Thouless Time Analysis of Anderson and Many-Body Localization Transitions

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Spectral statistics of disordered systems encode Thouless and Heisenberg timescales, whose ratio determines whether the system is chaotic or localized. We show that the scaling of the Thouless time with the system size and disorder strength is very similar in one-body Anderson models and in disordered quantum many-body systems. We argue that the two parameter scaling breaks down in the vicinity of the transition to the localized phase, signaling a slowing-down of dynamics.

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Introduction.—The phenomenon of many-body localization (MBL) [1,2], the robust mechanism of ergodicity breaking in the quantum world [3–5], has received a lot of attention over the last decade. Investigations of MBL in lattice models, pioneered in spin systems [6–8], were extended to bosonic models [9,10] and to systems of spinful fermions [11–14]. Remarkably, MBL, usually thought of as an Anderson localization [15] in the presence of interactions, was shown to occur in systems with completely delocalized single particle states, either due to random interactions [16–18] or in a quasiperiodic Fibonacci chain [19]. MBL was also found in disorder-free systems as a result of gauge invariance [20,21] or due to Wannier-Stark localization [22,23]. MBL is also present in systems with long range interactions [24–28], as well as for the Floquet states of driven systems [29]. Local integrals of motion [30–36] provide a common framework to understand features of MBL such as the area-law entanglement entropy of eigenstates [37,38], the logarithmic growth of bipartite entanglement entropy after quench from a separable state [39,40], or the Poisson statistics of energy levels.

The crossover between the level statistics of an ergodic system with time reversal symmetry that follow predictions of the Gaussian orthogonal ensemble (GOE) of random matrices [41,42] and the Poisson statistics of the MBL phase seems to be well understood [43–48]. However, a recent analysis [49] of the spectral form factor (SFF), $K(\tau)$, in the wide regime of slow thermalization on the ergodic side of the crossover [50–53] questions the very existence of the MBL phase in the thermodynamic limit. Instead, it predicts a two parameter scaling of Thouless time

$$t_{\mathrm{Th}} = t_0 e^{W/\Omega} L^2, \tag{1}$$

where L is the system size, W is the disorder strength, t_0 and Ω are constants. The Thouless time t_{Th} is defined as the timescale beyond which the SFF follows the universal GOE form. Another important timescale, the Heisenberg time $t_H = 2\pi/\Delta$, is defined by the average level spacing Δ that scales exponentially with a many-body system size of L, $\Delta \propto e^{cL}$. The Heisenberg time t_H is a limit beyond which the discrete nature of the energy spectrum manifests itself and where system dependent quantum effects are unavoidable. In the thermodynamic limit, Eq. (1) implies $t_{\mathrm{Th}}/t_H \to 0$. Hence, [49] arrives at the surprising conclusion that disordered quantum spin chains have spectral properties following the GOE predictions regardless of

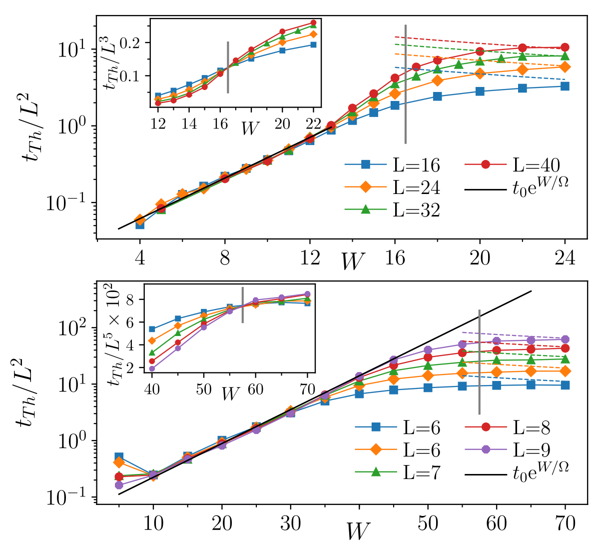

FIG. 1. Thouless time t_{Th} vs disorder strength W extracted from the SFF for 3D (upper plot) and 5D (lower plot) Anderson models for various system sizes L. The black solid lines denote the scaling of Eq. (1). The gray vertical lines denote the critical disorder strength $W_C^{3\mathrm{D}} = 16.54$ ($W_C^{5\mathrm{D}} = 57.3$) in the 3D (5D) model. The dashed lines denote the Heisenberg time t_H. The insets show t_{Th}/L^3 (t_{Th}/L^5) in the 3D (5D) case.

the disorder strength W and that the MBL is merely a finite-size effect.

