Resource Reduction for Distributed Quantum Information Processing Using Quantum Multiplexed Photons

Nicolò Lo Piparo,[1,*] Michael Hanks,[1] Claude Gravel,[1] Kae Nemoto,[1] and William J. Munro[2,1]

[1]*National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan*

[2]*NTT Basic Research Laboratories and NTT Research Center for Theoretical Quantum Physics, NTT Corporation, 3-1 Morinosato-Wakamiya, Atsugi, Kanagawa, 243-0198, Japan*

(Received 4 July 2019; accepted 14 May 2020; published 27 May 2020)

Distributed quantum information processing is based on the transmission of quantum data over lossy channels between quantum processing nodes. These nodes may be separated by a few microns or on planetary scale distances, but transmission losses due to absorption and/or scattering in the channel are the major source of error for most distributed quantum information tasks. Of course, quantum error correction (QEC) and detection techniques can be used to mitigate such effects, but error detection approaches have severe performance limitations due to the signaling constraints between nodes, and so error correction approaches are preferable—assuming one has sufficient high quality local operations. Typically, performance comparisons between loss-mitigating codes assume one encoded qubit per photon. However, single photons can carry more than one qubit of information and so our focus in this Letter is to explore whether loss-based QEC codes utilizing quantum multiplexed photons are viable and advantageous, especially as photon loss results in more than one qubit of information being lost. We show that quantum multiplexing enables significant resource reduction, in terms of the number of single-photon sources, while at the same time maintaining (or even lowering) the number of 2-qubit gates required. Further, our multiplexing approach requires only conventional optical gates already necessary for the implementation of these codes.

DOI: 10.1103/PhysRevLett.124.210503

There are many active approaches being pursued in the development of quantum technologies, including those associated with imaging and sensing [1–3], communication [4–9], and computation [10–15]. What has become clear is that many of these will be distributed in nature [5] and, as such, it will be essential to share quantum information between the remote nodes, regardless of whether those nodes are separated on the atomic or planetary scales [16–18]. This distributed nature means we are going to require both a quantum interface between matter and photonic qubits and a photonic bus to transfer such information between nodes [19]. However, real implementations will suffer from losses, which will dramatically affect the performance of the quantum protocols in which such devices are being used. Mechanisms must be developed to mitigate such detrimental effects.

There are quite a number of routes available to offset loss effects, ranging from the development of lower loss fibers to more efficient quantum information coding. The latter route is quite appealing as it can be used with current technology and is likely to be more compatible with our existing infrastructure. There is a well-known set of loss-based quantum detection and error correction codes that can be used in this situation. In [20] they discuss a simple quantum network scenario in which the quantum multiplexing (QM) of photonic degrees of freedom allows one to design a single-step combined entanglement distribution and error detection protocol with improved entanglement generation rates, using fewer physical (photons and quantum memories) and temporal resources. However, their performance is still limited by the probabilistic nature of the various quantum operations and the resulting necessary heralding signals.

Quantum error correction codes (ECCs) naturally avoid a heralding bottleneck, with example loss-based codes including the quantum parity [21], cat [22], binomial [23], Reed-Solomon [24], surface [25], and Gottesman-Kitaev-Preskill (GKP) codes [26]. They allow us to approach the deterministic transmission of quantum information over a lossy channel, as long as those total losses do not exceed a certain threshold (50% at most) [27,28]. Typical encodings use either the polarization or time bin degrees of freedom, but are not particularly resource efficient as they require a large number of single photons. The creation of reliable single-photon sources has proved challenging since the generation probability does not exceed 70% [29–32], whereas, on the contrary, single-qubit gate fidelities can reach 99% [33–35]. Another limiting factor comes from 2-qubit gates, which require longer times with fidelities below 90% [36–38]. However, single photons have the potential to carry much more information using different degrees of freedom (see Supplemental Material for further details [39]). Hence the natural question is whether using multiple degrees of

