sample contained in a glass cell, coils to excite a discharge in the ^{3}He, the NMR coil for excitation and pickup of free-precession, optical fibers to guide the incident optical pumping light and to monitor transmission through the cell for laser tuning, circular polarization optics, and the mounting structure. The design principles require removing all ferromagnetic materials including electronic components and cables, minimizing materials near the ^{3}He sample and fabricating as closely as possible cylindrically or spherically symmetric distributions of materials.

The magnetometer was assembled in a 3D printed mount with grooves that aligned the components. In our initial (Mark-I) assemblies the mount was polylactic acid thermoplastic (PLA) with a square cross section "clamshell" for ease of mounting, alignment, and access for cell orientation and material perturbation studies. The plastic mount was enclosed in 0.002-inch thick copper folded from a flat sheet and soldered at the joints. The ^{3}He cells were blown borosilicate (PYREX) glass, approximately spherical with 2.5 cm diameter and a stem several mm long (see Supplemental Material [9]). Several ^{3}He pressures were investigated, and 10 torr (20 °C) provided the best combination of polarization lifetime, free-precession relaxation time (T_2^*), and signal size.

Optical pumping of metastable ^{3}He or MEOP has been extensively studied, most recently at high magnetic fields relevant to this work [9,53–55]. A discharge was excited by radio frequency (4–6 MHz) applied to a 2-cm-diameter coil pair in contact with the outside of the glass cell. The input fiber provided optical pumping light from a 1083 nm, 2W fiber-amplified laser [56]. For development studies at 2–3 mT an optical polarimeter monitored the circular polarization of the 668 nm fluorescence indicating 30–50% ^{3}He polarization [57]. We did not directly measure the ^{3}He polarization at 1.45 T, however, NMR signal sizes and system parameters were consistent with 4%–8% in the 10-torr cell. The polarization (discharge on) time was typically 1–2 minutes, and the polarization lifetime was greater than five hours.

The ^{3}He NMR system consisted of a saddle coil surrounding the ^{3}He cell that produced a field along the y axis. The NMR coil, matched to 50 Ω and tuned to the ^{3}He resonant frequency $\omega'_h/(2\pi) \approx 47.1$ MHz, provided both the NMR pulse and inductive pickup. The coil was connected to the NMR controller described in the Supplemental Material [9]. The mixer reference frequency was set below the precession frequency, and the 100–300 Hz mixed-down signal was digitized. An identical NMR controller for the H_2O probes was tuned to 61.7 MHz and interfaced to the same data acquisition system. The NMR tip angle was ≈23° for ^{3}He leaving about 90% of the longitudinal polarization after each pulse and providing variation of the longitudinal and transverse magnetization for systematic studies. The 90° pulse for H_2O protons maximized the signal size. The pulse and mixer reference frequencies for both the ^{3}He and proton (H_2O) channels and the data acquisition sample trigger were generated by separate function generators all locked to a single rubidium clock. Details of the NMR signal processing and data blinding are discussed in the Supplemental Material [9].

Calibration of ^{3}He *and E821* H_2O *probes*—The two H_2O probes used in E821, one with a spherical sample and

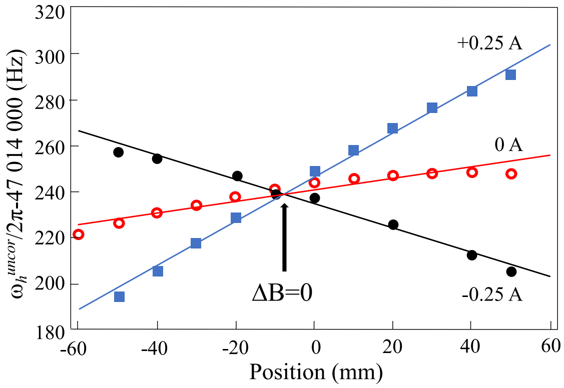

FIG. 2. Scan of the ^{3}He frequency as the magnetometer was translated along z with positive and negative gradient currents showing the $\Delta B = 0$ position z_0. The solenoid's higher order (quadratic) gradients are evident; straight lines are provided to guide the eye.