In this Letter, we analyze the SFF in the delocalized phase and its modifications when approaching the transition to the localized phase. We show that Thouless time scales like L^2, in agreement with Eq. (1), in the deep delocalized phase in Anderson models as well as in disordered many-body systems. The scaling with L evolves to a larger power at the critical point of the Anderson model, a phenomenon that we correlate with the diffusive and subdiffusive transport properties, respectively, in the delocalized phase and at the metal-insulator transition. The results obtained for 3D and 5D Anderson models with known localization properties put the conclusions of [49] about the scaling of Thouless time t_{Th} in considerable doubt, suggesting the presence of an MBL phase at sufficiently strong disorder strengths when finite-size effects are properly taken into account.

Thouless time.—In a noninteracting system, the Thouless time was introduced as the time to diffuse through the system and reach its boundary [54]. It determines the energy scale below which the level statistics are well described by GOE [55], whereas its ratio with the Heisenberg time fixes the dimensionless conductance of the system [56] and enters the scaling theory of the Anderson localization transition [57]. The Thouless time t_{Th} in disordered many-body systems can be probed by examining the behavior of the SFF [58–61] defined as

$$K(\tau) = \frac{1}{Z}\left\langle \left| \sum_{j=1}^{\mathcal{N}} g(\epsilon_j) e^{-i\epsilon_j \tau} \right|^2 \right\rangle, \tag{2}$$

where ϵ_j are eigenvalues of the system after the unfolding [62] (which sets their density to unity), $g(\epsilon)$ is a Gaussian function reducing the influence of the spectrum's edges, the average is taken over disorder realizations, and $\mathcal{N}$ is the dimension of the Hilbert space. For a GOE matrix, the SFF is known analytically: $K_{\mathrm{GOE}}(\tau) = 2\tau - \tau \log(1 + 2\tau)$ for $\tau \le 1$ and $K_{\mathrm{GOE}}(\tau) = 2\tau - \tau \log(1 + 2\tau)$ for $\tau > 1$. The linear ramp $K_{\mathrm{GOE}}(\tau) \approx 2\tau$ of SFF starting at $\tau = 0$ reflects correlations between all pairs of eigenvalues in a GOE matrix. In contrast, the SFF $K(\tau)$ calculated for a physical system follows the GOE predictions $K(\tau) = K_{\mathrm{GOE}}(\tau)$ only for $\tau > \tau_{\mathrm{Th}}$ defining τ_{Th}, which, in turn, is proportional to the Thouless time $t_{\mathrm{Th}} = \tau_{\mathrm{Th}} t_H$. The proportionality factor t_H comes from the fact that unfolded eigenvalues ϵ_i enter the definition of $K(\tau)$; it is equal to the Heisenberg time t_H, determined by the inverse level spacing.

For a diffusive transport, the mean square displacement $\langle r^2(t)\rangle$ is proportional to time t. Hence, the above definition of t_{Th} coincides with the original definition of the Thouless time in a diffusive system provided that the $t_{\mathrm{Th}} \sim L^2$ where L is the system size. For subdiffusion, the mean square displacement behaves as $\langle r^2(t)\rangle \sim t^\alpha$ with $0 < \alpha < 1$; thus we expect $t_{\mathrm{Th}} \sim L^{2/\alpha}$. In the deeply localized regime where the localization length is much smaller than the system size, a particle never explores the full system size, so that the original Thouless time eventually diverges and becomes larger than the Heisenberg time. In contrast, the Poisson statistics are characteristic for a localized regime where the SFF is independent of time; the Thouless time deduced from the SFF is thus equal to the Heisenberg time. This implies that the latter definition is applicable only in the delocalized regime. Before we consider interacting models, we examine first the Thouless time as defined by the SFF in Anderson models.

