Bare-Excitation Ground State of a Spinless-Fermion–Boson Model and *W*-State Engineering in an Array of Superconducting Qubits and Resonators

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This Letter unravels an interesting property of a one-dimensional lattice model that describes a single itinerant spinless fermion (excitation) coupled to zero-dimensional (dispersionless) bosons through two different nonlocal coupling mechanisms. Namely, below a critical value of the effective excitation-boson coupling strength, the exact ground state of this model is the zero-quasimomentum Bloch state of a bare (i.e., completely undressed) excitation. It is demonstrated here how this last property of the lattice model under consideration can be exploited for a fast, deterministic preparation of multipartite W states in a readily realizable system of inductively coupled superconducting qubits and microwave resonators.

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Sophisticated quantum-state engineering [1,2] is a prerequisite for the development of next-generation quantum technologies [3,4]. In this context, tantalizing progress was made in recent years by utilizing diverse physical platforms [5–7]. In particular, owing to their continuously improving scalability and coherence properties, superconducting (SC) circuits [8–11] and, among them, circuit-QED systems [12,13], allow accurate preparation of various quantum states of SC qubits and photons alike [14].

The most prominent classes of entangled many-qubit states are maximally entangled Greenberger-Horne-Zeilinger (GHZ) [15] and W states [16]. An N-qubit W state is the equal superposition of states with exactly one qubit in its "up" state, the remaining ones being in their "down" states. In particular, it is known that a W state and its GHZ counterpart cannot be transformed into each other via local operations and classical communication [17]. W states are also extremely robust with respect to particle loss, remaining entangled even if any $N-2$ parties lose the information about their particle [18]. They lend themselves to applications in quantum-information protocols [5,19–21], which motivates their preparation in various systems [22–28].

This Letter establishes a connection between a one-dimensional (1D) lattice model describing a nonlocal interaction of a spinless-fermion excitation with dispersionless bosons and multipartite W states. Its point of departure is the notion that in one of the relevant regimes this model —which includes excitation-boson (e-b) couplings of Peierls and breathing-mode types—has an unconventional ground state. Namely, below a critical value of the effective e-b coupling strength, its ground state is the zero-quasimomentum Bloch state of a bare excitation. It is shown here how this property of the model under consideration can be exploited for a fast, deterministic preparation of N-qubit W states in an array of inductively coupled SC qubits and resonators, an analog simulator of this model [29].

The state-preparation protocol proposed here, based on microwave pumping, allows one to obtain multipartite W states within time frames 3 orders of magnitudes shorter than the currently achievable coherence times of SC qubits. Unlike the situation in quantum-state control, where typical preparation times scale unfavorably with the system size, here they do not depend on the number of qubits at all. What makes this protocol particularly robust is the fact that its target state is the ground state of the system in a parametrically large window of values of its main experimental knob—an external dc flux.

Model and its ground state.—The 1D lattice model under consideration describes a single spinless-fermion excitation interacting with dispersionless bosons through two different nonlocal coupling mechanisms. The noninteracting part of its total Hamiltonian includes the excitation kinetic-energy and free-boson terms

$$H_0 = -t_e \sum_n (c_{n+1}^\dagger c_n + \mathrm{H.c.}) + \hbar\omega_b \sum_n b_n^\dagger b_n. \quad (1)$$

Here $c_n^\dagger$ (c_n) creates (destroys) an excitation at site n ($n = 1, \ldots, N$), $b_n^\dagger$ (b_n) a boson with frequency ω_b at the same site, while t_e is the excitation hopping amplitude. The interacting (e-b) part is given by

$$\begin{aligned} H_{e\text{-}b} = g\hbar\omega_b \sum_n [& c_n^\dagger c_n (b_{n-1}^\dagger + b_{n-1} - b_{n+1}^\dagger - b_{n+1}) \\ & + (c_{n+1}^\dagger c_n + \mathrm{H.c.})(b_{n+1}^\dagger + b_{n+1} - b_n^\dagger - b_n)], \end{aligned} \quad (2)$$

where g is the dimensionless e-b coupling strength. The first term on the right-hand side (rhs) of the last equation captures the antisymmetric coupling of the excitation density at site n with the local boson displacements on the neighboring sites $n \pm 1$ (breathing-mode-type coupling) [30]. The second term accounts for the linear dependence of the effective excitation-hopping amplitude between sites n and $n+1$ on the respective boson displacements (Peierls-type coupling) [31–33].