one with a cylindrical sample, are described in detail in Ref. [58]. The calibration of each H_2O probe to ^{3}He used a precision-shimmed superconducting solenoid magnet. The magnetic field drift was dominated by a diurnal cycle and was less than 60 ppb over the five hour calibration measurements. The ^{3}He magnetometer and H_2O probes were mounted on a translation stage, and a positive or negative linear gradient along each axis was applied by rapidly ($\lesssim$1 min) reversing the gradient-coil currents to determine a unique position (x_0, y_0, z_0), the $\Delta B = 0$ position. Adjusting each probe to find x_0 and y_0 then required only translation along the z axis to position each probe at the $\Delta B = 0$ position indicated in Fig. 2.

Calibration studies of the two H_2O probes were undertaken on separate days, with small modifications to the ^{3}He setup between the two studies. Figure 3 shows the uncorrected frequencies $\omega_3^{\rm uncor}$ and $\omega_p^{\rm uncor}$ as a function of time during calibration of the cylindrical H_2O probe showing the effect of the magnetic-field drift. All frequency measurements were interpolated to the average time of the ^{3}He measurements indicated by the vertical dotted line in Fig. 3. Corrections were applied to both species' interpolated frequencies to provide the two shielded frequencies ω_3' and ω_p' for each H_2O probe at the specified position and time.

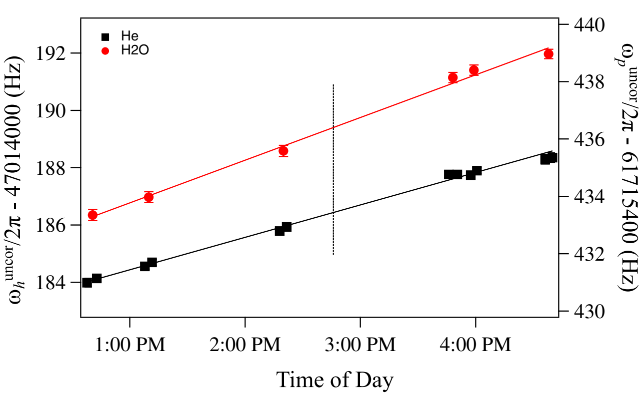

FIG. 3. Uncorrected frequencies for ^{3}He (left axis) and the cylindrical H_2O probe (right axis); the solid lines are linear fits vs time. The χ^2_ν, 1.0 and 1.2, respectively, for ^{3}He and H_2O, are accounted for in the magnetic drift uncertainties. The vertical dotted line indicates the time to which all frequencies are interpolated. Both scales span 10 Hz. The different slopes are due to the different magnetic moments.

Corrections and uncertainties—Corrections, determined independently within one day of each calibration study, are presented in Table I. Corrections fall into four categories: materials external to the sample, sample container materials, sample-material magnetization effects, and temperature dependence. Though all of these are in principle applicable to both species, the size and method of determining the corrections differed significantly.

External materials include all components of a probe, cables, translation stage and mounting structure other than the sample and glass sample container. All effects of external materials were measured with an auxiliary vaseline-sample NMR probe at the $\Delta B =$ position with and without the ^{3}He assembly or H_2O probe. The auxiliary probe temperature was not monitored ($|\frac{d\omega_{aux}}{dT}| \approx 2.5$ ppb/°C [59]), and uncertainties including those due to temperature variations were estimated from the scatter of multiple measurements. Because the NMR coil inhibited positioning of the auxiliary probe at the exact position of the ^{3}He cell, and because the $\Delta B = 0$ positioning could not be checked with the ^{3}He cell removed, the auxiliary probe was moved back and forth a few millimeters over the cell position to estimate the uncertainty due to position misalignment.

