Family-Vicsek Scaling of Roughness Growth in a Strongly Interacting Bose Gas

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Family-Vicsek scaling is one of the most essential scale-invariant laws emerging in surface-roughness growth of classical systems. In this Letter, we theoretically elucidate the emergence of the Family-Vicsek scaling even in a strongly interacting quantum bosonic system by introducing a surface-height operator. This operator is comprised of a summation of local particle-number operators at a simultaneous time, and thus the observation of the surface roughness in the quantum many-body system and its scaling behavior are accessible to current experiments of ultracold atoms.

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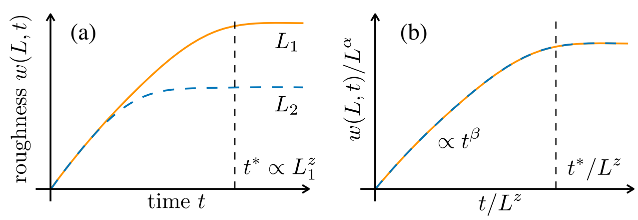
Introduction.—Dynamic scaling is a hallmark of spatiotemporal scale invariance, which plays a pivotal role in uncovering universal aspects behind complicated nonequilibrium phenomena [1,2]. The typical examples are critical and coarsening dynamics [3–5], in which essential information such as dimension and symmetry of a model classifies universality of the dynamics. Such universal dynamics has been widely observed in both classical [6–9] and quantum systems [10–17], being an arena for foundations of nonequilibrium statistical mechanics.

Stochastic surface growth is one of the long-standing universal dynamics discussed in classical nonequilibrium phenomena, where the roughness of the growing surface shows universal spatiotemporal scale invariance [18]. Consider a surface height $h(x,t)$ in a one-dimensional (1D) system with the linear size L. Then, the roughness $w(L,t)$ is quantified as the standard deviation of $h(x,t)$ from its spatial average. For a wide variety of stochastic processes, the roughness obeys a dynamic scaling law called the Family-Vicsek (FV) scaling [19,20] [see Figs. 1(a) and 1(b)]

$$w(L,t) = s^{-\alpha} w(sL, s^z t) \propto \begin{cases} t^\beta & (t \ll t^*); \\ L^\alpha & (t^* \ll t) \end{cases} \qquad (1)$$

with a constant s. Here, t^* is a saturation time proportional to L^z, and α, β, and $z = \alpha/\beta$ are power exponents featuring the universality of a stochastic surface growth model. The typical models are the Kardar-Parisi-Zhang (KPZ) model [21] and the Edwards-Wilkinson (EW) model [22], whose universal exponents are shown in Fig. 1(c). This kind of universality in stochastic surface growth has been extensively explored in various classical systems in the community of, not only physics [23–29], but also mathematics [30] and biology [31,32].

It is then natural to ask whether or not such universal fluctuation dynamics appears in a relaxation process triggered by a parameter quench in quantum systems. Recent theoretical works study the KPZ universality class in quantum magnets by using the spin spatiotemporal correlation function [33–35]. The similar scaling behavior emerges for the spatiotemporal correlation function of the density and phase fluctuations in a Bose-Einstein condensate, calculated by means of the (stochastic) Gross-Pitaevskii equation [36–43]. However, the effect of quantum fluctuations is



(c)

Model	α	β	z
KPZ	1/2	1/3	3/2
EW	1/2	1/4	2
BHM ($\nu \gg 1$)	0.517 ± 0.030	0.255 ± 0.012	2.07 ± 0.20
BHM ($\nu \simeq 1/2$)	0.500 ± 0.003	0.489 ± 0.004	1.00 ± 0.01

this Letter

FIG. 1. Family-Vicsek scaling and its exponents. (a) Time evolution of the surface roughness $w(L,t)$ for two different system sizes ($L_1 > L_2$). The surface roughness grows in time and is finally saturated after the saturation time t^*. (b) Dynamic scaling of $w(L,t)$. When we normalize the ordinate and the abscissa by L^α and L^z, all curves collapse to a single one. The saturation time t^* is scaled as L^z with $z = \alpha/\beta$. (c) Exponents of Kardar-Parisi-Zhang (KPZ), Edwards-Wilkinson (EW), and Bose-Hubbard models (BHM) ($\nu \gg 1$ and $\nu \simeq 1/2$) with the filling factor ν. This Letter finds the exponents of the BHM under the strong repulsive interaction.

yet to be clarified in these works: the former considers maximally mixed states, i.e., infinite-temperature states, whereas the latter are within the mean-field approximation. In addition, it is nontrivial whether the universal FV scaling of the surface roughness occurs for far-from-equilibrium relaxation dynamics. This is important for our understanding of quantum thermalization in isolated systems, which is related to the foundation of statistical mechanics. Experiments of ultracold atoms [44–51] have observed the thermalization processes, but such long-time universal growth of fluctuations is little known.