The coupling Hamiltonian $H_{e\text{-}b}$ can be recast in the generic momentum-space form

$$H_{e\text{-}b} = \frac{1}{\sqrt{N}} \sum_{k,q} \gamma_{e\text{-}b}(k,q) c^{\dagger}_{k+q} c_k (b^{\dagger}_{-q} + b_q). \quad (3)$$

Its corresponding e-b vertex function depends on both the excitation and boson quasimomenta (k and q, respectively, here expressed in units of the inverse lattice period) and is given by

$$\gamma_{e\text{-}b}(k,q) = 2ig\hbar\omega_b[\sin k + \sin q - \sin(k+q)]. \quad (4)$$

The ground state of $H = H_0 + H_{e\text{-}b}$ undergoes a sharp level-crossing transition [34] at a critical value $\lambda^c_{e\text{-}b} \sim 1$ [cf. Fig. 2] of the effective coupling strength $\lambda_{e\text{-}b} \equiv 2g^2\hbar\omega_b/t_e$. For $\lambda_{e\text{-}b} < \lambda^c_{e\text{-}b}$ the ground state is the $K=0$ eigenvalue of the total quasimomentum operator $K_{\text{tot}} = \sum_k k c^{\dagger}_k c_k + \sum_q q b^{\dagger}_q b_q$. For $\lambda_{e\text{-}b} \geq \lambda^c_{e\text{-}b}$, on the other hand, the ground state is twofold degenerate and corresponds to $K = \pm K_{\text{gs}}$ ($K_{\text{gs}} \neq 0$).

A ground state with $K=0$ is by no means unusual—in fact, an overwhelming majority of coupled e-b models have such ground states. Yet, the model at hand has the peculiar property that its $K=0$ ground state for $\lambda_{e\text{-}b} < \lambda^c_{e\text{-}b}$ is the $k=0$ bare-excitation Bloch state $|\Psi_{k=0}\rangle \equiv c^{\dagger}_{k=0}|0\rangle_e \otimes |0\rangle_b$, where $|0\rangle_e$ and $|0\rangle_b$ are the excitation and boson vacuum states. In what follows, it will first be demonstrated explicitly that $|\Psi_{k=0}\rangle$ is an exact eigenstate of H for an arbitrary value of $\lambda_{e\text{-}b}$. It will subsequently be shown numerically (see Fig. 2 below) that for $\lambda_{e\text{-}b} < \lambda^c_{e\text{-}b}$ this state is the ground state of H.

Given that $|\Psi_{k=0}\rangle$ is an eigenstate of H_0, to prove that it is an eigenstate of the total Hamiltonian H it suffices to show that it is also an eigenstate of $H_{e\text{-}b}$. Indeed, by acting with $H_{e\text{-}b}$ [cf. Eq. (3)] on this state and making use of the fact that $c_k c^{\dagger}_0|0\rangle_e \equiv \delta_{k,0}|0\rangle_e$, one obtains

$$H_{e\text{-}b}|\Psi_{k=0}\rangle = \frac{1}{\sqrt{N}} \sum_q \gamma_{e\text{-}b}(k=0,q) c^{\dagger}_q |0\rangle_e \otimes b^{\dagger}_{-q}|0\rangle_b. \quad (5)$$

Because here $\gamma_{e\text{-}b}(k=0,q)=0$ for an arbitrary q [cf. Eq. (4)], each term in the sum on the rhs of Eq. (5) vanishes, implying that $H_{e\text{-}b}|\Psi_{k=0}\rangle = 0$. Therefore, $|\Psi_{k=0}\rangle$ is an eigenstate of $H_{e\text{-}b}$ (for an arbitrary $\lambda_{e\text{-}b}$), the corresponding eigenvalue being equal to zero. This concludes the proof that $|\Psi_{k=0}\rangle$ is an exact eigenstate of H.