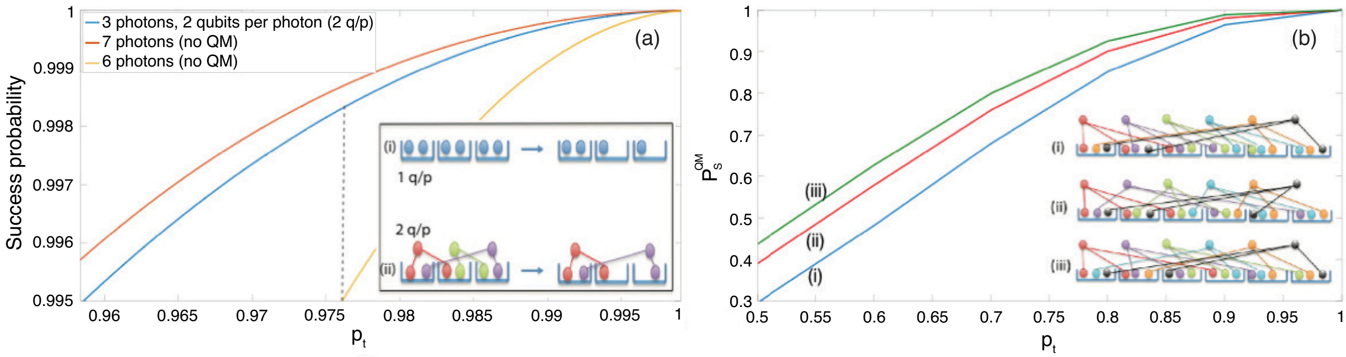

FIG. 1. (a) Plot of the overall success probability versus photon transmission probability p_t of the redundant quantum parity code with (blue curve) and without (red and yellow curves) quantum multiplexed photons. (Inset) Schematic illustration of a particular instance of the 6-qubit quantum redundancy parity code, in which (i) each photon carries one qubit and (ii) three photons carry two qubits of information each $(2q/p)$ (ii). (b) Similarly, we show the success probability P_S^{QM} versus p_t for three different configurations of a quantum multiplexed system. Here six photons carry three qubits each, distributed over six blocks (each block containing three qubits).

freedom is advantageous, in terms of reducing the number of photons while maintaining the same number of 2-qubit gates.

Here we investigate the potential of quantum multiplexing to reduce the resources required to implement loss-based error correction codes. We take as a central figure of merit the required number of single photons as well as qubits. We analyze two well-known ECCs, the redundant quantum parity [21] and quantum Reed-Solomon codes [24], determining the number of photons and qubits required to reach a threshold success probability with the multiplexing method.

Let us begin by exploring the redundant quantum parity code [21] in a photon transmission regime, for which both the number of qubits (memories within the node) and the number of photons can be reduced using our quantum multiplexing approach, all while maintaining the near-deterministic transmission of information between the two nodes. In the redundant quantum parity code, the information α, β in our encoded state $|\psi\rangle_{(n,m)} = \alpha|+\rangle_1 \cdots |+\rangle_n + \beta|-\rangle_1 \cdots |-\rangle_n$ (each block term $|\pm\rangle_i = |H\rangle^{\otimes m} \pm |V\rangle^{\otimes m}$ containing m photons) is successfully transmitted over the channel when at least one block of m qubits arrives intact (no losses) and each other block retains at least one photon [see Fig. 1(a) inset]. The success probability is given by [40]

$$P_S = [1 - (1 - p_t)^m]^n - [1 - p_t^m - (1 - p_t)^m]^n, \quad (1)$$

where p_t is the single-photon transmission probability through the channel. Our first observation is that this concatenated code is not particularly resource efficient, as the number of qubits at the first logical layer m grows inversely with the transmission probability p_t. Further, n grows inversely with p_t^m and so (m, n) grow exponentially with distance between nodes. Our quantum multiplexer is a natural solution [20]: here we encode multiple qubits onto a single photon, meaning less photons in total need to be transmitted. More specifically, we enact a 2-qubit gate between the first degree of freedom (polarization) and a second photonic or matter qubit. Then, swapping the polarization of the initial photon with another degree of freedom (time bin in this case), a third system can then interact independently with the polarization of this same photon (for further details, see Supplemental Material [39]). The quantum multiplexer has many potential benefits, including deterministic operations between different degrees of freedom—especially important when single-photon sources are probabilistic in nature.

