Quantum State Engineering by Shortcuts to Adiabaticity in Interacting Spin-Boson Systems

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We present a fast and robust framework to prepare nonclassical states of a bosonic mode exploiting a coherent exchange of excitations with a two-level system ruled by a Jaynes-Cummings interaction mechanism. Our protocol, which is built on shortcuts to adiabaticity, allows for the generation of arbitrary Fock states of the bosonic mode, as well as coherent quantum superpositions of a Schrödinger cat-like form. In addition, we show how to obtain a class of photon-shifted states where the vacuum population is removed, a result akin to photon addition, but displaying more nonclassicality than standard photon-added states. Owing to the ubiquity of the spin-boson interaction that we consider, our proposal is amenable for implementations in state-of-the-art experiments.

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Introduction.—Quantum state engineering, i.e., the manipulation and control of a quantum system to attain a target state with high fidelity, lies at the core of quantum-based technologies [1]. In this realm, nonclassical states are of key significance to exploit quantum resources and find numerous applications in different areas, such as quantum information processing [2], sensing [3], and fundamental physics inquiries [4,5]. In particular, hybrid quantum systems are well suited to operate as fundamental building block for the engineering of nonclassical states and the implementation of the aforementioned tasks [6–15]. Such systems can be controlled and manipulated with a very high accuracy in distinct state-of-the-art experiments, such as setups based on trapped ions [16,17], ensembles of NV centers embedded in a single-crystal diamond nanobeam [18], and superconducting qubits [19]. Since the first generation of a nonclassical state, attained in a trapped-ion experiment [20], other realizations have been achieved in different experimental setups, e.g., [21]. However, the preparation of nonclassical states is challenging as they are prone to decoherence, and thus fragile against noise sources. Fast and robust protocols are therefore valuable for their successful preparation, such as the application of stimulated Raman adiabatic passages [22,23] or dynamical decoupling schemes [24–26]. Yet, dissipation and distinct noise sources can have a significant impact in their performance, typically requiring a trade-off with slow evolution times.

To circumvent these drawbacks, the current efforts are geared towards the design of protocols at the coherent level, dubbed as shortcuts to adiabaticity (STA), aiming at speeding up the quantum adiabatic process [27–29] (see Refs. [30,31] for fast quasiadiabatic dynamics protocols). Owing to short evolution times, these protocols are intrinsically resilient to decoherence effects. The counter-diabatic driving requires an additional term that suppresses nonadiabatic transitions between instantaneous eigenstates [32]. This active field of research is finding numerous applications in distinct areas, ranging from aspects of many-body physics [33,34] to the design of super-efficient quantum engines [35,36], allowing for the design of robust protocols [cf. Ref. [37] for a review].

In this Letter, we present a scheme that allows for a fast and robust preparation of nonclassical states built on STA and making use of the ubiquitous Jaynes-Cummings (JC) interaction between a two-level system and a single bosonic mode. To illustrate the performance and versatility of the reported protocol, we show how to generate Fock states, Schrödinger cat states and strongly nonclassical states akin to excitation-added states [38], with very high fidelity and in a short evolution time compared to their adiabatic preparation. Finally, we comment on the robustness and noise resilience of our protocol, and its experimental implementation, which is amenable in state-of-the-art quantum optics setups, such as cavity or circuit quantum electrodynamics, trapped ions, and optomechanics.

General framework.—At the heart of quantum optics, the JC model [39] describes the coupling between light and matter through a simple mechanism connecting a two-level system and a bosonic mode. The relevance of this model goes beyond the scope of pure light-matter interaction and correctly describes spin-phonon coupling, essential for example in trapped ions [16,17] and electromechanical setups [40]. The Hamiltonian of this model reads (we choose units such that $\hbar = 1$)

$$H_{\rm JC}(t) = \omega_q(t)\sigma_z/2 + \omega a^\dagger a + \lambda(t)(a\sigma^+ + a^\dagger\sigma^-), \qquad (1)$$