In this Letter, we study fluctuation growth dynamics in a 1D strongly interacting Bose-Hubbard model (BHM) from the perspective of the FV scaling of the surface roughness. We use the roughness instead of the correlation function because the roughness in the quantum system can be defined only by a local quantity and hence is easy to be observed experimentally. In fact, employing analogy between fluctuating hydrodynamics and stochastic surface growth [52–55], we can introduce a surface-height operator composed of local particle-number operator in the BHM. By using the surface height extended to the quantum system, we calculate the surface roughness, demonstrating the emergence of the FV scaling in the isolated quantum many-body system. All the initial states used in this Letter are pure states, and thus our findings are obtained from the quantum dynamics triggered by purely quantum fluctuations. We have demonstrated two possibilities of the FV scaling exponents depending on the filling factor ν, which are summarized in Fig. 1(c). We argue that the exponents of the high-filling system follow the EW class, while the low-filling (close to $1/2$) system belongs to an unconventional class. Furthermore, considering the isotropic Heisenberg spin chain as a related model, we obtain a signature of the KPZ class.

We comment on the relation between this Letter and previous studies on quantum transport [33–35]. The previous works on transport phenomena have evaluated z (which also appears in the FV scaling) using different-time correlation functions and have discussed ballistic, diffusive, and anomalous transport. On the other hand, these correlation functions do not directly exhibit the FV scaling. This is in stark contrast with our roughness, which can be expressed as a sum of equal-time correlation functions as described later. To the best of our knowledge, the FV scaling shown here has, in fact, not been reported in the context of quantum transport.

Theoretical model and setup.—We consider an N-boson system trapped in a 1D optical lattice, which is well described by the BHM [56–58]. The Hamiltonian is given by

$$\hat{H} = -J\sum_{j=1}^{M}(\hat{b}_{j+1}^{\dagger}\hat{b}_j + \mathrm{H.c.}) + \frac{U}{2}\sum_{j=1}^{M}\hat{b}_j^{\dagger}\hat{b}_j^{\dagger}\hat{b}_j\hat{b}_j, \quad (2)$$

where $\hat{b}_j$ and $\hat{b}_j^{\dagger}$ are the annihilation and creation operators at the jth site, respectively, J is a hopping parameter, U is an interaction coupling parameter, and M is the number of lattice sites. We assume the periodic boundary condition.

We introduce a surface-height operator to define surface roughness in the BHM. The key idea is the emergence of the KPZ scaling in classical fluctuating hydrodynamics, where the correlation function of the density fluctuation $\delta\rho(x,t)$ shows a similar scaling law in a correlation function of $\partial_x h(x,t)$ in the KPZ equation [52–55]. From this analogy, we propose the following integral quantity as a surface height in the fluctuating hydrodynamics:

$$\int^{x}\delta\rho(y,t)dy. \quad (3)$$

Extending Eq. (3) to the 1D BHM, we introduce the following surface-height operator for the quantum discrete system:

$$\hat{h}_j(t) = \sum_{k=1}^{j}[\hat{b}_k^{\dagger}(t)\hat{b}_k(t) - \nu], \quad (4)$$

where $\nu = N/M$ is a filling factor. Then, the surface roughness $w_q(t)$ for the fluctuation of $\hat{h}_j(t)$ is defined by

$$w_q(M,t) = \sqrt{\frac{1}{M}\sum_{j=1}^{M}\langle[\hat{h}_j(t) - h_{\mathrm{av}}(t)]^2\rangle} \quad (5)$$

with the spatially averaged surface height $h_{\mathrm{av}}(t) = \sum_{j=1}^{M}\langle\hat{h}_j(t)\rangle/M$. Here, the bracket means a quantum average with an initial state. This roughness can be expressed as a summation of the equal-time correlation functions for the particle-fluctuation operator $\hat{b}_k^{\dagger}(t)\hat{b}_k(t) - \nu$.