Let us explore this in a little more detail. In the inset of Fig. 1(a)(i) we illustrate a six-photon redundant quantum parity code realization without the use of quantum multiplexing in which three blocks of two photons each are used. After the photons are transmitted over the lossy channel, the code is successful if at least one block contains two photons and the other two blocks each contain one or more photons. One can think of substituting those six photons with three quantum multiplexed photons each carrying two qubits of information. In Fig. 1(a)(ii) these are represented by the colored lines connected to the dots contained in the blocks (see Supplemental Material for further details [39]). In this case, the ECC only tolerates the loss of one photon. Therefore, it would seem logical that we can reduce the number of photons by using the multiplexing approach, provided that the success probability is above the desired threshold value. This raises the question as to what the success probability P_S^{QM} will be in this quantum multiplexed approach. One can show for n_{tot} transmitted photons that

$$P_S^{\mathrm{QM}} = \sum_{i=0}^{n^*} \left[\binom{n_{\mathrm{tot}}}{i} - (U_i + E_i) \right] p_t^{n_{\mathrm{tot}} - i} (1 - p_t)^i, \quad (2)$$

where U_i and E_i are the number of events in which losing i photons will leave none of the blocks with the initial

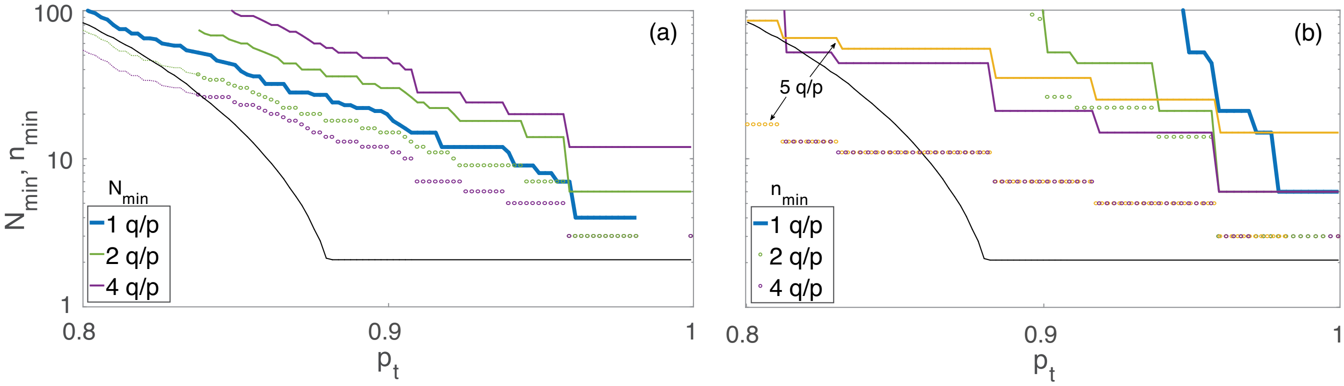

FIG. 2. Plot of the minimum number of qubits (solid lines) and photons (dotted lines) for the (a) redundant quantum parity and (b) Reed-Solomon codes required to reach a threshold success probability of $\overline{P_S} = 0.995$ versus the photon transmission probability p_t using a quantum multiplexed encoding of 2–5 qubits per photon (q/p), respectively. Also shown is the nonmultiplexed situation of 1 q/p for all codes including the hexagonal GKP code (black curves).

number of qubits or at least one empty block, and n^* is the number of lost photons the ECC can tolerate. We need to determine both U_i and E_i, which are highly dependent on how the quantum multiplexed photons are connected to the blocks [see Fig. 1(b)]. Different configurations lead to different success probabilities. We can also release the constraints of all blocks having to have the same number of qubits (an *unbalanced* configuration), which is typically not utilized in error correction schemes. This enables us to further reduce the number of qubits (and photons) even in the nonmultiplexed case (see Supplemental Material [39]).