where $\omega_q(t)$ (ω) is the two-level (bosonic) frequency and $\lambda(t)$ is the interaction strength between such subsystems. The two-level system is characterized by the ladder operators $\sigma^+ = \sigma^{-\dagger} = |e\rangle\langle g|, \sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, with $|g\rangle$ and $|e\rangle$ as the fundamental and excited state of the two-level system, while the bosonic mode is described by the annihilation and creation operators a and $a^\dagger$ with $[a, a^\dagger] = 1$. Without loss of generality, we assume that the driving is performed on the frequency of the two-level system $\omega_q(t)$ and the coupling rate $\lambda(t)$, while the bosonic frequency remains constant. As the total number of excitations $N_e \equiv |e\rangle\langle e| + a^\dagger a$ is conserved, $H_{\rm JC}(t)$ can be diagonalized in the subspace spanned by $\{|e,n\rangle, |g,n+1\rangle\}$, where $|n\rangle$ ($n = 0, 1, \ldots$) is the n-excitation Fock state of the mode. We thus have $H_{\rm JC}(t) = -\omega_q(t)/2|g,0\rangle\langle g,0| + \bigoplus_n H_n(t)$ with the Landau-Zener-like terms $H_n(t) = (n+1/2)\omega\mathbb{I} + [\delta(t)/2]\bar{\sigma}_z + \lambda(t)\sqrt{n+1}\bar{\sigma}_x$ and the spin-like operators $\bar{\sigma}^- = |g,n+1\rangle\langle e,n|$, $\bar{\sigma}^+ = |e,n\rangle\langle g,n+1|$, $\bar{\sigma}_z = |e,n\rangle\langle e,n| - |g,n+1\rangle\langle g,n+1|$ [$\delta(t) = \omega_q(t) - \omega$ is the detuning from atomic resonance].

Shortcut to adiabaticity.—In general, driving under $H_{\rm JC}(t)$ leads to a nonadiabatic evolution. Adiabatic evolution is achieved when $\omega_q(t)$ and $\lambda(t)$ vary slowly, i.e., in a time much larger than the typical timescale of the system given by the inverse of the minimum energy gap of $H_{\rm JC}(t)$ [41]. This process can be sped up by introducing an additional term to the bare Hamiltonian, whose form is given by $H_{\rm CD}(t) = i\sum_{n,\sigma=\pm}[\partial_t\Phi_{n,\sigma}(t), \Phi_{n,\sigma}(t)]$ with $\Phi_n(t) = |n,\sigma(t)\rangle\langle n,\sigma(t)|$ and $|n,\sigma(t)\rangle$ denoting the dressed-atom eigenstates of the $H_{\rm JC}(t)$ [32,42]. The resulting counterdiabatic Hamiltonian reads $H_{\rm CD}(t) = i\theta(t)(a^\dagger\sigma^- - a\sigma^+)$ [43] with

$$\theta(t) = \frac{\delta(t)\dot{\lambda}(t) - \lambda(t)\dot{\omega}_q(t)}{\Omega_n^2(t) + \delta^2(t)}, \qquad (2)$$

where the parameter $\Omega_n(t) = 2\lambda(t)\sqrt{n+1}$ accounts for a time-varying Rabi frequency in the n subspace. This additional driving suppresses nonadiabatic excitations allowing for an arbitrarily fast adiabatic evolution. To ensure that the effective Hamiltonian $H_{\rm CD}^{\rm STA}(t) = H_{\rm JC}(t) + H_{\rm CD}(t)$ equals the original $H_{\rm JC}(t)$ at the start and end of the protocol, we impose the condition $\dot{\lambda}(0) = \dot{\lambda}(\tau) = 0$ as well as $\dot{\omega}_q(0) = \dot{\omega}_q(\tau) = 0$. These conditions ensure $H_{\rm CD}^{\rm STA}(t = 0, \tau) = H_{\rm JC}(t = 0, \tau)$, which can be recast in finding protocols such that $H_{\rm CD}(t = 0, \tau) = 0$.