Thouless time in 3D and 5D Anderson models.—The Hamiltonian of the Anderson model describes the hopping of a particle on a D-dimensional lattice with disorder and reads

$$\hat{H} = -t \sum_{\langle i,j\rangle} (\hat{c}_i^\dagger \hat{c}_j + \mathrm{H.c.}) + \sum_i \epsilon_i \hat{c}_i^\dagger \hat{c}_i, \tag{3}$$

where $\hat{c}_i^\dagger$ is the creation operator for particle at site i, $\langle .,. \rangle$ denotes the sum over neighboring lattice sites, $t \equiv 1$ is the tunneling amplitude, and $\epsilon_i \in [-W, W]$ denotes uniformly distributed uncorrelated random variables forming the on-site potential. Numerical studies of transport properties of the 3D Anderson model [63–66] indicate that transport is diffusive for disorder strengths $W < W_C^{\mathrm{3D}} \approx 16.54$ [67] and that the system remains insulating for $W > W_C$. Exactly at the transition, the 3D Anderson model is characterized by subdiffusion [68] and multifractal wave functions [69,70]. Studies of transport in 5D Anderson model [71] find a localization transition, which is consistent with the findings of studies of level statistics [72] giving the critical disorder $W_C^{\mathrm{5D}} = 57.3$, confirmed in [73,74].

Level spacing distribution in the 3D Anderson model was studied in [55,75–78]. The Thouless times presented in Fig. 1 unveil a long-range correlation aspect of level statistics in Anderson models. Examples of SFF and details on the Thouless time estimation are given in [79].

At the small disorder strength W, the Thouless times depend quadratically on system size L (Fig. 1), following precisely the scaling Eq. (1), which simply means the dynamics are diffusive. For the 3D model, the $t_{\mathrm{Th}}/L^2 \propto e^{W/\Omega}$ behavior persists up to $W \approx 12$. For bigger disorder strengths, the quadratic scaling with the system size is no longer valid. Directly at the transition, $W = W_C^{\mathrm{3D}}$, the Thouless time should scale as the Heisenberg time, i.e., $t_{\mathrm{Th}} \propto L^3$. This is indeed the case, as the inset in the upper plot in Fig. 1 demonstrates. A further increase of the disorder strength leads to a slow increase of the Thouless time t_{Th} with eventual saturation to the Heisenberg time t_H.

In the deep delocalized phase where t_{Th} scales with L^2, the ratio t_{Th}/L^2 is nothing—up to a constant multiplicative factor—but the inverse of the diffusion coefficient $D(W)$, in accordance with the original definition of the Thouless time. The dependence of $D(W)$ with W is not known analytically, but it is known that it decreases quickly with W, vanishing at the critical point and scaling like $(W_c - W)^s$ below it, with the critical exponent $s \approx 1.574$. In any case, it is definitely not $e^{-W/\Omega}$ as in Eq. (1). It may be that, in a limited range of W values, $D(W)$ can be approximately fitted by an exponential decrease, but other forms could do the job as well.

The 5D case is essentially identical, except that the Thouless time scales like L^5 instead of L^3 at the critical point. The growth of the Hilbert space size as L^5 prevents reaching system sizes $L \geq 10$. Nevertheless, the obtained Thouless times t_{Th}, when rescaled by L^5 as suggested by the relation $t_{\mathrm{Th}} \sim t_H$ valid at the transition, lead to a clear crossing of the t_{Th}/L^5 curves at $W_C^{5\mathrm{D}}$. The W dependence of t_{Th}/L^2 in the deeply delocalized regime is again approximately reproduced by an exponential, although it certainly fails near the critical point.

