## Thermodynamically Consistent Equation of State for an Accreted Neutron Star Crust

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We study the equation of state (EOS) of an accreting neutron star crust. Usually, such an EOS is obtained by assuming (implicitly) that the free (unbound) neutrons and nuclei in the inner crust move together. We argue that this assumption violates the condition  $\mu_n^{\infty}$  = const, required for hydrostatic (and diffusion) equilibrium of unbound neutrons  $(\mu_n^{\infty})$  is the redshifted neutron chemical potential). We construct a new EOS respecting this condition, working in the compressible liquid-drop approximation. We demonstrate that it is close to the catalyzed EOS in most of the inner crust, being very different from EOSs of accreted crust discussed in the literature. In particular, the pressure at the outer-inner crust interface does not coincide with the neutron drip pressure, usually calculated in the literature, and is determined by hydrostatic (and diffusion) equilibrium conditions within the star. We also find an instability at the bottom of the fully accreted crust that transforms nuclei into homogeneous nuclear matter. It guarantees that the structure of the fully accreted crust remains self-similar during accretion.

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Introduction.—Neutron stars (NSs) are the densest objects in the Universe. The composition of their deepest layers (inner core) is uncertain and is considered as the main mystery of NS physics [\[1\]](#page-4-0). In contrast, it is believed that the composition of their outer layers, the so-called NS crust, is relatively well known. The outer region of the neutron star crust, referred to as the outer crust, is composed of atomic nuclei on the neutralizing background of degenerate, almost ideal electron gas. In the deeper layers, called the inner crust, unbound neutrons are also present [\[1,2\]](#page-4-0). With the subsequent increase of the density, the crust ends and we reach the outer core, which (in the vicinity of the crust-core boundary) is composed of neutrons  $(n)$ , protons  $(p)$ , and electrons  $(e)$ .

The only way to check whether theoretical models of superdense matter are reliable is to confront them with NS observations. One of the most promising possibilities in this regard is to study accreting NSs, which are observed in binary systems with active mass transfer from a companion star. For some of these sources, the accretion process is transient, and, in quiescent periods, x-ray telescopes are able to detect the thermal emission from the NS surface, revealing that it is heated up by accretion [\[3](#page-4-1)–6]. It is generally believed that the heating is caused by nonequilibrium nuclear reactions, which are initiated in the crust as it is compressed under the weight of newly accreted material. Obviously, an adequate interpretation of observations requires a reliable model describing this process (see [\[7\]](#page-4-2) for a recent review). A similar process can also be important for reheating of millisecond pulsars [\[8\]](#page-4-3).

Starting with Ref. [\[9\],](#page-4-4) a number of authors study the evolution of an accreted element as it compresses and sinks deeper and deeper toward the NS core in the course of accretion. Some of them used a one-component approximation [10–[14\],](#page-4-5) while others used reaction networks, allowing for mixtures of different nuclei [15–[18\].](#page-4-6) They applied either liquid-drop models [10–[13,15,18\],](#page-4-5) up-to-date theoretical atomic mass tables [\[16,17\],](#page-4-7) or detailed extended Thomas-Fermi calculations, allowing for the existence of unbound neutrons [\[14\].](#page-4-8) The main common feature of all these works is that they follow compositional changes associated with reactions induced by the increasing pressure inside an accreted fluid element ("traditional approach"). Such a consideration would be clearly applicable if we compressed uniform infinite matter. But in reality the inner crust is not uniform, so that unbound neutrons can travel between different layers to lower the system energy [\[19\]](#page-4-9).