In what follows, we consider the BHM with the strong repulsive interaction ($J \ll \nu U$), which allows one to truncate the local Fock states into a few ones because the fluctuations of the particle number should be suppressed. Then, the BHM can be effectively described by spin models depending on the filling factor ν. Below, we investigate the roughness dynamics for high- ($\nu \gg 1$) and low-filling ($\nu < 1$) cases. The high-filling case can be solved by means of the SU(3) truncated Wigner approximation (TWA) method if ν satisfies $\nu J \ll U$, whereas the low-filling case is exactly solvable using the Jordan-Wigner transformation.

Results (high-filling case).—We study the surface-roughness growth when the filling factor ν is much higher than unity. Because of the strong interaction, we can truncate the local Fock states into $|\nu+1\rangle$, $|\nu\rangle$, $|\nu-1\rangle$. Employing the truncated states, we can rewrite the original Hamiltonian (2) as the effective spin-1 model [59–62]

$$\hat{H}_{S1} = -\nu J \sum_{j=1}^{M} (\hat{S}^x_{j+1}\hat{S}^x_j + \hat{S}^y_{j+1}\hat{S}^y_j) + \frac{U}{2}\sum_{j=1}^{M}(\hat{S}^z_j)^2 \quad (6)$$

with the spin-1 operator $\hat{S}^\mu_j (\mu = x, y, z)$. The derivation of this model is given in the Supplemental Material [63]. In the spin representation, the particle-number fluctuation at the jth site is expressed by $\hat{S}^z_j$, and thus the surface-height operator (4) reads $\hat{h}_j(t) = \sum_{k=1}^{j} \hat{S}^z_k(t)$. In a restricted solid-on-solid model [66], the same surface height is introduced for mapping from the model to a quantum spin chain in an imaginary-time formalism.

The surface-height distribution is constructed by the mapping rule that the eigenvalues 1,0, and -1 of $\hat{S}^z_j$ are assigned to diagonally upward, horizontal, and diagonally downward lines, respectively (see Supplemental Material [63]). This kind of mapping is originally developed in the simple exclusion processes [67–70], where the surface height is given by a time integral of currents. Defining the current operator $\hat{I}_j = iJ(\hat{b}^\dagger_j \hat{b}_{j-1} - \hat{b}^\dagger_{j-1}\hat{b}_j)/\hbar$, we can derive a similar relation for the BHM: $\hat{h}_j(t) = \int_0^t [\hat{I}_0(t_1) - \hat{I}_{j+1}(t_1)]dt_1$, which is obtained by integrating the Heisenberg equation for $\hat{h}_j$. This is almost same as the surface height in the simple exclusion processes except for $\hat{I}_0$ and suggests that there is a closer connection between the quantum roughness dynamics and the classical processes.

We numerically solve the Schrödinger equation with (6) using the SU(3) TWA method [71] and calculate the surface roughness $w_q(M, t)$. The numerical method works well under the condition $\nu J \ll U$, which is discussed in the Supplemental Material by comparing with the exact numerical result [63]. The initial condition is chosen as the Mott state

$$|\psi_{\text{ini}}\rangle = \prod_{j=1}^{M} \frac{1}{\sqrt{\nu!}} (\hat{b}^\dagger_j)^\nu |0\rangle \quad (7)$$

with the vacuum $|0\rangle$.

Figure 2 shows the snapshots of $\hat{S}^z_j$ and $\hat{h}_j$ at different times in a single trajectory of the TWA calculation, from which we find that the surface-height distribution shows clear growth of the large-scale fluctuations in time, whereas the spin distributions (particle-number fluctuations) do not grow. The surface-height dynamics looks similar to stochastic surface growth in classical models.

We numerically calculate $w_q(M, t)$ and find that the roughness grows with increasing t and M as shown in Fig. 3. The FV scaling is expressed by $w_q(M, t) = s^{-\alpha} w_q(sM, s^z t)$. Substituting $s = M_{\text{ref}}/M$ with the reference system size $M_{\text{ref}} = 32$ into this scaling relation, we obtain

$$w_q(M, t) = (M/M_{\text{ref}})^\alpha w_q[M_{\text{ref}}, t(M/M_{\text{ref}})^{-z}]. \quad (8)$$