Diffusion and subdiffusion in Anderson models.—To demonstrate that the obtained behaviors of the Thouless time t_{Th} are related to the time dynamics in Anderson systems, we consider the initial state $|\psi_0\rangle$ with a particle located at a given lattice site with periodic boundary conditions. The time evolved state $|\psi_0(t)\rangle = e^{-i\hat{H}t}|\psi_0\rangle$ is obtained employing the Chebyshev technique [80], which allows us to get results for system sizes up to $L = 240$ and $L = 30$ for the 3D and 5D cases, respectively. The mean square displacement

$$\langle r^2(t)\rangle = \langle\psi_0(t)|\sum_{i=1}^{D}(\hat{r}_i - \bar{r}_i)^2|\psi_0(t)\rangle, \qquad (4)$$

where r_i is the ith component of the position operator $\hat{\mathbf{r}}$ and $\bar{r}_i = \langle\psi_0(t)|\hat{r}_i|\psi_0(t)\rangle$, allows us to distinguish (considering first the $L \to \infty$ limit and then, looking at times $t \gg 1$) diffusive $\langle r^2(t)\rangle \propto Dt$, subdiffusive $\langle r^2(t)\rangle \propto t^\alpha$, and localized behaviors. The latter occurs when $\langle r^2(t)\rangle$ saturates after the initial expansion of the wave packet. The time dependence of the mean square displacement is reflected by the function $\alpha(t) \equiv d\log\langle r^2(t)\rangle/d\log t$. In the case of diffusion, $\alpha(t) = 1$. For subdiffusion, $0 < \alpha(t) = \alpha < 1$, and in the localized case $\alpha(t) \to 0$.

On the delocalized side of the transition in 3D and 5D models, for $W < W_C^{3\mathrm{D}}$ and $W < W_C^{5\mathrm{D}}$, respectively, we observe (Fig. 2) that $\alpha(t)$ initially increases over time, reaching larger maximal values for increasing system sizes. Assuming that this trend persists with increasing system size, taking the thermodynamic limit $L \to \infty$ we end up with a diffusive behavior of $\alpha(t) = 1$ for $t \gg 1$. The decrease of $\alpha(t)$ observed at the delocalized side of the transition for a given system size L occurs when the wave packet ceases to spread as its size approaches the system size. The situation is different at the transition, where, regardless of the system size, $\alpha(t)$ approaches a constant

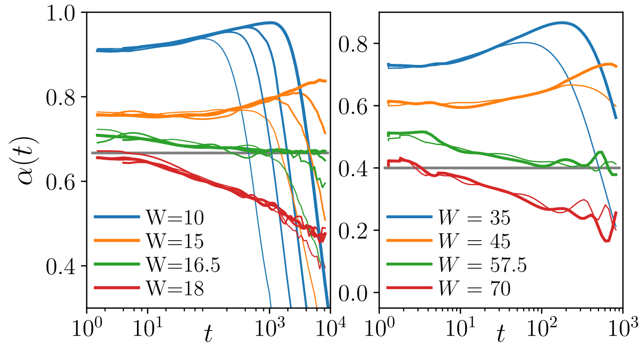

FIG. 2. Time dependent $\alpha(t)$ function for 3D (left) and 5D (right) Anderson models for various disorder strengths W. In the 3D case, the results for the system size $L = 80, 120, 160, 240$ are denoted by progressively thicker lines, whereas in the 5D case the thin (thick) lines correspond to $L = 20$ ($L = 30$).

value $\alpha_{3\mathrm{D}} = 2/3$ in the 3D case [68,81] or $\alpha_{5\mathrm{D}} = 2/5$ in the 5D case. Subsequently, $\alpha(t)$ decreases when the size of the wave packet approaches the system size L. This indicates that, in the thermodynamic limit $L \to \infty$, for $t \gg 1$, there is a subdiffusion of $\alpha(t) \to \alpha_{3\mathrm{D}}(\alpha_{5\mathrm{D}})$ at the transition in the 3D (5D) Anderson model. Finally, for $W > W_C^{3\mathrm{D}}$ ($W_C^{5\mathrm{D}}$), $\alpha(t)$ decreases with the time being nearly independent of the system size—a sign of localization.

The observed diffusion and subdiffusion for the 3D and 5D models agree with the results obtained for the Thouless time t_{Th}. In the diffusive system, $\langle r(t)^2\rangle \propto Dt$, which means that the time for reaching the boundary of the system is $t_{\mathrm{Th}}^B \propto L^2$. For subdiffusion, $\langle r(t)^2\rangle \propto t^\alpha$ implies that $t_{\mathrm{Th}}^B \propto L^{2/\alpha}$. Given the values for $\alpha_{3\mathrm{D}}$ and $\alpha_{5\mathrm{D}}$, we see that the obtained scalings of t_{Th}^B on the delocalized side of the transition and at the transition agree with the scalings $t_{\mathrm{Th}} \propto L^2$ and $t_{\mathrm{Th}} \propto L^3$ (or $t_{\mathrm{Th}} \propto L^5$ in the 5D case) obtained from the SFF.