For the H_2O probe, ± 10 ppb variations when rotating the probe around the probe axis were reported by [58] and confirmed for this work. Rotations of the stem with respect to the axis of the magnet with polar angle $\theta = 0$ and 90° and with $\phi = 0$ and 90° were consistent with modeling the stem as a magnetic dipole, which predicted $P_2(\cos\theta)$ dependence at the cell center. For the H_2O calibration measurements, the stem was oriented along y ($\theta = 90°$); extrapolating to the magic angle $\theta = 54°$ resulted in the correction of -0.61 ± 0.20 Hz. The stem also caused magnetic gradients, which resulted in a factor of 2 shorter T_2^* with $\theta = 0°$. Rotation of the cell around the stem axis showed variations of 0.35 Hz, included as an uncertainty. The same ^{3}He cell and stem orientation corrections were applied for both H_2O probes. In principle, there was also a correction for the different displacement of air ($\approx$20% paramagnetic O_2) by the sample and the auxiliary probe. For a spherical ^{3}He sample, this effect would vanish. Modeling the air displaced by the stem as a dipole, the magnetic field at the center of the cell has an amplitude less than about 20 fT and correction $<1\,\mu$Hz. For the H_2O probes, the sample was not removed, and there was no correction.

TABLE I. Corrections and uncertainties for cross-calibration studies of both H_2O probes. For entries with correction < 0.01 Hz, the correction is indicated with a 0; entries that do not apply for a specific probe are indicated with $\cdots$; the * indicates that one correction or uncertainty applies to both studies. The last two lines provide new determinations from this work to be compared to $|R'_{hp}| = 0.7617861313(33)$ and $|\mu'_{hp}| = 1.074553090(13)$ based on Ref. [33]. To convert frequency uncertainties to approximate ppb, divide by 47.1×10^{-3} for ^{3}He and 61.7×10^{-3} for H_2O.

	Spherical H_2O (Hz)				Cylindrical H_2O (Hz)			
	H_2O		^{3}He		H_2O		^{3}He	
Source	Corr.	Unc.	Corr.	Unc.	Corr.	Unc.	Corr.	Unc.
External materials	10.61	0.49	21.38	1.22	13.82	0.49	23.64	1.13
Probe materials	2.71	0.62	…	…	2.90	0.62	…	…
H_2O Probe-material asymmetry*	0	1.24	…	…	0	1.24	…	…
^{3}He glass stem*	…	…	−0.61	0.20	…	…	−0.61	0.20
^{3}He cell rotation*	…	…	0	0.35			0	0.35
Sample magnetization	0	0	0	0.20	−93.10	0.26	0	0.18
Radiation damping*	…	…	0	0.18	…	…	0	0.18
$\sigma_p^{H_2O}$ Temperature dependence	−0.48	0.64	…	…	−1.27	0.64	…	…
χ^{H_2O} Temperature dependence	0	0	…	…	0	0.02	…	…
Total of corrections	12.84	1.60	20.77	1.31	−77.65	1.62	23.05	1.23
Frequency extraction unc.	0.01		…		0.10		…	
Magnet drift uncertainty	0.05		0.09		0.18		0.12	
Position uncertainty*	0.07		0.05		0.07		0.05	
Total unc.	1.61		1.32		1.64		1.23	
$\omega'/2\pi$	61 710 229.90		47 009 998.24		61 715 758.45		47 014 209.45	
$B - 1\,449\,000\,000$ (nT)	400 419 (41)		400 448 (44)		530 269(41)		530 287 (42)	
$\lvert R'_{hp}\rvert = \lvert\omega'_3/\omega'_p\rvert$	0.761 786 147 (29)				0.761 786 141 (28)			
Combined $\lvert R'_{hp}\rvert$	0.761786144(20)							
$\lvert\mu'_h\rvert$ (10^{-27} J T^{-1})	1.074553107(31)							

Sample magnetization dependence arises due to magnetic susceptibility and due to ^{3}He nuclear polarization. For perfectly spherical samples the average long-range contribution to the field anywhere in the sample vanishes [60]. Modeling the hyperpolarized ^{3}He gas in the stem as a magnetic dipole predicts ≈0.5 pT for 10% ^{3}He polarization, corresponding to a shift less than 15 μHz. The H_2O shape dependence is given by $\Delta\omega'_p/\omega'_p = \chi^{H_2O}(T)(1/3-\epsilon)$ (SI units), where $\chi^{H_2O}(T)$ is the susceptibility. For an infinitely long cylinder perpendicular to $\vec{B}$, $\epsilon = 1/2$ and $\Delta\omega'_p = 93.10$ Hz; for a sphere, $\epsilon = 1/3$. Additional magnetization-dependent shifts have been revealed in recent studies with hyperpolarized ^{3}He-^{129}Xe mixtures used in a comagnetometer configuration [61–64]. Scaling these effects from Ref. [65] suggests shifts much less than 1 mHz.