We can however circumvent the difficulty in the implementation of an additional driving by performing a unitary transformation on $H_{\rm CD}^{\rm STA}(t)$ so as to obtain a local counterdiabatic Hamiltonian with the same form of the original $H_{\rm JC}(t)$ [43], namely, $H_{\rm LCD}(t) = \tilde{\omega}_q(t)\sigma_z/2 + \omega a^\dagger a + \tilde{\lambda}(t)(a\sigma^+ + a^\dagger\sigma^-)$, but with the new parameters

$$\tilde{\omega}_q(t) = \omega_q(t) - 2\sqrt{(n+1)}\frac{\lambda(t)\dot{\theta}(t) - \theta(t)\dot{\lambda}(t)}{\theta^2(t) + \Omega_n^2(t)}, \qquad (3)$$

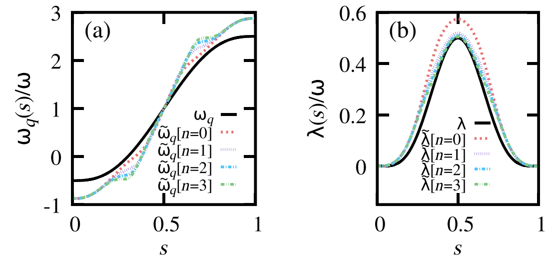

FIG. 1. Panel (a) [(b)], solid black line: profile of the time-dependent parameter $\omega_q(s)$ [$\lambda(s)$] plotted against $s = t/\tau$ for $\omega\tau = 10$, $\omega_q(0) = -\lambda_m = -\omega/2$ and $\omega_q(\tau) = 5\omega/2$ with $\lambda_0 = 0$. STA is attained using $\tilde{\omega}_q(s)$ and $\tilde{\lambda}(s)$, shown here for the first four n subspaces (dotted and dashed lines).

and $\tilde{\lambda}(t) = \sqrt{\lambda^2(t) + \theta^2(t)}$. Note that the driving must also fulfill $\ddot{\lambda}(0) = \ddot{\lambda}(\tau) = 0$ and $\ddot{\omega}_q(0) = \ddot{\omega}_q(\tau) = 0$ to ensure that $H_{\rm LCD}(t = 0, \tau) = H_{\rm JC}(t = 0, \tau)$. For that, we consider the protocols $\omega_q(t) = \omega_q(0) + 10\Delta\omega_q s^3 - 15\Delta\omega_q s^4 + 6\Delta\omega_q s^5$, and $\lambda(t) = (\lambda_m - \lambda_0)\cos^4[\pi(1 + 2s)/2] + \lambda_0$, with $s = t/\tau$, $\Delta\omega_q = \omega_q(\tau) - \omega_q(0)$, and where λ_0 is the initial coupling constant, while λ_m denotes its maximum value. As for a Landau-Zener problem, a population transfer between $|e,n\rangle$ and $|g,n+1\rangle$ requires that $\omega_q(t)$ changes its sign during the evolution while $\lambda(t) \neq 0$ for some t with $\lambda(0) = \lambda(\tau_q) = 0$, which also applies to the modified frequencies. In Fig. 1, we illustrate the time-dependent behavior of the modified frequency and coupling parameters. It is worth stressing that having control on $\omega_q(t)$ and $\lambda(t)$ allows for a perfect state transfer in the JC ladder for an arbitrary time τ, while it is hindered when either $\dot{\omega}_q(t) = 0$ or $\dot{\lambda}(t) = 0\,\forall\, t$. We remark that while $H_{\rm CD}^{\rm STA}(t)$ and $H_{\rm LCD}(t)$ perform in a similar manner, their associated energetic cost may differ [44].