The results shown in Fig. 2 highlight the importance of finite-size and finite-time effects. The limit $L \to \infty$ followed by $t \to \infty$ has to be carefully examined to reveal the trend toward diffusion or subdiffusion in the system. For instance, if the data for 3D model at $W = 15$ were available only up to time $t = 10^2$, one could incorrectly assume a subdiffusion with $\alpha \approx 0.75$. It seems plausible that the case of interacting systems is analogous, suggesting that the claims about subdiffusion on the ergodic side of an MBL transition [50,51,82,83] might be invalid in the asymptotic limit $L \to \infty$, $t \gg 1$ [84,85].

Thouless time in disordered many-body systems.—Consider 1D disordered spin-1/2 chains with Hamiltonian:

$$H = J_1\sum_{i=1}^{L}(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z) + \sum_{i=1}^{L} h_i S_i^z + J_2\sum_{i=1}^{L}(S_i^x S_{i+2}^x + S_i^y S_{i+2}^y + \Delta S_i^z S_{i+2}^z), \qquad (5)$$

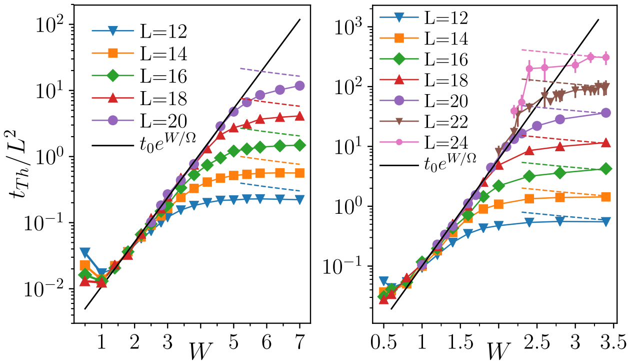

FIG. 3. Thouless time t_{Th} for $J_1 - J_2$ model (left) and XXZ model (right) extracted from the SFF. t_{Th} is divided by L^2 to emphasize the scaling with system size L. The dashed lines show the Heisenberg time t_H.

where $\vec{S}_i$ are spin-1/2 matrices, $J_1 = 1$ is the energy unit, periodic boundary conditions are assumed, and $h_i \in [-W, W]$ are independent, uniformly distributed random variables. Setting $J_2 = 0$ and $\Delta = 1$, we arrive at a disordered XXZ model, which has been widely studied in the MBL context [86–92]. In particular, an analysis of mean gap ratio $\bar{r}$ [93] predicts the critical value of disorder strength $W_C = 3.72(6)$ [94] for the transition to an MBL phase. Similar reasoning leads to $W_C \approx 9$ for the $J_1 - J_2$ model. For details of our calculations of Thouless times, see [79].