In Fig. 1(a) we plot the overall success probability P_S versus p_t for two nonmultiplexed (equal and unbalanced) configurations alongside one quantum multiplexed situation with a minimum threshold success probability requirement of $\bar{P}_S = 0.995$ (typical for many quantum computation-based tasks). It is clear that our three-photon quantum multiplexed case (three blocks with $2q/p$) dramatically outperforms the traditional six-photon nonmultiplexed case (three blocks with two qubits, photons each). In the region $0.958 \lesssim p_t \lesssim 0.976$, the six-photon case does not reach our threshold target, while the three-photon multiplexed approach does. The seven-photon configuration (with the first block containing three photons, while the second and third blocks contain two photons each) performs slightly better than the multiplexed case. However, both are above the threshold and the multiplexed situation uses fewer photons, qubits, and 2-qubit gates. The multiplexed approach also halves the number of photons in the region $0.976 \lesssim p_t \leq 0.995$. These are critical resource savings.

It is clear that the lower p_t is, the more qubits (2-qubit gates) and photons we will need to reach $\overline{P_S}$. It is important, in reducing the total numbers of these resources, to also explore unbalanced quantum multiplexing configurations. In Fig. 2(a) we plot the minimal number of qubits $N_{\rm min}$ and photons $n_{\rm min}$ versus p_t for resource-optimal configurations with two and four qubits per photon. Quantum multiplexed systems utilize fewer photons, however, the number of qubits is either the same or slightly higher, except in a small region near $p_t \sim 0.97$ [Fig. 1(a)]. In fact, we can almost halve the number of photons being transmitted over the channel—quite an advantage, especially as single-photon sources are currently not as efficient as quantum gates or measurements.

The number of qubits can be maintained equal to the nonmultiplexing case, while reducing the number of photons, with a mixed strategy, in which each photon can carry an arbitrary number of qubits (from one to four). Table I shows the total number of photons and the total number of qubits needed for reaching $\overline{P_S}$ at $p_t = 0.916$ using the pure and the mixed strategies. We observe that we can reach the required $\overline{P_S}$ with a lower number of photons (12) given the same number of qubits (15) when we apply the mixed strategy. The number of 2-qubit gates required is therefore the same as for the nonmultiplexing case, even for bigger codes. This further highlights the potential advantages of quantum multiplexing. Can these improvements be generalized to other loss-based quantum error correction codes?

TABLE I. Minimum number of photons and qubits required to reach our overall information transfer success probability threshold of $\overline{P_S} = 0.995$ with $p_t = 0.916$. Similar results are seen for most values of p_t. The asterisk corresponds to the *optimal* case, in which, by using the mixing strategy, for a given $N_{\rm min}$ we reach the lowest $n_{\rm min}$ for a specific value of p_t.

Total number of photons	Total number of qubits
15 ($1q/p$)	15
14, 13, 12* (mixed)	15
11 ($2q/p$)	22
11, 10, 9, 8 (mixed)	21
7 ($3q/p$)	21

In the quantum Reed-Solomon $[[d, 2k-d, d-k+1]]_d$ code information is encoded in d qudits, with the code failing on the loss of $d-k+1$ out of d qudits. For comparative purposes, we will express the degree of multiplexing as q qubits of information per photon. When we encode the qudits in these q degrees of freedom of quantum multiplexed photons, any qudit of information depends upon the successful transmission of $\lceil \log_2(d)/q \rceil$ photons [24]. The probability of failure is therefore

$$P_{\text{fail}} = \sum_{j=d-k+1}^{d} \binom{d}{j} \left(1 - p_t^{\lceil \frac{\log_2(d)}{q} \rceil}\right)^j p_t^{\lceil \frac{\log_2(d)}{q} \rceil (d-j)}. \quad (3)$$