Fock state generation.—As briefly mentioned, the implementation of the control $\tilde{\omega}_q(t)$ and $\tilde{\lambda}(t)$ allows for a perfect state transfer between $|e,n\rangle$ and $|g,n+1\rangle$, which can be used to generate an arbitrary Fock state $|N\rangle$ of the bosonic mode. Needless to say, for this specific case a time-independent evolution under $H_{\rm JC}$ may perform in a similar manner as our superadiabatic protocol [43], and the following example is given on a mere illustrative ground. We assume the initial state $|e,0\rangle$, then driven to $|g,1\rangle$ using a STA protocol. Upon a π pulse on the spin, a STA is performed such that $|g,2\rangle$ is obtained. Concatenating this N times, state $|e,N\rangle$ is achieved [cf. Fig. 2(a)]. In order to illustrate the performance of this protocol, we show the evolution of the Mandel parameter $Q(t) = (\langle n^2(t)\rangle - \langle n(t)\rangle^2)/\langle n(t)\rangle - 1$ with $\langle n(t)\rangle = \langle\psi(t)|a^\dagger a|\psi(t)\rangle$, which accounts for the nonclassicality of the resulting state. We also compute the purity $p(t) = {\rm Tr}[\rho_s^2(t)]$ of the reduced two-level state $\rho_s(t) = {\rm Tr}_b[\rho(t)]$, with $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$ and ${\rm Tr}_b[\bullet]$ denoting the trace over the bosonic mode. Both $Q(t)$ and $p(t)$ showcase a perfect population transfer

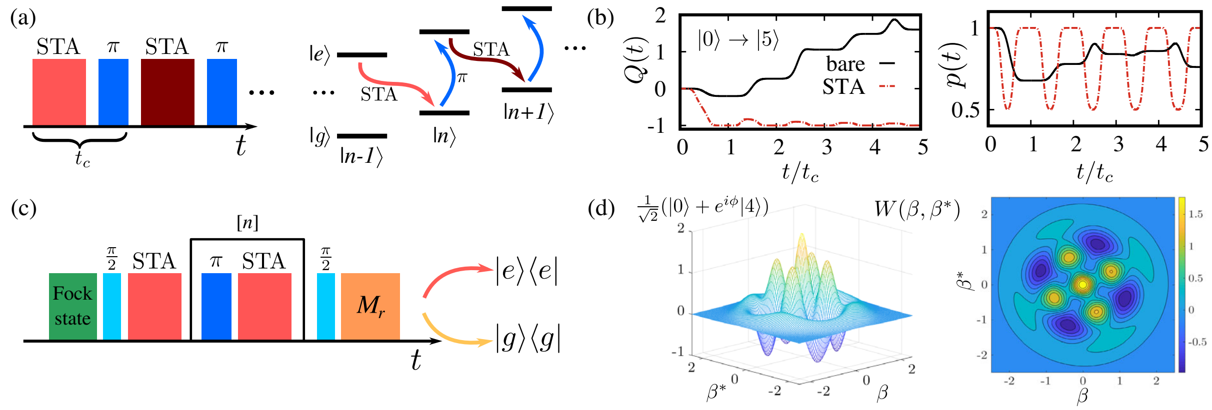

FIG. 2. (a) Scheme for the generation of a Fock state $|N\rangle$ using STA and a π pulse, and the transitions in the JC ladder. (b) Mandel parameter $Q(t)$ and purity $p(t)$ for the preparation of a $|N=5\rangle$ Fock state. We have used the STA drives $\tilde{\omega}_q(t)$ and $\tilde{\lambda}(t)$ with $\omega\tau = \omega t_\pi = 5$ and $\omega\sigma_\pi = 1$, with $\lambda_0 = 0$, $\lambda_m = \omega/4$ and $\omega_q(0) = 3\omega_q(\tau) = 3\omega/2$. (c) Scheme for the preparation of a cat state based on Fock state preparation, $\pi/2$ pulses, STA and a projective measurement onto the spin. The pulses within the box can be performed n times to generate superpositions comprising Fock states separated by $2(n+1)$. (d) Wigner function of the state ρ_f resulting from the application of the previous scheme to achieve $(|0\rangle + e^{i\phi}|4\rangle)/\sqrt{2}$ with same parameters as above but $\omega\tau = 30$.