The traditional approach was known to lead to jumps of the neutron chemical potential  $\mu_n$  at the phase transitions, which are especially pronounced in the one-component approximation and considerably soften if mixtures of nuclei are allowed for [\[15\].](#page-4-6) However, these effects were typically considered as a local inconsistency, which, likely, does not affect the global properties of the accreted crust. In this Letter, we show that it is not the case and that allowing unbound neutrons to move independently of nuclei has a dramatic effect on the crust composition and equation of state (EOS). We construct the corresponding EOS within the compressible liquid-drop model (CLDM), which ignores pairing and shell effects. This EOS is fully thermodynamically consistent; in particular,  $\mu_n$  in the inner crust is continuous and, moreover, satisfies the hydrostatic (and diffusion) equilibrium condition  $\mu_n^{\infty} =$ const (see below), where  $\mu_n^{\infty} \equiv \mu_n \exp(\nu/2)$  is the redshifted  $\mu_n$  and  $\nu = 2\phi/c^2$  ( $\phi$  is the gravitational potential and  $c$  is the speed of light  $[1]$ ).

The calculated EOS significantly differs from EOSs obtained within the traditional approach, being very close to the EOS of catalyzed crust.

In Ref. [\[20\]](#page-4-10), we demonstrate an additional inconsistency of the traditional approach: It leads to a strong violation of the force balance equation for nuclei (gravitational and electric forces are both directed downward) in a few rather extended regions of the inner crust, thus revealing inconsistency of the traditional approach from another point of view.

Equilibrium condition for unbound neutrons.— Neutrons, not bound to nuclei, exist in the inner NS crust. Except for a narrow layer of width  $L$  near the outer-inner crust interface ( $L \le 5$  m for  $T = 5 \times 10^8$  K), they are superfluid (e.g., Ref. [\[2\]\)](#page-4-11) and move with velocity  $V_{sn}$ , governed by the (linearized) superfluid equation  $m_n \partial V_{sn}/\partial t = -\nabla \mu_n^{\infty}$ , where  $m_n$  is the neutron mass (see, e.g., Refs. [\[21](#page-4-12)–23]). The hydrostatic equilibrium implies  $\mu_n^{\infty}$  = const as a necessary condition in the whole region of neutron superfluidity.

In the narrow nonsuperfluid layer, the typical diffusion time  $\tau^D \sim L^2/D \lesssim 3 \times 10^6$  s (the neutron diffusion coefficient  $D$  is estimated in analogy to Ref. [\[24\]](#page-4-13)) is much smaller than the replacement timescale of this layer by accretion,  $~\sim \rho L/\dot{M} \gtrsim 2\times10^9$  s (we take  $\rho \sim 4.3\times10^{11}$  gcm<sup>-3</sup> for the density and assume that  $\dot{M}$  equals the local Eddington accretion rate  $\dot{M} \sim 10^5$  g cm<sup>-2</sup> s<sup>-1</sup>). As a result, unbound neutrons in the nonsuperfluid layer should be, to a good approximation, in diffusion equilibrium:  $\mu_n^{\infty} = \text{const.}$ 

Three crustal EOSs.—The typical temperature in the crust of accreting NSs is  $T \le 5 \times 10^8$  K and has a minor effect on the EOS [\[1\]](#page-4-0), so below we shall work in the approximation of  $T = 0$ . As discussed above, neutron hydrostatic and diffusion (nHD) equilibrium conditions imply  $\mu_n^{\infty}$  = const in the whole inner crust (we assume that the region of neutron superfluidity extends to the crust-core boundary). To illustrate the importance of this condition, let us consider three EOSs: catalyzed (ground state) EOS, which is believed to describe a pristine NS crust and two EOSs for the accreted NS crust: (i) traditional, which completely disregards neutron diffusion (denoted as "Trad" EOS), and (ii) a new EOS that respects the nHD condition (denoted as "nHD" EOS). For simplicity, we apply the CLDM, in which nuclei are described as liquid drops, located at the center of the spherical Wigner-Seitz (WS) cells [\[1,2,25\].](#page-4-0) We ignore a possible layer of nonspherical nuclei in the vicinity of the crust-core boundary (for EOSs based on the SLy4 energy density functional, employed here, this layer is absent [\[26,27\]\)](#page-4-14). We also assume that the proton drip does not occur in the crust, which is true for all numerical models discussed here. The model is parametrized by the number densities  $n_{ni}$ ,  $n_{pi}$ , and  $n_{no}$  for, respectively, neutrons and protons inside and neutrons outside nuclei; the neutron skin surface density  $\nu_s$ ; and the volume  $V_c$  of WS cell, as well as by the (proton) radius  $r_p$  of a nucleus. In addition, it is useful to introduce the volume fraction occupied by nucleus inside the WS cell,  $w = 4\pi r_p^3/(3V_c)$ ; the surface area of a nucleus,  $\mathcal{A} = 4\pi r_p^2$ ; and the electron number density  $n_e$ , determined from the quasineutrality condition  $n_e = w n_{pi}$ . Within the CLDM, the energy density can be written as