Qubit-resonator system.—The analog simulator of the model under consideration [see Fig. 1(a)] consists of SC qubits (Q_n) with the energy splitting ε_z, microwave resonators (R_n) with the photon frequency ω_c, and coupler

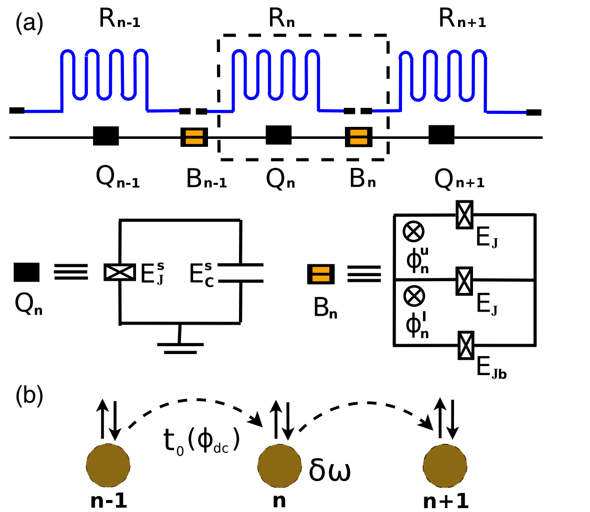

FIG. 1. (a) Schematic of the qubit-resonator system, whose nth repeating unit (indicated by the dashed rectangle) comprises SC qubit Q_n, resonator R_n, and coupler circuit B_n. The fluxes from the resonator modes n and $n+1$ thread the upper loops of the coupler circuit B_n, effectively giving rise to an indirect inductive qubit-resonator coupling. (b) Pictorial illustration of the effective lattice model of the system, with the excitation hopping amplitude $t_0(\phi_{\text{dc}})$ and each lattice site hosting dispersionless bosons with the frequency $\delta\omega = \omega_c - \omega_0$.

circuits (B_n) [35], which mediate both qubit-qubit and qubit-resonator interactions in this system. The simulator can be realized with transmons [12] ($E^s_J/E^s_C \sim 100$, where E^s_C and E^s_J are the single-qubit charging and Josephson energies) or gatemons [36] ($E^s_J/E^s_C \sim 25$). Its nth repeating unit is described by the free Hamiltonian $H^0_n = (\varepsilon_z/2)\sigma^z_n + \hbar\omega_c b^{\dagger}_n b_n$, where the pseudospin-1/2 operators $\boldsymbol{\sigma}_n$ represent qubit n and the bosonic operators $(b_n, b^{\dagger}_n)$ photons in the nth resonator.

The upper and lower loops of B_n are threaded by magnetic fluxes ϕ^u_n and ϕ^l_n, respectively [both are expressed in units of $\Phi_0/2\pi$, where $\Phi_0 \equiv hc/(2e)$ is the flux quantum]. In particular, the upper loop is subject to ac driving with the flux $\pi\cos(\omega_0 t)$. The other contribution to ϕ^u_n originates from the modes of resonators n and $n+1$ and is given by $\phi_{n,\text{res}} = \delta\theta[(b_{n+1} + b^{\dagger}_{n+1}) - (b_n + b^{\dagger}_n)]$, where $\delta\theta = [2eA_{\text{eff}}/(\hbar d_0 c)] \times (\hbar\omega_c/C_0)^{1/2}$, with A_{eff} being the effective coupling area, C_0 the resonator capacitance, and d_0 the effective spacing in the resonator [37]. Therefore, unlike the much more common capacitive coupling [38], the qubit-resonator coupling in the system at hand is inductive [11]. Similarly, ϕ^l_n includes an ac contribution, given by $-(\pi/2)\cos(\omega_0 t)$, and a dc part ϕ_{dc}, the main experimental knob in this system.