resulting from each STA $+\,\pi$ cycle of duration t_c as we have $Q(t_c) = -1$ and $p(t_c) = 1$ where $t_c = \tau + 2t_\pi$ with τ and $2t_\pi$ being the time spent in the STA evolution and the π pulse, respectively. The latter is modeled as a Gaussian function with standard deviation σ_π (cf. [43] for further details). In Fig. 2(b) we show the evolution of $Q(t)$ and $p(t)$ under STA for the target state $|N=5\rangle$ and compare them to the results obtained using the bare $H_{\rm JC}(t)$. The Mandel parameter $Q(t)$ clearly unveils the sub-Poissonian behavior of the boson statistics (i.e., $Q(t) < 0$) for STA. Indeed, the STA protocol results in $Q(nt_c) = -p(nt_c) = -1$ and $p((n-1)t_c + \tau/2) = 1/2$ for $n = 1, 2, \ldots, 5$, since $|\psi(nt_c)\rangle = |e, n\rangle$ by construction [43]. Under $H_{\rm JC}(t)$, the statistics becomes super-Poissonian, unless the protocol is performed sufficiently slow [43].

Cat-state preparation.—The so-called Schrödinger cat state, one of the paradigmatic examples of nonclassical states, not only pose interest in fundamental quantum physics, but are highly valuable for quantum information processing applications. These states have been observed in numerous physical systems, including electronic [45], photonic [46–48], and atomic degrees of freedom [49,50]. A scheme for the deterministic creation of Schrödinger's cat states has been recently demonstrated using a single three-level system trapped in an optical cavity [51]. Nevertheless, it remains a big challenge to create superpositions of macroscopically distinct coherent states in nanomechanical systems [52,53]. In order to realize a cat state we start from a particular Fock state (see discussion above) and first apply a $\pi/2$ pulse to split the quantum state in two different n subspaces, upon which a fast state transfer (STA) is performed. In particular, for an initial state $|e, N\rangle$, the previous steps lead approximately to $|e, N-1\rangle + e^{i\phi}|g, N+1\rangle$ where ϕ denotes a relative phase acquired during the STA. Upon application of another $\pi/2$ pulse, followed by a projective measurement $M_r = |r\rangle\langle r| \otimes \mathbb{I}_b$ onto the spin state ($r \in \{e, g\}$), the resulting bosonic state becomes $|\psi_{N-1,N+1}\rangle \sim (|N-1\rangle + e^{i\phi}|N+1\rangle)/\sqrt{2}$. One can easily extend the previous sequence to generate cat states of a larger size by simply introducing π pulses and additional STA evolution [cf. Fig. 2(c)]. Note that, as the STA protocols depend on the addressed n subspace, any given choice of $\tilde{\omega}_q(t)$ and $\tilde{\lambda}(t)$ cannot achieve perfect population transfer in two or more distinct n subspaces simultaneously. However, this obstacle can be overcome by choosing parameters such that $\tilde{\omega}_q(t)$ and $\tilde{\lambda}(t)$ are similar in each of the required subspaces [43]. As an example, in Fig. 2(d) we show the Wigner function $W(\beta, \beta^*) = 2{\rm Tr}[\rho_f D(\beta) e^{i\pi a^\dagger a} D^\dagger(\beta)]$ [54], with $D(\beta) = e^{\beta a^\dagger - \beta^* a}$ the displacement operator, for an attained final state ρ_f involving $|0\rangle$ and $|4\rangle$, thus displaying the hallmarks of a cat state: distinguishable local-state components whose strong quantum interference results in negativity of $W(\beta, \beta^*)$. We benchmark the quality of our state-engineering protocol using the fidelity $F = \langle\psi_{0,4}|\rho_f|\psi_{0,4}\rangle \gtrsim 0.999$ with $|\psi_{0,4}\rangle = (1/\sqrt{2})(|0\rangle + e^{i\phi}|4\rangle)$ and $\phi \approx \sqrt{2}\pi$. It is worth stressing that higher fidelities can be achieved depending on the choice of the parameters, while a time-independent evolution leads to $F \approx 0.7$ [43].