$$
\epsilon = w\epsilon^{\text{bulk}}(n_{ni}, n_{pi}) + (1 - w)\epsilon^{\text{bulk}}(n_{no}, 0)
$$

$$
+ E_s(\nu_s, r_p) / V_c + E_C(n_{pi}, r_p, w) / V_c + \epsilon_e(n_e). \quad (1)
$$

Here  $\epsilon^{\text{bulk}}(n_n, n_p)$  is the energy density of homogeneous nuclear matter;  $E_s$  is the surface energy of a nucleus. The Coulomb energy of a WS cell is given by  $E_C =$  $(16\pi^2/15)(n_{pi}e)^2r_p^5f(w)$ , where  $f(w) = 1 - 1.5w^{1/3} + 0.5w$ and  $\epsilon_e$  is the energy density of degenerate electron gas [\[1\]](#page-4-0).

Taking the baryon number density  $n_b = w(n_{pi} + n_{ni}) +$  $(1 - w)n_{no} + A v_s/V_c$  and number density of nuclei,  $n_N = V_c^{-1}$ , to be fixed, we minimize  $\varepsilon$  with respect to other independent variables and obtain the betaequilibrium, mechanical, and local neutron diffusion equilibrium (within one unit cell) conditions. Using these conditions (see Supplemental Material [\[28\]](#page-4-15)), we arrive at the two-parameter equation of state,  $\epsilon = \epsilon(n_b, n_N)$ , with the second law of thermodynamics presented as

$$
d\epsilon = \mu_n dn_b + \mu_N dn_N, \qquad (2)
$$

where  $\partial \epsilon(n_b, n_N)/\partial n_b$  is denoted as  $\mu_n$ , because it equals the chemical potential of free (unbound) neutrons, as follows from the minimization procedure discussed above. The effective chemical potential  $\mu_N$  describes the energy change due to addition of an extra nuclear cluster to the system at fixed  $n_b$ ,  $\mu_N = (\sigma \mathcal{A} - 2E_C)/3$ , where  $\sigma$  is the surface tension [\[1\]](#page-4-0) (see Supplemental Material [\[28\]](#page-4-15)). The catalyzed EOS corresponds to the absolute minimum of  $\varepsilon$  at fixed  $n_b$ ; hence, it is given by the condition  $\mu_N = 0$ . With this condition, the EOS becomes one-parametric, i.e., specified in a unique way for a given  $n_b$ .

For accreted crust,  $T$  is not high enough to allow for nuclear reactions that minimize  $\varepsilon$  by choosing  $n_N$  in an optimum way; thus,  $\mu_N$  is, generally, nonzero. To make the EOS one-parametric, we need an additional equation. In the traditional approach (e.g., Ref. [\[10\]\)](#page-4-5), the equation follows from the requirement that the total baryon number in the WS cell is conserved:  $A_c = n_bV_c$  = const. (Note that this equation should be modified in the regions where pycnonuclear reactions proceed and  $A_c$  doubles [\[10,12](#page-4-5)–14].)