In this code, the block is given by the total number of photons encoding a single qudit, and if a block is incomplete, the qudit is not successfully transmitted. Therefore, the performance can be improved by maintaining independence between these blocks and by reducing the chances for loss events within any single block. Adding additional quantum multiplexing will help so long as it preserves independence between qudit loss events. For the quantum Reed-Solomon code, we can also determine the lowest number of qubits and photons required to reach $\overline{P_S}$, as shown in Fig. 2(b). Here, the advantage of using quantum multiplexed photons is evident in terms of a reduction of the number of qubits, 2-qubit gates, and photons compared to the no quantum multiplexing case. In particular, the higher the quantum multiplexing degree, the less qubits and photons we require. For instance, at $p_t = 0.85$, we have that for $q = 4$, $N_{\min} \simeq 40$ and $n_{\min} \simeq 10$, whereas when no quantum multiplexed photons are in use, we have that both $N_{\min}$ and $n_{\min}$ are over 1000. As p_t gets lower, the number of photons and qubits increases considerably, hence, we need to use higher degrees of quantum multiplexing. Furthermore, by comparing Fig. 2(a) with Fig. 2(b), we infer that there is always a specific value of q for which the Reed-Solomon code requires a lower number of resources compared to the parity code [for $q = 4$, at $p_t = 0.85$, $N_{\min}(n_{\min})$ is 72% (75%) lower for the Reed-Solomon code than the parity code]. For other error correction codes based on the transmission of qudits, we expect the same reduction in the number of qubits, 2-qubit gates, and photons when quantum multiplexing is in use.

There are other loss codes based on encoding information in superposition of photon number (bosonic [23] and GKP [26], for instance), in which quantum multiplexing is ineffective. In these cases, this would correspond to the assignment of information about multiple excitation to the various degrees of freedom of a single mode. However, any quantum multiplexed photon mode is equivalent, in this case, to a no quantum multiplexed mode. There is always, therefore, a code using fewer excitations and a higher number of modes than the original that will perform as well as the quantum multiplexed case.

It is essential to compare these quantum multiplexed codes to the best loss codes currently known—namely, the GKP codes [41,42]. In particular, in [42] the authors show that the hexagonal GKP code is optimal among all single-mode bosonic codes against loss errors expressed as a Gaussian displacement channel. In Figs. 2(a) and 2(b), we plot (black curves) the average number of photons for the hexagonal GKP code [43]. This suggests that there are regions where the GKP code has a better performance and other regions where this is reversed. The multiplexed codes operate better in the higher loss regimes. Further, a critical consideration has to be the near-deterministic implementation of the code itself. Our quantum multiplexing approach requires the same basic 2-qubit or qudit gates needed for quantum logic (and the original codes themselves) with the addition of high efficiency optical switches to swap state between the different degrees of freedom. On the other hand, the initialization of the GKP code is quite demanding to achieve in a near-deterministic way and necessitates a more complex continuous variable procedure, though Gaussian operations are sufficient for subsequent qubit control. Generating such codes in a heralded but probabilistic fashion has been achieved, but unfortunately increases the resources required [44,45] (see Supplemental Material [39]). This indicates that additional resources will be required at the end nodes to process the quantum data transmitted over the communication channel. We note that a proof-of-principle demonstration of deterministic preparation was performed in [45] and look forward to the development of this promising approach going forward.

To summarize, we have shown how quantum multiplexed loss codes have the potential to significantly decrease the resources required to transfer quantum information between two adjacent nodes. This is achieved while maintaining or even lowering the required number of 2-qubit gates. Two primary error correction codes were considered: the redundant quantum parity code and the quantum Reed-Solomon code. For the former, we found that the total number of single photons that need to be transmitted through the channel can be dramatically reduced (near 50%) without significantly increasing the number of qubits. Further, we found it advantageous for individual photons to have different degrees of quantum multiplexing, as well as for blocks to contain different numbers of qubits. The quantum Reed-Solomon code significantly outperforms the redundant quantum parity code and, using quantum multiplexed qudits, has the potential to reduce simultaneously the number of photons, qubits, and gates used. These improvements should be possible in many (but not all) of the other loss-based error correction codes when quantum multiplexing is used. Quantum multiplexing has the potential to be a new resource saving tool especially for near-term implementations. Our findings can be applied to any communication system that needs error correction to improve its efficiency, such as in quantum repeaters, quantum computation, and quantum sensing.