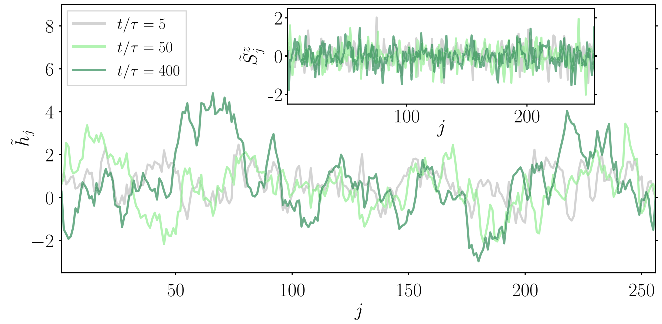


FIG. 2. Snapshots for the Weyl representations of (main panel) $\hat{h}_j$ and (inset) $\hat{S}^z_j$ at different times in a single trajectory of the TWA calculation with $\nu J/U = 0.1$. We denote these representations by $\tilde{S}^z_j$ and $\tilde{h}_j$. The time is normalized by $\tau = \hbar/\nu J$. While the spin distributions exhibit no significant structures by eye, the distribution of the surface height clearly grows with the large-scale fluctuations.

Normalizing the ordinate and the abscissa in the inset of Fig. 3 by $(M/M_{\text{ref}})^\alpha$ and $(M/M_{\text{ref}})^z$, respectively, we find that the curves for different system sizes collapse to a single function as in the main panel of Fig. 3, which is the definite hallmark of the FV scaling. The extracted exponents are given by $(\alpha, \beta, z) = (0.517 \pm 0.030, 0.255 \pm 0.012, 2.07 \pm 0.20)$, which are almost identical to the exponents of the EW class [see Fig. 1(c)]. Here, we obtain the values of the exponents by using Eq. (8) and fitting the numerical data for $2 < t/\tau < 100$ to ct^β with a constant c. The details of extracting the exponents are given in the Supplemental Material [63]. Here, we emphasize that the roughness growth in isolated quantum systems free from noises is nontrivial because, in classical systems, stochastic noises play an essential role. For example, when the noise term is absent,

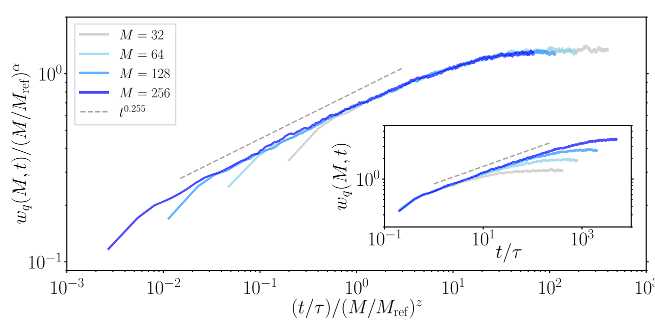


FIG. 3. Time evolution of $w_q(M, t)$ for the effective spin-1 Hamiltonian (6) with $\nu J/U = 0.1$. The system sizes are $M = 32$, 64, 128, and 256, and the curves are obtained using 1000 samples. The ordinate and the abscissa are normalized by $(M/M_{\text{ref}})^\alpha$ and $(M/M_{\text{ref}})^z$. All the curves show the FV scaling with the extracted exponents $\alpha = 0.517 \pm 0.030$, $\beta = 0.255 \pm 0.012$, and $z = 2.07 \pm 0.20$. The way to extract them is described in the Supplemental Material [63]. The roughness $w_q(M, t)$ obeys the power-law growth and the exponent is close to 1/4. (Inset) Raw numerical data calculated by the SU(3) TWA method.

the classical EW model becomes a diffusive equation, which does not show the roughness growth.

We also investigate the dependence of the dynamics on the initial state and still find the same power-law growth, details of which are given in the Supplemental Material [63]. Such a FV scaling is not found for the fluctuations of $\hat{S}_j^z$.

Note that the initial dynamics ($t/\tau \lesssim 0.5$) shows the different type of growth. We expect that this regime strongly depends on the initial state and is nonuniversal because the timescale is shorter than the hopping time τ. Thus, it is natural that the data in the region do not obey the FV scaling.

Next, we consider dynamics under the condition $\nu J \simeq U$, in which the SU(3) TWA calculation is not valid. In small systems, we find a signature of the power-law growth in the roughness. However, because the time region having the power-law-like behavior is short, we cannot confirm the clear FV scaling. This result is described in the Supplemental Material [63].