The Josephson energy of B_n is given by $H^J_n = -\sum_{i=1}^3 E^i_J \cos\varphi^i_n$, with φ^i_n being the respective phase drops on the three Josephson junctions within B_n and E^i_J their energies; it is henceforth assumed that $E^1_J = E^2_J \equiv E_J$ and

$E_J^3 = E_{Jb} \neq E_J$. Using the flux-quantization rules [9], the total Josephson energy $\sum_n H_n^J$ can be expressed in terms of the gauge-invariant phase variables φ_n of SC islands of different qubits. The latter enter this energy through terms of the type $\cos(\varphi_n - \varphi_{n+1})$, which in the regime of interest for transmons and gatemons ($E_J^s \gg E_C^s$) can be recast (up to an additive constant) as $\delta\varphi_0^2[\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+ - (\sigma_n^z + \sigma_{n+1}^z)/2]$, where $\delta\varphi_0^2 \equiv (2E_C^s/E_J^s)^{1/2}$ [38].

Further analysis is carried out in the rotating frame of the drive, where $\delta\omega \equiv \omega_c - \omega_0$ is the effective boson frequency and the Josephson coupling term becomes time dependent. This time dependence can, however, be disregarded due to its rapidly oscillating character, in line with the rotating-wave approximation (RWA). The remaining part of H_n^J can succinctly be written as $\bar{H}_n^J = -\mathcal{E}_n^J(\phi_{\rm dc}, \phi_{n,\rm res})\cos(\varphi_n - \varphi_{n+1})$, where

$$\mathcal{E}_n^J = E_{Jb}(1 + \cos\phi_{\rm dc}) - E_J J_1(\pi/2)\phi_{n,\rm res}, \tag{6}$$

and $J_m(x)$ are Bessel functions of the first kind whose presence in this expression stems from the use of the Jacobi-Anger expansion [39] in conjunction with the RWA. In what follows, without significant loss of generality E_{Jb} is chosen to be given by $2E_J J_0(\pi/2)$.

The above expression for $\cos(\varphi_n - \varphi_{n+1})$ in terms of the operators $\boldsymbol{\sigma}_n$ implies that the effective interaction between adjacent qubits in this system is of XY type. Through the flux $\phi_{n,\rm res}$, the interaction strength acquires a dependence on the boson displacements $u_n \propto b_n + b_n^\dagger$ whose form is equivalent to that of the XY spin-Peierls model [40]. The spinless-fermion–boson coupling that results from this interaction via Jordan-Wigner (JW) transformation is nonlocal in nature, in contrast to other examples of such couplings in various solid-state systems [28,41].

Effective Hamiltonian and its ground states.—To show that the effective system Hamiltonian consists of contributions akin to H_0 and $H_{e\text{-}b}$ [cf. Eqs. (1) and (2)], one switches to the spinless-fermion representation using the JW transformation. The latter reads $\sigma_n^z = 2c_n^\dagger c_n - 1$, $\sigma_n^+ = 2c_n^\dagger e^{i\pi\sum_{l<n} c_l^\dagger c_l}$, where the sum in the last exponent defines the JW string [42]. In this representation, the noninteracting part of the effective system Hamiltonian comprises the excitation-hopping and free-photon terms. [Note that $c_n^\dagger c_n$ terms resulting from the σ_n^z terms in H_n^0 and $\overline{H}_n^J$ are largely immaterial for further discussion as they only lead to a constant energy offset (band-center energy).] It assumes the form of H_0, with $\omega_b \to \delta\omega$ and $t_e \to t_0(\phi_{\rm dc}) \equiv E_{Jb}\delta\varphi_0^2(1 + \cos\phi_{\rm dc})$, the latter being the effective $\phi_{\rm dc}$-dependent hopping amplitude [cf. Fig. 1(b)]. At the same time, the interacting part adopts the form of $H_{e\text{-}b}$. In particular, via the JW transformation the Peierls coupling term is obtained in a manner familiar from the XY spin-Peierls model [40], while the breathing-mode term originates from the σ_n^z terms in the above expression for $\cos(\varphi_n - \varphi_{n+1})$.