We thank Koji Azuma for useful discussions. This project was made possible through the support of the MEXT KAKENHI Grant-in-Aid for Scientific Research on Innovative Areas "Science of Hybrid Quantum Systems" Grant No. 15H05870, the MEXT Quantum Leap Flagship Program (MEXT Q-LEAP) Grant No. JPMXS0118069605, and a grant from the John Templeton Foundation (JTF Grant No. 60478). The opinions expressed in this publication are those of the authors and do not necessarily reflect the views of the John Templeton Foundation.

*nicopale@gmail.com

[1] D. S. Simon, G. Jaeger, and A. V. Sergienko, Int. J. Quantum. Inform. **12**, 1430004 (2014).

[2] L. A. Lugiato, A. Gatti, and E. Brambilla, J. Opt. B **4**, S176 (2002).

[3] C. L. Degen, F. Reinhard, and P. Cappellaro, Rev. Mod. Phys. **89**, 035002 (2017).

[4] C. H. Bennett and G. Brassard, Theor. Comput. Sci. **560**, 7 (2014).

[5] W. J. Munro, K. Azuma, K. Tamaki, and K. Nemoto, IEEE J. Sel. Top. Quantum Electron. **21**, 6400813 (2015).

[6] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).

[7] H. Lo, Science **283**, 2050 (1999).

[8] W.-Y. Hwang, Phys. Rev. Lett. **91**, 057901 (2003).

[9] N. Sangouard, C. Simon, H. de Riedmatten, and N. Gisin, Rev. Mod. Phys. **83**, 33 (2011).

[10] M. Nielsen and I. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2000).

[11] C. Bennett and D. DiVincenzo, Nature (London) **404**, 247 (2000).

[12] S. J. Devitt, A. M. Stephens, W. J. Munro, and K. Nemoto, Nat. Commun. **4**, 2524 (2013).

[13] R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. **86**, 5188 (2001).

[14] E. Knill, Nature (London) **434**, 39 (2005).

[15] L.-M. Duan and R. Raussendorf, Phys. Rev. Lett. **95**, 080503 (2005).

[16] K. Nemoto, M. Trupke, S. J. Devitt, B. Sharfenberger, K. Buczak, J. Schmiedmayer, and W. J. Munro, Sci. Rep. **6**, 26284 (2016).

[17] L. Childress and R. Hanson, MRS Bull. **38**, 134 (2013).

[18] M. S. Blok, N. Kalb, A. Reiserer, T. H. Taminiau, and R. Hanson, Faraday Discuss. **184**, 173 (2015).

[19] E. Saglamyurek, N. Sinclair, J. Jin, J. A. Slater, D. Oblak, F. Bussieres, M. George, R. Ricken, W. Sohler, and W. Tittel, Nature (London) **469**, 512 (2011).

[20] N. Lo Piparo, W. J. Munro, and K. Nemoto, Phys. Rev. A **99**, 022337 (2019).

[21] T. C. Ralph, A. J. F. Hayes, and A. Gilchrist, Phys. Rev. Lett. **95**, 100501 (2005).

[22] P. T. Cochrane, G. J. Milburn, and W. J. Munro, Phys. Rev. A **59**, 2631 (1999).

[23] M. H. Michael, M. Silveri, R. T. Brierley, V. V. Albert, J. Salmilehto, L. Jiang, and S. M. Girvin, Phys. Rev. X **6**, 031006 (2016).

[24] M. Grassl, W. Geiselmann, and T. Beth, in *Applied Algebra, Algebraic Algorithms and Error-Correcting Codes. AAECC 1999.*, Lecture Notes in Computer Science, edited by M. Fossorier, H. Imai, S. Lin, and A. Poli (Springer, Berlin, Heidelberg, 1999), Vol. 1719.

[25] A. G. Fowler, D. S. Wang, C. D. Hill, T. D. Ladd, R. Van Meter, and L. C. L. Hollenberg, Phys. Rev. Lett. **104**, 180503 (2010).

[26] D. Gottesman, A. Kitaev, and J. Preskill, Phys. Rev. A **64**, 012310 (2001).

[27] A. S. Holevo and R. F. Werner, Phys. Rev. A **63**, 032312 (2001).