Photon-shifted states.—An interesting class of nonclassical states is generated by the combination of addition and subtraction of bosonic excitations [38]. Their most basic embodiments consists of the addition or subtraction of a single quantum, which results in $|\psi_{\text{ph-add}}\rangle \propto a^\dagger|\psi\rangle$ and $|\psi_{\text{ph-sub}}\rangle \propto a|\psi\rangle$, respectively. These arithmetic operations are important in quantum-based technologies [55–61]. Building on our scheme, we now show how to produce nonclassical states—which we term photon-shifted states—achieved by transferring the population of the field vacuum to excitation-bearing Fock states. Such state manipulation has recently been demonstrated in a trapped ion system via an anti-JC interaction [62]. The step forward embodied by our proposal is that photon-shifted states can be generated in a fast and controllable manner using a single STA driving as follows. Let us consider an initial coherent state $|e,\alpha\rangle = D(\alpha)|e,0\rangle$. By applying a STA driving for the $n = 0$ subspace, one removes exactly the population of the vacuum state and shifts it to $|1\rangle$. The scheme is repeated, after a π pulse, to progressively transfer population to higher-excitation Fock states. Remarkably, provided $|\alpha| \lesssim 1$, the previous protocol approximately corresponds to a photon addition. However, our scheme yields in general states that are more nonclassical than $|\psi_{\text{ph-add}}\rangle$. To prove such claim, we use the negativity of the Wigner function $\mathcal{N} = (1/2\pi)\int d^2\beta[|W(\beta,\beta^*)| - W(\beta,\beta^*)]$ [63], which is shown in Fig. 3(a) against the value of α of the initial state. Clearly, a photon-shifted state achieves a larger value of $\mathcal{N}$—and thus more nonclassicality—than a photon-added state. The removal of the vacuum and population-shift has a profound impact on the Wigner function of the mode and on the corresponding state ρ_f [cf. Figs. 3(b)–3(c)]. Although not explicitly shown, similar results are obtained for other initial field states, such as thermal states [43].

Robustness.—As our scheme is built on STA protocols allowing for short evolution times, the method is naturally robust against decoherence effects. In particular, we can achieve a desired nonclassical target state under a broad range of noise rates of typical decoherence processes, such as spin dephasing, spontaneous emission, mode heating, and damping (see [43] for more details and numerical

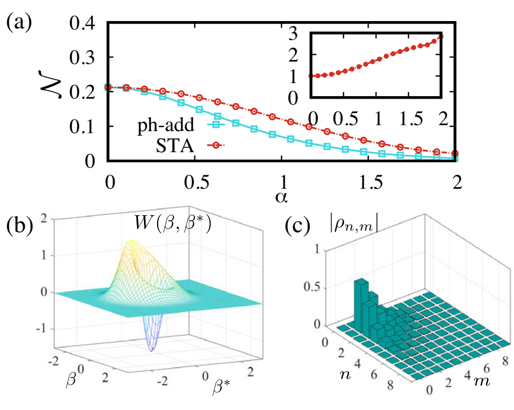

FIG. 3. (a) Nonclassicality $\mathcal{N}$ after a STA evolution removing the vacuum of $|e,\alpha\rangle$ and the value associated to photon addition $|\psi_{\text{ph-add}}\rangle$. The inset shows the ratio $\mathcal{N}_{\text{STA}}/\mathcal{N}_{\text{ph-add}}$. (b) Wigner function of the mode state after the protocol to remove the vacuum for $\alpha = 3/4$, and (c) its associated state for the first 10 Fock states, $|\rho_{n,m}| = |\langle n|\rho_{\text{f}}|m\rangle|$ [same parameters as in Fig. 2(b) with $\omega\tau = 8$].

results). Moreover, we have checked the robustness of the method to pulse-shape variations, which is a relevant step towards the actual implementation of the STA protocols. In order to evaluate the effect of such imperfections, we considered the preparation of $|\psi_{0,4}\rangle$ for $\tilde{\omega}_q(t)$ and $\tilde{\lambda}_q(t)$ approximated as $\tilde{x}_{\mathsf{F}}(t) = \sum_{k=0}^{N_{\mathsf{F}}} c_k\cos(k\omega_{\mathsf{F}}t) + s_k\sin(k\omega_{\mathsf{F}}t)$ with $\tilde{x} \in \{\tilde{\omega}_q, \tilde{\lambda}\}$. The cat state shown in Fig. 2(d) is achieved with $F \gtrsim 0.99$ already for $N_{\mathsf{F}} = 2$ (see [43] for further examples and details). This demonstrates the robustness of the proposed protocols.