Results (low-filling case).—We consider the BHM with the half filling $\nu = 1/2$. Owing to the strong repulsive interaction, the bases of the local Fock states can be reduced to $|0\rangle$ and $|1\rangle$. As a result, the Hamiltonian (2) becomes the XX model [72]

$$\hat{H}_{XX} = -2J \sum_{j=1}^{M} (\hat{s}_{j+1}^x \hat{s}_j^x + \hat{s}_{j+1}^y \hat{s}_j^y) + \text{const.} \tag{9}$$

The spin-1/2 operators $\hat{s}_j^\alpha (\alpha = x, y, z)$ are given by $\hat{s}_j^x = (\hat{b}_j^\dagger + \hat{b}_j)/2$, $\hat{s}_j^y = -i(\hat{b}_j^\dagger - \hat{b}_j)/2$, and $\hat{s}_j^z = \hat{b}_j^\dagger \hat{b}_j - 1/2$, which satisfy the commutation relation $[\hat{s}_i^\alpha, \hat{s}_j^\beta] = i\delta_{ij} \sum_\gamma \epsilon_{\alpha\beta\gamma} \hat{s}_j^\gamma$. The particle-number fluctuation is given by $\hat{s}_j^z$, and thus the surface-height operator (4) reduces to $\hat{h}_j = \sum_{k=1}^{j} \hat{s}_k^z$. Similar to the high-filling case, we can construct the surface-height distribution by assigning an up (down) spin to a diagonally upward (downward) line [63]. As an initial state, we use a staggered state given by

$$|\psi_{\text{ini}}\rangle = \prod_{j=1}^{M/2} \hat{b}_{2j-1}^\dagger |0\rangle, \tag{10}$$

where M is assumed to be even. Under this setup, we exactly solve the Heisenberg equation with Eq. (9) by employing the Jordan-Wigner transformation [72] and calculate the exact time evolution of the surface roughness (5). The details of the algebraic calculations is given in the Supplemental Material [63].

Figure 4 shows the exact time evolution of $w_q(M, t)$ for different system sizes, which demonstrates growth of the surface roughness. Normalizing the ordinate and the abscissa in a similar manner to Eq. (8), we find that all the different curves collapse to a single function except for the very early and late stages of the dynamics.

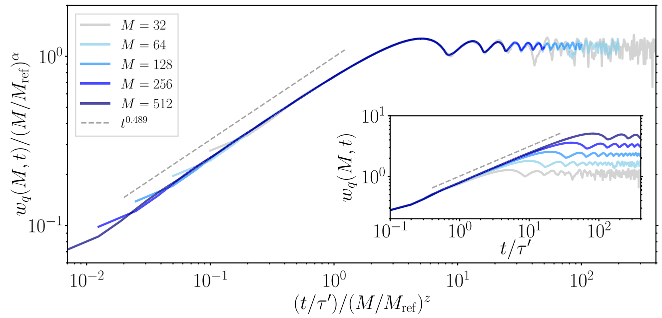


FIG. 4. Time evolution of $w_q(M, t)$ in the XX model (9) starting from the staggered initial state (10). The ordinate and the abscissa are normalized by $(M/M_{\text{ref}})^\alpha$ and $(M/M_{\text{ref}})^z$. The surface roughness shows the power-law growth up to the saturation time, after which small oscillations emerge. Except for the very early stage, the growth of the surface roughness shows the FV scaling. The extracted exponents are $\alpha = 0.500 \pm 0.003$, $\beta = 0.489 \pm 0.004$, and $z = 1.00 \pm 0.01$. (Inset) Raw data calculated by the exact solution. The time is normalized by $\tau' = \hbar/2J$.

The extracted power-law exponents are $(\alpha, \beta, z) = (0.500 \pm 0.003, 0.489 \pm 0.004, 1.00 \pm 0.01)$. We also investigate the dependence of the exponents on the filling factor ν and confirm that the similar exponents emerge in the late dynamics, unless ν is far from 1/2, as described in the Supplemental Material [63]. As far as we know, any classical models do not have these exponents, which suggests that this FV scaling belongs to an unconventional universality class.

Here, we particularly consider the specific staggered state (10) as an initial condition because the initial roughness should be small. We also calculate for other initial states and still find that the FV scaling robustly appears as long as the initial roughness is small enough and the filling fraction is not too small [63].