And what about the nHD EOS?  $A_c$  is not conserved now, because neutrons can move independently of nuclei. Instead, this EOS should respect the nHD condition  $\mu_n^{\infty}$  = const, as well as the general hydrostatic equilibrium condition  $P'(r) = -(P+\epsilon)\nu'(r)/2$  [\[1\]](#page-4-0), where P is the pressure and the prime means derivative with respect to the <span id="page-2-0"></span>radial coordinate  $r$ . Combining these two equations with the Gibbs-Duhem relation  $dP = n_b d\mu_n + n_N d\mu_N$ , one arrives at the requirement  $\mu_N^{\infty} = \mu_N e^{\nu/2} = \text{const.}$  In other words (because  $\mu_n e^{\nu/2}$  is also a constant), the ratio  $\mu_N/\mu_n$ must be fixed in the inner crust, i.e.,  $\mu_N/\mu_n = C$ , or, recalling the definition of  $\mu_N$ ,

$$
\sigma \mathcal{A} - 2E_C = 3C\mu_n,\tag{3}
$$

where  $C$  is some constant. This condition parametrizes a family of nHD EOSs. It allows one, in particular, to present  $\mu_n$  as a function of P and C:  $\mu_n = \mu_n(P, C)$ . The catalyzed EOS is a member of this family (hence, neutrons are in the diffusion equilibrium in catalyzed matter—an expected result); it corresponds to the choice  $C = 0$  (i.e.,  $\mu_N = 0$ ). As shown below, only one particular C corresponds to the fully accreted NS crust, which we shall be mostly interested in what follows.

nHD EOS for a fully accreted crust.—In this case, C can be determined from two requirements: (i) P and  $\mu_n$  at the crust-core boundary must be continuous; and (ii) the structure and composition of the fully accreted crust should not change in the course of accretion. In particular, the latter condition means that the total number of nuclei in the crust should be conserved. However, accretion permanently brings nuclei to the crust. Clearly, the stationary situation is possible only if the same number of nuclei disintegrate somewhere in the crust.

The nHD EOS provides a natural mechanism of nuclei disintegration due to a specific instability discussed below. Namely, numerical calculations show that for each C there is a maximum pressure  $P_{\text{max}}$ , such that the solution to Eq. [\(3\)](#page-2-0) does not exist at  $P > P_{\text{max}}$  (in Supplemental Material [\[28\]](#page-4-15), we argue that it is a general feature of nHD EOSs).

To demonstrate the physical mechanism behind the instability, we, first of all, combine the equation  $P'(r) =$  $-(P+\epsilon)\nu'(r)/2$  and condition  $\mu_n^{\infty} = \text{const}$  to derive a relation,  $d\mu_n = \mu_n/(P+\epsilon) dP$ , which is valid in the nHDequilibrated inner crust and is equivalent to Eq. [\(3\).](#page-2-0) It states that  $\mu_n$  in a given volume is fixed if P is fixed, independently of nuclear transformations occurring in this volume. Now, let us consider a layer, initially located at  $P_{\text{max}}$ , but compressed slightly by newly accreted material, so that  $P$  is a bit larger than  $P_{\text{max}}$ . The absence of stationary solutions at  $P > P_{\text{max}}$  means that the layer should be out of beta equilibrium at such a pressure (otherwise, it is impossible to remain in the hydrostatic equilibrium). Then beta captures come into play trying to return the system to beta equilibrium, but, as we checked numerically, they are accompanied by neutron emissions and the emitted neutrons diffuse out of the layer in order to preserve  $\mu_n$  at a given P. As a result, the layer begins to shrink, and nuclei in the layer start to "evaporate" (A and Z decrease) until disintegration—the required instability.

This instability is, in fact, similar to the mechanism discussed in Ref. [\[29\].](#page-4-16) Namely, at  $P > P_{\text{max}}$ , nuclei become unstable with respect to electron capture accompanied by emission of neutrons. Each electron capture makes the nucleus even more unstable, leading to a series of subsequent electron captures and neutron emissions until complete disintegration. The instability is also analogous to the superthreshold electron capture cascades studied in Refs. [\[16,17,30\]](#page-4-7), but, in contrast to these works, disintegration is complete and takes place at fixed P and  $\mu_n$ .