In the case of $J_1 - J_2$ model, Thouless times obtained for available system sizes seem to follow the scaling Eq. (1): for increasing system size L the point $\tilde{W}(L)$ where $t_{\mathrm{Th}}/L^2(W)$ deviates from the $e^{W/\Omega}$ behavior, shifts to larger disorder strength, as shown in the left panel of Fig. 3. One interpretation of this behavior along the lines of [49] is that one assumes that system size dependence of $\tilde{W}(L)$ continues indefinitely so that the scaling Eq. (1) holds in the thermodynamic limit. This would imply that there is no transition to an MBL phase. However, the scaling of the Thouless time for available system sizes in the 5D Anderson model, exhibited in the lower panel of Fig. 1, is very similar: the curves for the larger system sizes deviate from Eq. (1) at increasing disorder strength. As such a behavior occurs in the 5D Anderson model despite the localization transition taking place at W_C^{5D}, we may give a second possible interpretation of the result: the scaling Eq. (1) is not broken at available system sizes because of strong finite-size effects. While it is still possible to devise the location of the critical point W_C^{5D} provided one knows the correct value of the exponent α governing the subdiffusion at the Anderson transition, it is not clear how to rescale the Thouless times t_{Th} in the many-body case. Indeed, the transport properties on the delocalized side are not fully understood. For example, Ref. [95] suggests a subdiffusive behavior with exponent α vanishing close to the transition. Presumably, a sensible criterion for the transition in the many-body case would be $t_{\mathrm{Th}} \propto t_H \propto e^{cL}$. In any case, the main observation in [49] is that t_{Th}/L^2 is approximately equal to $e^{W/\Omega}$ in the deeply delocalized regime of the $J_1 - J_2$ model. This implies, in turn, that the diffusion coefficient $D(W)$ decreases as $e^{-W/\Omega}$, exactly like in the 3D and 5D Anderson models. Concluding that $D(W)$ never vanishes is a dangerous extrapolation, which leads to incorrect results for the Anderson models. The similarity of the scaling of the Thouless time for 5D Anderson and $J_1 - J_2$ models suggests that the conclusion of [49] about $D(W) \propto e^{-W/\Omega}$ in the $J_1 - J_2$ model for any disorder strength in the thermodynamic limit is misleading. Our results show that the apparent scaling Eq. (1) is probably a finite-size effect.

The finite-size effects in the $J_1 - J_2$ model are necessarily enhanced by the next-to-nearest neighbor coupling term. Thus, we may expect weaker finite-size effects for the XXZ model. The scaling of the Thouless time for this model is presented in the right panel of Fig. 3. The scaling follows Eq. (1) only for disorder strengths $W \in [1, 2]$. We observe two important differences with the results for the $J_1 - J_2$ model. First, at weak disorder W, the exponential dependence of the Thouless time t_{Th} on W is weaker than in the interval $W \in [1, 2]$. This is due to the proximity of the integrable point $W = 0$ [96,97] with Poisson level statistics and $t_{\mathrm{Th}} = t_H$. Second and more important, we see a breakdown of Eq. (1) for the XXZ model at $W \gtrsim 2$, where the data for $L = 22$ and $L = 24$ exceed the $t_0 e^{W/\Omega}$ line even though the Thouless time is still an order of magnitude smaller than the Heisenberg time t_H. This indicates that the exponential scaling with W is a numerical observation explicitly broken in the XXZ model and likely valid only in a limited range in other systems. The data for $L = 22$ and $L = 24$ are available only for $W \geq 2$ and $W > 2.2$ [79]. Nevertheless, the breakdown of the scaling Eq. (1) for $L = 22$, 24 at $W \approx 2.2$ is apparent, indicating that the L^2 scaling of Thouless time breaks down. This reflects the slow-down of transport as the MBL transition is approached.

Conclusions.—Our results show that the Thouless time, defined by the behavior of the SFF, reflects the transport properties in disordered noninteracting models, as we have shown in the examples of 3D and 5D Anderson models. In particular, the scaling of the Thouless time t_{Th} at the transition encodes the subdiffusive behavior of the mean square displacement $\langle r^2(t) \rangle \sim t^{\alpha}$ with the exponents $\alpha_{\mathrm{3D}} = 2/3$ and $\alpha_{\mathrm{5D}} = 2/5$, leading to scaling $t_{\mathrm{Th}} \sim L^{2/\alpha}$ with system size at the transition.

The scaling of the Thouless time for the $J_1 - J_2$ model seems to follow $t_{\mathrm{Th}} \sim t_0 L^2 e^{W/\Omega}$. However, the behavior of t_{Th} is directly analogous to the case of the 5D Anderson model. The latter undergoes a transition to a localized phase and the Thouless time does not exceed the $t_0 L^2 e^{W/\Omega}$ curve only because of strong finite-size effects at available system sizes. It is plausible that the situation is the same in the $J_1 - J_2$ model, raising doubts about the claims of [49].

Our results for XXZ model demonstrate that the L^2 scaling of the Thouless time t_{Th}, which is valid deep in the delocalized phase, is evidently broken at $W \approx 2.2$, signaling a transition to an MBL phase at a strong disorder strength.

Finally, let us mention alternative definitions of Thouless time [98–104]. A comparison of these different approaches is in progress. While finalizing this manuscript, we became aware of the related works [105,106].

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