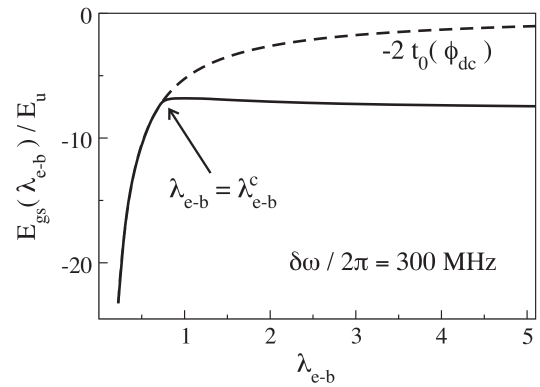

FIG. 2. Ground-state energy of the system with $\delta\omega/2\pi = 300$ MHz as a function of the effective coupling strength $\lambda_{e\text{-}b}$. For $\lambda_{e\text{-}b} < \lambda_{e\text{-}b}^c \approx 0.72$ (i.e., $\phi_{\rm dc} < 0.972\pi$) the ground state of the system corresponds to a bare excitation, while for $\lambda_{e\text{-}b} \geq \lambda_{e\text{-}b}^c$ it corresponds to a heavily dressed (polaronic) one.

The dimensionless coupling strength g is determined by the system parameters through the relation $g\hbar\delta\omega = \delta\varphi_0^2 E_J J_1(\pi/2)\delta\theta$, while the $\phi_{\rm dc}$-dependent—thus *in situ* tunable—effective coupling strength is given by

$$\lambda_{e\text{-}b}(\phi_{\rm dc}) = g\frac{J_1(\pi/2)\delta\theta}{J_0(\pi/2)(1 + \cos\phi_{\rm dc})}. \tag{7}$$

For a typical resonator $\delta\theta \sim 3.5 \times 10^{-3}$ [29]. In addition, for $\delta\omega$ it is pertinent to take $\delta\omega/2\pi = 200$–300 MHz and also choose E_J such that $\delta\varphi_0^2 E_J/2\pi\hbar = 100$ GHz.

The ground-state energy of the system, expressed in units of $E_u \equiv 10^{-3}\delta\varphi_0^2 E_J$, was evaluated through Lanczos-type exact diagonalization [29,43] and illustrated (without the constant-energy contribution) in Fig. 2. For $\lambda_{e\text{-}b} \geq \lambda_{e\text{-}b}^c$, the system has a polaronlike ground state (strongly boson-dressed excitation), with its energy showing a rather weak dependence on $\lambda_{e\text{-}b}$. On the other hand, for $\lambda_{e\text{-}b} < \lambda_{e\text{-}b}^c$, the ground state corresponds to $|\Psi_{k=0}\rangle$, i.e., a bare excitation with $k = 0$. Its energy $E_{\rm gs} = -2t_0(\phi_{\rm dc})$ is the minimum of a 1D cosine-shaped dispersion. The energy separation of this ground state from the first excited state exactly equals $\hbar\delta\omega$ for any $\phi_{\rm dc}$ below the critical value. This is consistent with the fact that coupled e-b systems with dispersionless bosons invariably have one-boson continua separated from their ground states by the single-boson energy and in the weak coupling regime typically feature only one bound state below those continua [44].