During accretion, the number of nuclei in the (initially catalyzed) crust is increasing until the instability sets in at  $P = P_{\text{max}}$ . Since at  $P > P_{\text{max}}$  stable crust does not exist,  $P_{\text{max}}$  should coincide with the pressure at the crust-core boundary [\[31\]](#page-4-17). Thus, the parameter  $C$  and, hence, nHD EOS for a fully accreted crust (hereafter, simply "nHD EOS") can be determined by matching  $\mu_n$  at  $P = P_{\text{max}}$  in the crust and in the core:  $\mu_n(P_{\text{max}}, C) = \mu_n^{\text{core}}(P_{\text{max}})$ , where  $\mu_n^{\text{core}}(P)$  stands for  $\mu_n$  in the core.

Now we have everything at hand to find where the outerinner crust interface is located. To this end, we note that, by definition, one has  $m_n = \mu_n$  at the interface; thus, the pressure  $P_{oi}$  there can be found from the condition:  $m_n = \mu_n(P_{oi}, C)$ . Note that it should not necessarily coincide (for nHD EOS) with the neutron drip pressure  $P_{\text{nd}}$  of Trad EOS, because the latter is obtained neglecting possible redistribution of neutrons in the star.

Numerical example.—To illustrate our results, we employ the SLY4 energy density functional [\[32\];](#page-4-18) the corresponding surface energy and tension  $\sigma$  are adopted from Ref. [\[33\].](#page-4-19) We find that the interface between the inner and outer crust is located at  $8.0 \times 10^{29}$ ,  $8.1 \times 10^{29}$ , and  $9.1 \times 10^{29}$  dyn cm<sup>-2</sup> for catalyzed, nHD, and Trad EOSs, respectively (for the nHD EOS, such  $P_{oi}$  leads to  $C \approx 0.0025$ ). The corresponding pressures at the crust-core boundary equal  $4.93 \times 10^{32}$ ,  $5.20 \times 10^{32}$ , and  $5.14 \times 10^{32}$  dyn cm<sup>-2</sup>. For simplicity, when considering the Trad model, we assume that the pycnonuclear reactions take place at  $Z = 10$ .

Figure [1](#page-2-1) demonstrates three EOSs described in this work: catalyzed (solid line), nHD (long dashes), and

<span id="page-2-1"></span>

FIG. 1. Pressure versus density for different crustal EOSs discussed in the text.

Trad (dots); the EOS of pure neutron matter (dashed line) is added for comparison. One can see that the nHD EOS, suggested here, significantly differs from the Trad EOS, obtained within the traditional approach, and is much closer to the catalyzed EOS (cf., e.g.,  $P_{oi}$  for catalyzed and nHD EOSs:  $8.0 \times 10^{29}$  and  $8.1 \times 10^{29}$  dyn cm<sup>-2</sup>, respectively).

The flat region at  $\rho \sim 1.1 \times 10^{12}$  g cm<sup>-3</sup> for the Trad EOS corresponds to a pycnonuclear reaction. These reactions are also clearly visible as jumps in Fig. [2](#page-3-0), which demonstrates profiles of nuclear charges Z and mass numbers A for the same EOSs as in Fig. [1](#page-2-1) [in Supplemental Material [\[28\]](#page-4-15), we also show the function  $A_c(P)$ ]. In addition, dot-dashed lines show profiles obtained in Ref. [\[14\]](#page-4-8) ignoring the condition  $\mu_n^{\infty} = \text{const.}$ The corresponding EOSs are calculated for the Sly4 functional in the extended Thomas-Fermi approach and for the liquid-drop model of Ref. [\[34\].](#page-4-20) One can see that for the traditional approach our CLDM reproduces the results of Ref. [\[14\]](#page-4-8) reasonably well; in particular, pycnonuclear reactions occur three times in the inner crust.

Crust composition (i.e., Z and A) for the nHD EOS is determined by Eq. [\(3\).](#page-2-0) One may note that it is remarkably different from that for the Trad EOS, being rather close (at not too large  $P$ ) to the composition of catalyzed crust. The latter fact is not surprising, since at not too large  $P$  two terms in the lhs of Eq. [\(3\)](#page-2-0) are much larger than the term  $C\mu_n$ in its rhs; hence, Eq. [\(3\)](#page-2-0) is quite similar to its "catalyzed" counterpart  $\mu_N = 0$ . At larger P, surface tension decreases, because matter inside and outside nuclear clusters becomes more and more similar, while the term  $C_{\mu_n}$  increases and, eventually, all three terms in Eq. [\(3\)](#page-2-0) become comparable; as a result, Z and A for the nHD EOS substantially differ from those for the catalyzed EOS at such P.