Effects of the free-induction-decay signal (FID) frequency evolution due to dephasing in the nonuniform field are discussed by Refs. [66,67]. Studies with simulated FIDs using the measured magnetic field gradients indicate that these effects were less than 0.1 Hz and less than 0.01 Hz for the cylindrical and spherical H_2O probes and negligible for ^{3}He. Radiation damping shifts due to the current induced in the pickup coil were studied through the frequency dependence of $\omega_3^{\mathrm{uncor}}$ over a set of five 23° pulses. Extrapolating the frequency dependence to zero longitudinal polarization sets an upper limit of 0.18 Hz on the shift. Radiation damping for the H_2O probes, studied by the longitudinal magnetization dependence over the five-pulse sequences and as the probe-tuning drifted, was negligible.

Temperature dependence of the H_2O diamagnetic shielding was corrected to 25 °C by measuring the temperature on the outside of the H_2O probe with a 1000 Ω platinum resistor (PT1000). Conservatively estimating an uncertainty of 5 Ω corresponding to 1°C resulted in 0.64 Hz uncertainty. From [68], $\chi^{H_2O}(T)$ was measured to change by 0.5% over 40°, which we interpret as $\partial\chi^{H_2O}/\partial T = 1.1 \times 10^{-9}/°C$, which adds an uncertainty of 0.02 Hz for the H_2O cylindrical probe.

Other corrections and uncertainties including higher-order time dependence and field fluctuations, for example moving equipment or tools, were estimated as the standard deviation of the residuals of the linear fits in Fig 3. Clock stability was found to be better than 0.01 Hz. The position reproducibility was 0.2 mm contributing an uncertainty 0.07 Hz for H_2O and 0.05 Hz for ^{3}He.

Conclusions—The final results of measurements of the corrected frequencies and corresponding absolute magnetic field determined by the ^{3}He and both H_2O probes are given in Table I. The absolute magnetic fields determined with ^{3}He and H_2O differ by 20 ± 41 ppb and 12 ± 41 ppb for the spherical and cylindrical H_2O probes, respectively. Since the two H_2O probes with corrections consistent with those applied here [58] were used in the analysis of the E821 $g_\mu - 2$ measurements [48,49], this can be interpreted as confirming the calibration of the E821 magnetic-field measurement system to 16 ± 29 ppb, i.e., agreement better than 32 ppb (68% C.L.) compared to the 2.4 ppm tension of $a_\mu^{\rm exp}$ from E821 with the standard model $a_\mu^{\rm SM}$.

We can also use the corrected frequencies to determine $|R'_{hp}| = |\mu'_h/\mu'_p(25\,°{\rm C})|$ and $|\mu'_h| = |R'_{hp}||\mu'_p(25°)|$. The combined results for the two probes presented in Table I are consistent with Ref. [33] but with a 6.8 times larger uncertainty on $|R'_{hp}|$. (Both determinations of $|\mu'_h|$ use the shielded proton moment introducing a common uncertainty.) Straightforward improvements to the ^{3}He magnetometer materials, structure, and improved measurement of the corrections should lead to determination of $|R'_{hp}|$ at the few ppb level and provide a new method for absolute calibration of H_2O probes.

Most importantly, this work and improvements to our first-generation absolute ^{3}He magnetometer establish the technical basis for practical absolute magnetometry with ^{3}He and the establishment of a new magnetic field standard. Though this new standard would currently trace to measurements of μ'_h/μ'_p, the anticipated independent measurement of the helion moment [35] would provide a completely new magnetometry standard.

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