W states and their preparation.—Bearing in mind that JW strings act trivially on $|0\rangle_e$, so that $c_n^\dagger|0\rangle_e \equiv S_n^+|0\rangle_e$ (where $\mathbf{S}_n \equiv \hbar\boldsymbol{\sigma}_n/2$), it holds that

$$c_k^\dagger|0\rangle_e = N^{-1/2}\sum_{n=1}^{N} e^{-ikn}S_n^+|0\rangle_e. \tag{8}$$

The last equation is equivalent to $|\Psi_k\rangle = |W_N(k)\rangle \otimes |0\rangle_b$, where $|W_N(k)\rangle$ is a "twisted" N-qubit W state. Thus, bare-excitation Bloch states coincide with generalized W states, while, in particular, $|\Psi_{k=0}\rangle$—the ground state of the system at hand for $\lambda_{e\text{-}b} < \lambda^c_{e\text{-}b}$—corresponds to the ordinary N-qubit W state

$$|W_N\rangle = \frac{1}{\sqrt{N}}(|10\ldots0\rangle + |01\ldots0\rangle + \cdots + |00\ldots1\rangle). \qquad (9)$$

In the following, a microwave-pumping-based protocol for the preparation of an N-qubit W state is proposed assuming that the system is initially in the vacuum state $|0\rangle \equiv |0\rangle_e \otimes |0\rangle_{\mathrm{ph}}$. The external driving required for this purpose is assumed to be represented by the operator

$$\Omega_{q_d}(t) = \frac{\hbar\beta(t)}{\sqrt{N}}\sum_{n=1}^{N}(\sigma_n^+ e^{-iq_d n} + \sigma_n^- e^{iq_d n}), \qquad (10)$$

where $\beta(t)$ describes its time dependence and the factors $e^{\pm iq_d n}$ account for the possibility that flip operations on different qubits are applied with a phase difference. It is important to stress that the general form of driving in Eq. (10) allows one to prepare—through different choices of q_d and $\beta(t)$—various states of the proposed system, including its strongly boson-dressed ground states realized when ϕ_{dc} is above the critical value.

The transition matrix element of the operator $\Omega_{q_d}(t)$ between the initial state $|0\rangle$ and the target state $|\Psi_{k=0}\rangle \equiv |W_N\rangle \otimes |0\rangle_{\mathrm{ph}}$ evaluates to $\hbar\beta(t)\delta_{q_d,k=0}$, which indicates that the preparation of this particular target state requires only a global driving field [i.e., $q_d = 0$ in Eq. (10)]. Thus, in contrast to some other schemes for W-state preparation [22], the present one does not require a local qubit control [45,46]. By assuming that $\beta(t) = 2\beta_p\cos(\omega_d t)$, where $\hbar\omega_d$ is the energy difference between the two relevant states, in the RWA these states are Rabi coupled with the effective Rabi frequency β_p [47,48]. Thus, starting from the state $|0\rangle$, the desired state N-qubit W state will be prepared within a time interval of duration $\tau_{\mathrm{prep}} = \pi\hbar/(2\beta_p)$, which does not depend on N.

Taking the pumping amplitude to be $\beta_p/(2\pi\hbar) = 10$ MHz, one finds $\tau_{\mathrm{prep}} \approx 25$ ns, which is 3 orders of magnitude shorter than typical coherence times of SC qubits (e.g., for transmons $T_2 \sim 20$–100 μs [10]). Thus, the proposed protocol should not be affected by a loss of coherence in the system. At the same time, the obtained τ_{prep} is sufficiently long that a leakage outside of the computational subspace of a single qubit can be neglected. Namely, due to the multilevel character of SC qubits, a finite anharmonicity $\alpha \equiv E_{12} - E_{01}$ (where E_{ij} is the energy difference between qubit states j and i) is required. In order to avoid such a leakage, the minimal pulse duration of $t_p \sim \hbar/|\alpha|$ is necessary. For transmons ($\alpha \sim -200$ MHz), even a few-nanoseconds-long pulse is frequency selective enough that such a leakage is negligible [13]. The obtained $\tau_{\mathrm{prep}} \sim 25$ ns suffices even in the case of gatemons, whose typical anharmonicity is by a factor of 2 smaller than that of transmons [49].