Discussion.—As summarized in Fig. 1(c), the exponent $\alpha = 1/2$ seems to be model independent. It can be analytically derived using the eigenstate thermalization hypothesis (ETH) [73–76] and the cluster decomposition: $\lim_{t\to\infty} w_q^2(M, t) \simeq C(M + 1)/2$, where C is the constant. Thus, in a large system, we obtain $\alpha = 1/2$. The details of the derivation are given in the Supplemental Material [63]. Note that this argument itself cannot explain $\alpha = 1/2$ in the XX model since the derivation is based on the ETH, which is not valid in the free-fermion model. However, the essence of the derivation is the translational invariance and no long-range order, and thus we derive $\alpha = 1/2$ even in the XX model if the two assumptions are valid [63]. As for the exponent β, we have not analytically obtained the value and leave it for future work.

Next, we discuss the KPZ class in the isotropic Heisenberg (IH) model, which is a simple extension of the XX model (9). While this model is outside the framework of the BHM, it appears in many condensed-matter contexts and is a prototypical model for statistical mechanics. The Hamiltonian is given by

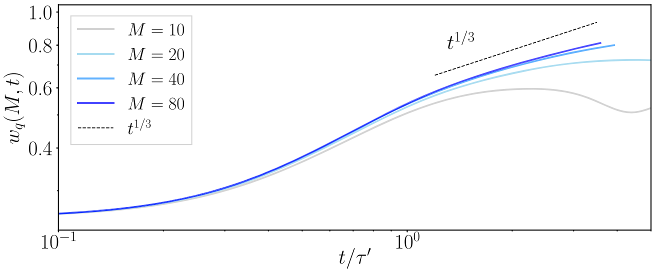


FIG. 5. Time evolution of $w_q(M, t)$ in the isotropic Heisenberg (IH) model (11) starting from the staggered initial state. When M is larger than 40, the roughness growth shows a signature of $1/3$ power law. We perform the calculations using the ITensor Library [77].

$$\hat{H}_{\rm IH} = -2J \sum_{j=1}^{M} (\hat{s}^x_{j+1}\hat{s}^x_j + \hat{s}^y_{j+1}\hat{s}^y_j + \hat{s}^z_{j+1}\hat{s}^z_j). \quad (11)$$

We numerically solve the model using the matrix product state technique and then find a signature of the KPZ class as shown in Fig. 5, which shows the time evolution of the roughness calculated by the surface-height operator $\hat{h}_j = \sum_{k=1}^{j} \hat{s}^z_k$. The roughness growth obeys a power-law-like behavior with $\beta = 1/3$ in the time region [1, 3], where the results for $M = 40$ and 80 overlap. We leave it for a future study to confirm the exponents for larger system sizes and longer timescales.

Finally, we discuss possible experiments for observing the FV scaling. The surface-height operator (4) is the summation of the local particle-number operator at a simultaneous time. Thus, in the low-filling case, the observation of the roughness is easier than that of spatio-temporal correlation functions by utilizing quantum gas microscopes. Experiments in ultracold atomic gases have already observed thermalization processes in the low-filling case starting from the staggered state [49], and thus the FV scaling in Fig. 4 can be detectable. Another promising test bed is a Rydberg system, in which the XX model is realizable in a highly controlled manner and the particle-number fluctuations can be observed [78,79]. On the other hand, in the high-filling case, current experiments may not have adequate resolution for detecting one particle dynamics, and thus it may be challenging to observe the FV scaling.

Conclusions and prospects.—We have theoretically studied the surface-roughness dynamics in the strongly interacting 1D Bose gas by introducing the surface-height operator in the BHM and then have demonstrated the emergence of the FV scaling in an isolated quantum many-body system. The extracted exponents in the high-filling case correspond to the EW class, while in the low-filling case the exponents are found to be unconventional with no corresponding classical models.

As future works, it is interesting to consider quantum thermalization in isolated systems from the viewpoint of the surface-roughness growth. The relation between simple exclusion processes and the quantum roughness growth can open an interesting avenue for connecting quantum thermalization dynamics and classical stochastic processes. It is also interesting to investigate the relation with universal dynamics for certain nonlocal quantities such as entanglement entropy and operator spreading [80–83], which are in stark contrast to our finding because the surface-height operator is a summation of local operators. As another direction, it is important to pursue further connections between classical and quantum roughness growth by focusing on higher-order cumulants of surface fluctuations, which may reveal the Tracy-Widom random matrix universality characteristic of the KPZ classes [23–29].

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