Experimental feasibility.—Our scheme can be realized in a variety of physical systems where a JC interaction between a two-level system and a bosonic field can be controlled, such as in superconducting qubits [64,65], trapped ions [16,17,66], or spin-mechanical systems [40,67,68], among others. Here we focus on an ion-trap implementation [62,69,70]. In this setup, a well-controllable qubit can be encoded on the two magnetically insensitive hyperfine states of the $S_{1/2}$ manifold of a $^{171}\text{Yb}^+$ ion [71], whose frequency is $\omega_{\text{hf}}/2\pi = 12.6428$ GHz. The trapped-ion is confined in a harmonic potential with frequency $\omega_X/2\pi \approx 2$ MHz [62,69,70], such that the free Hamiltonian reads $H_0 = \omega_{\text{hf}}\sigma_z/2 + \omega_X a^\dagger a$. Applying two counterpropagating Raman laser beams, the internal levels of the ion can be coupled with the vibrational mode as $H_{\text{int}} = \Omega\cos(\Delta kx - \omega_l t - \phi)\sigma_x$, where Ω, Δk, ω_l, and ϕ are the Rabi frequency, net wave vector on the x axis, frequency and phase of the laser fields, respectively, while $x = (2m\omega_X)^{-1/2}(a + a^\dagger)$ is the position operator of the ion with mass m. In an interaction picture with respect to H_0, upon the optical and vibrational rotating wave approximations, and within the Lamb-Dicke regime, one obtains $H_{\text{int}}^I = U_0^\dagger(t)H_{\text{int}}U_0(t) \approx \lambda(a\sigma^+ e^{i\delta t} + \text{H.c.})$ with $U_0(t) = e^{-itH_0}$, $\lambda = \Omega\Delta k(2m\omega_X)^{-1/2}/2$, and $\phi = \pi/2$, and where we have selected $\omega_l = \omega_{\text{hf}} - \omega_X - \delta$ with $\delta \ll \omega_X$ [62,69,70]. Note that H_{int}^I already corresponds to $H_{\text{JC}}(t)$ [cf. Eq. (1)] in the interaction picture of $\omega_q(t)\sigma_z/2 + \omega a^\dagger a$, requiring a detuning $\delta(t) = \omega_q(t) - \omega$ and a modulated laser intensity $\Omega(t)$, such that the proposed protocols $\lambda(t)$ and $\omega_q(t)$ can be realized. Indeed, $\lambda/2\pi \approx 12.5$ kHz and $\delta(t)$ can be varied within $|\delta(t)|/(2\pi) \sim 0$–$100$ kHz while ensuring the correct functioning of the required approximations [62,66,69,70]. A possible set of realistic parameters to implement the scheme is given by $\omega/2\pi \approx 50$ kHz, such that $\lambda_m \approx \omega/4$, which leads to $\tau \approx 0.2$ ms for $\omega\tau = 10$. For such short τ, decoherence effects are not expected to play a relevant role [62,70], and one may still rely on suitable dynamical decoupling schemes to further protect the system against decoherence processes [25,26,72–75].

Conclusions.—We have developed a general framework for a fast, robust, and accurate preparation of nonclassical states in spin-boson systems that are highly desirable in, for example, quantum information processing tasks [38] and fundamental physics inquiries [4,5]. In particular, the proposed pulses allow for a perfect state transfer in a JC model, which are built relying on STA. As an illustration of the potential and versatility of the method, we show how to generate arbitrary Fock states and cat states. In addition, we show how to obtain a class of photon-shifted states where the vacuum population can be removed, thus similar to photon addition but featuring more nonclassicality. These protocols, intrinsically robust against decoherence thanks to their arbitrarily short evolution time, are also resilient to imperfect implementation or modifications to their actual shape profiles. Our results may open new routes and possibilities for an efficient preparation of non-classical states in a variety of settings, amenable for their experimental realization in state-of-the-art setups.

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