[28] M. M. Wolf, D. Perez-Garcia, and G. Giedke, Phys. Rev. Lett. **98**, 130501 (2007).

[29] F. Kaneda and P. G. Kwiat, Sci. Adv. **5**, eaaw8586 (2019).

[30] X.-L. Wang, L.-K. Chen, W. Li, H.-L. Huang, C. Liu, C. Chen, Y.-H. Luo, Z.-E. Su, D. Wu, Z.-D. Li, H. Lu, Y. Hu, X. Jiang, C.-Z. Peng, L. Li, N.-L. Liu, Y. A. Chen, C. Y. Lu, and J.-W. Pan, Phys. Rev. Lett. **117**, 210502 (2016).

[31] H. Wang, Y. He, Y.-H. Li, Z.-E. Su, B. Li, H.-L. Huang, X. Ding, M.-C. Chen, C. Liu, J. Qin, J.-P. Li, Y.-M. He, C. Schneider, M. Kamp, C.-Z. Peng, S. Hofling, C.-Y. Lu, and J.-W. Pan, Nat. Photonics **11**, 361 (2017).

[32] C. Xiong, X. Zhang, Z. Liu, M. J. Collins, A. Mahendra, L. G. Helt, M. J. Steel, D.-Y. Choi, C. J. Chae, P. H. W. Leong, and B. J. Eggleton, Nat. Commun. **7**, 108853 (2016).

[33] K. Itoh and H. Watanabe, MRS Commun. **4**, 143 (2014).

[34] T. D. Ladd and M. S. Carroll, in *Encyclopedia of Modern Optics*, 2nd ed. (Elsevier, New York, 2018), pp. 467–477.

[35] J. M. Nichol, L. A. Orona, S. P. Harvey, S. Fallahi, G. C. Gardner, M. J. Manfra, and A. Yacoby, npj Quantum Inf. **3**, 3 (2017).

[36] T. F. Watson, S. G. J. Philips, E. Kawakami, D. R. Ward, P. Scarlino, M. Veldhorst, D. E. Savage, M. G. Lagally, M. Friesen, S. N. Coppersmith, M. A. Eriksson, and L. M. K. Vandersypen, Nature (London) **555**, 633 (2018).

[37] W. Huang, C. H. Yang, K. W. Chan, T. Tanttu, B. Hensen, R. C. C. Leon, M. A. Fogarty, J. C. C. Hwang, F. E. Hudson, K. M. Itoh, A. Morello, A. Laucht, and A. S. Dzurak, Nature (London) **569**, 532 (2019).

[38] D. M. Zajac, A. J. Sigillito, M. Russ, F. Borjans, J. M. Taylor, G. Burkard, and J. R. Petta, Science **359**, 439 (2018).

[39] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.124.210503 for derivation of the 3-photon quantum multiplexing approach to the redundancy code. It also describes the gates required to create such a state.

[40] W. J. Munro, A. M. Stephens, S. J. Devitt, K. A. Harrison, and K. Nemoto, Nat. Photonics **6**, 777 (2012).

[41] V. V. Albert, K. Noh, K. Duivenvoorden, D. J. Young, R. T. Brierley, P. Reinhold, C. Vuillot, L. Li, C. Shen, S. M.

Girvin, B. M. Terhal, and L. Jiang, Phys. Rev. A **97**, 032346 (2018).

[42] K. Noh, V. V. Albert, and L. Jiang, IEEE Trans. Inf. Theory **65**, 2563 (2019).

[43] Our calculations are based on the recovery of a finite GKP logic state that undergoes through a Gaussian channel, reported in Eq. (7.21) of Ref. [41].

[44] C. Flühmann, T. L. Nguyen, M. Matinelli, V. Negnevitsky, K. Mehta, and J. P. Home, Nature (London) **566**, 513 (2019).

[45] P. Campagne-Ibarcq, A. Eickbush, S. Touzard, E. Zalys-Geller, N. E. Frattini, V. V. Sivak, P. Reinhold, S. Puri, S. Shankar, R. J. Shoelkopf, L. Frunzio, M. Mirrahimi, and M. H. Devoret, arXiv:1907.12487.