<span id="page-3-0"></span>

FIG. 2. The nuclei charge Z and atomic mass number A as a function of the pressure for different crustal EOSs.

Discussion and conclusions.—We construct the model of the inner crust of an accreting NS, which respects the nHD condition  $\mu_n^{\infty} = \text{const}$  imposed by the requirement of hydrostatic equilibrium with respect to the superfluid equation in most of the inner crust and by the diffusion equilibrium in a thin layer near the outer-inner crust interface. We find that the resulting nHD EOS is rather close to the catalyzed one, being significantly different from the Trad EOS obtained in the traditional approach, which ignores the condition  $\mu_n^{\infty} = \text{const}$  and implicitly assumes that both nuclei and unbound neutrons move together with one and the same velocity. Our another important result is that we found an instability that allows one to transform nuclei into npe matter at the crust-core boundary and explain its physical meaning. We also demonstrate that the interface  $P = P_{oi}$  between the (accreted) outer and inner crust is not associated with the "standard" neutron drip pressure  $P_{\text{nd}}$ , at which neutrons "drip out" of nuclei [\[35\]](#page-4-21). Instead,  $P_{oi}$  is determined by the nHD equilibrium condition inside the star. As a result,  $P_{\text{o}i}$ in the accreted crust appears to be just a bit higher than in the catalyzed crust, and nuclei at the  $P = P_{oi}$  interface absorb neutrons rather than emit them, as in the Trad EOS. Neutron absorptions (accompanied by electron emissions) lead to a jump of A at the upper boundary of the inner crust (see Fig. [2](#page-3-0)). Neutrons, necessary for such absorptions, are supplied by upward neutron flow, which originates at the crust-core boundary, where nuclei disintegrate into neutrons as a result of the instability discussed above. Then these neutrons redistribute over the inner crust and core in order to maintain nHD equilibrium.

The similarity of nHD and catalyzed EOSs suggests that accretion should have a less pronounced effect on the crust thickness and tidal deformability than in the traditional approach. It also suggests that the heat release due to nonequilibrium nuclear reactions in the accreted crust should be much smaller than it is usually thought to be, and this idea agrees with apparently very different reaction flows for the nHD EOS (e.g., pycnonuclear reactions for the nHD EOS are absent [\[36\]\)](#page-4-22). The heat release problem is considered in our forthcoming publication [\[37\]](#page-4-23). According to preliminary estimates, the net heat release is ∼0.5–0.7 MeV/nucleon (i.e., 2–3 times smaller than in the traditional approach), with significant fractions released at the outer-inner crust interface and crust-core boundary. These findings, along with the modification of the transport properties and heat capacity (due to changed nuclear composition), should noticeably affect the interpretation of transiently accreting NSs and may shed new light on the shallow heating and superburst ignition problems (e.g., [\[7\]](#page-4-2)).

The crucial role of the neutron hydrostatic and diffusion equilibrium for an accreted crust EOS, revealed in this Letter, is a general feature, which cannot be disregarded (see also [\[20\]](#page-4-10)). However, we should warn the reader that our results are illustrated within the simplified CLDM, which treats nuclear mass and charge numbers as continuous variables and neglects pairing and shell effects. In Ref. [\[14\],](#page-4-8) the latter are shown to be important for the energy release in the traditional approach. According to our preliminary results, obtained within the nHD approach, shell effects mainly influence the profile of the heat release and composition of the crust; at the same time, the  $P(\rho)$ dependence is not strongly affected.

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- <span id="page-4-9"></span>[19] The only exception which allows for diffusion of unbound neutrons is a series of works [\[24,38\],](#page-4-13) but it mainly focuses on the crust properties of newly born NSs (see also the discussion section in Ref. [\[15\]\)](#page-4-6).
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