Importantly, the large energy separation $\hbar\delta\omega$ between the target state and the lowest-lying excited state of the system ensures that the proposed W-state preparation will not be hampered by an inadvertent population of undesired states. For instance, for $\delta\omega/2\pi = 200(300)$ MHz, this energy separation is equal to $2E_u(3E_u)$, which represents a significant fraction of the energy difference between the initial and target states (cf. Fig. 2).

The proposed protocol is deterministic in nature and generates W-type entanglement of all the qubits in parallel. Moreover, in contrast to the typical situation in quantum-state control, where state-preparation times often scale unfavorably with the system size, here τ_{prep} does not depend on the system size at all. Finally, because they represent ground states of the system, multipartite W states prepared by this protocol can be expected to be extremely robust.

Besides allowing W-state preparation, the proposed system features an XY-type qubit-qubit interaction, which opens the possibility for a universal quantum computation [50,51]. Because the strength of this interaction depends dynamically on the boson degrees of freedom (photons), this system bears a formal similarity to certain trapped-ion systems in which the role of bosons is played by collective motional modes (phonons) [52]. Compared to its trapped-ion counterparts, this system has an added advantage that it merely involves dispersionless bosons of one single frequency, which circumvents the spectral crowding problem resulting from the quasicontinuous character of phonon spectra in large trapped-ion chains [53].

Robustness to losses and feasibility.—It is pertinent to briefly address the robustness of the system at hand to possible deleterious effects of losses. To this end, it is worthwhile to first note that qubit-state flips and displacements of the resonator modes are the two leading sources of decoherence in this system. In addition to the very long T_2 times of transmon (gatemon) qubits, the damping time of microwave photons in coplanar waveguide resonators can reach the same order of magnitude as T_2, with the corresponding quality factor being larger than 10^7 [54]. Additionally, the relevant excitation and photon energy scales in this system ($\delta\omega$, $g\delta\omega$, $t_0/\hbar$), expressed in frequency units, are all of the order of several $2\pi \times 100$ MHz. Thus, they far exceed the decoherence rates whose state-of-the-art values in these types of systems are $\gamma \sim 0.01$ MHz [10]. Finally, in this system, thermal excitations—which at temperatures typical for such SC-qubit setups ($T \sim 100$ mK) have characteristic energies of a few gigahertz—can be safely neglected. Therefore, the loss mechanisms do not pose obstacles to realizing the proposed system.

Conclusions.—The present Letter proposes a scheme for a fast, deterministic creation of a large-scale W-type entanglement [55] in a system of inductively coupled superconducting qubits and microwave resonators. The mechanism behind this scalable entanglement resource—which allows one to engineer W states with the preparation times independent of the system size—stems from the unconventional ground-state properties of a one-dimensional model describing a nonlocal coupling of a spinless fermion to zero-dimensional bosons. The feasibility and robustness of the underlying state-preparation protocol—which only requires a global driving field—is demonstrated with realistic system parameters.

This study can be viewed as being complementary to that of Ref. [22], where the preparation of W states of photons—rather than qubits—was proposed. The common denominator of these two proposals is that they both rely on superconducting systems and an *in situ* tunability of a hopping amplitude, albeit being based on completely different physical mechanisms. These schemes are far more scalable than the conventional ones in which resonator-mediated qubit-qubit interactions are utilized to controllably entangle multiple qubits; such an approach was recently used to prepare a GHZ state of 10 superconducting qubits [7]—the largest entanglement demonstrated so far in solid-state architectures. Thus, the need to demonstrate the envisioned W-state preparation is compelling.

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