Bootstrapping the Simplest Correlator in Planar $\mathcal{N}=4$ Supersymmetric Yang-Mills Theory to All Loops

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(Received 27 July 2019; revised manuscript received 16 December 2019; accepted 25 February 2020; published 29 April 2020)

We present the full form of a four-point correlation function of large BPS operators in planar $\mathcal{N}=4$ supersymmetric Yang-Mills theory to any loop order. We do this by following a bootstrap philosophy based on three simple axioms pertaining to (i) the space of functions arising at each loop order, (ii) the behavior in the operator product expansion (OPE) in a double-trace dominated channel and (iii) the behavior under a double null limit. We discuss how these bootstrap axioms are in turn strongly motivated by empirical observations up to nine loops unveiled through integrability methods in our previous work [F. Coronado, J. High Energy Phys. 01 (2019) 056.] on this simplest correlation function.

DOI: 10.1103/PhysRevLett.124.171601

Introduction.—Integrability methods have shaped a new path for the explicit evaluation of correlators of local operators in planar $\mathcal{N}=4$ supersymmetric Yang-Mills theory [1–5] and also nonplanar [6–8], especially for four-point functions of large protected single-trace operators. In [9] we used integrability-based methods to find the loop corrections to the polarized four-point function we named as the simplest. This correlator consists of four external protected operators with R-charge polarizations chosen as shown in Fig. 1. In the limit of long operators ($K \gg 1$) [10], we argued that this four-point function admits a factorization into the tree level part which carries all the dependence on the external scaling dimension K and the loop corrections which are given by the squared of the function $\mathbb{O}$ (the octagon)

$$\langle O_1 O_2 O_3 O_4 \rangle = \left[\frac{1}{x_{12}^2 x_{13}^2 x_{24}^2 x_{34}^2}\right]^{\frac{K}{2}} \times \mathbb{O}^2(z, \bar{z}), \qquad (1)$$

where the cross ratios are defined in terms of the spacetime positions as:

$$z\bar{z} = u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \quad \text{and} \quad (1-z)(1-\bar{z}) = v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

In this Letter we present some of the analytic properties of the octagon $\mathbb{O}$ which follow from the explicit nine-loop results in [9]. These properties include a restriction on the space of functions that appear at any loop order and the remarkable simplicity of the octagon in two different kinematical limits: the OPE limit $(z \to 1, \bar{z} \to 1)$ and the double light-cone limit $(z \to 0, \bar{z} \to \infty)$.

We also state that these three analytic properties can be used to uniquely define the octagon and with that also the

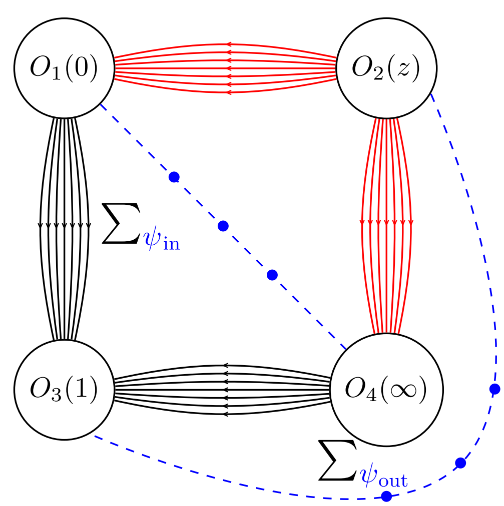

FIG. 1. The simplest four-point function with external operators $O_1(0,0) = \mathrm{Tr}(Z^{(K/2)}\bar{X}^{(K/2)}) +$ cyclic permutations, $O_2(z,\bar{z}) = \mathrm{Tr}(X^K)$, $O_3(1,1) = \mathrm{Tr}(\bar{Z}^K)$, and $O_4(\infty,\infty) = \mathrm{Tr}(Z^{(K/2)}\bar{X}^{(K/2)}) +$ cyclic permutations. The Wick contractions form a perimeter with four bridges of width $K/2$. According to hexagonalization [3] in the limit $K \gg 1$ the loop corrections are obtained by summing over $2D$ intermediate multiparticle states ψ_{in} and ψ_{out} on mirror cuts 1-4 and 2-3 respectively, with both sums evaluating to $\mathbb{O}$. Alternatively the octagon $\mathbb{O}$ represents the resummation of planar Feynman diagrams drawn inside (outside) the perimeter.

simplest correlator (1). We show how to solve this bootstrap problem by first introducing a Steinmann basis of ladders which resolve two of the aforementioned analytic properties. Then using the third property to completely fix the coefficients in an ansatz constructed with the Steinmann basis.

Our approach is reminiscent of the bootstrap program to constraint perturbative scattering amplitudes [11,12] and Feynman integrals [13] using Steinmann relations. While for off-shell correlators this relation is not a natural assumption, our data indicates that the octagon function $\mathbb{O}$ has a vanishing double discontinuity. In this Letter we call this property a Steinmann relation, as in the S-matrix context, although we associate its origin to the existence of a OPE channel dominated by double-trace operators in our simplest four-point function, see discussion below in a later section.

Our bootstrap approach reproduces the explicit results obtained from perturbation theory and integrability and allows us to easily extend them to arbitrary loop order. We accompany this Letter with an ancillary file [14] with our explicit results up to 24 loops.

Analytic properties of octagon.—The following analytic properties were observed up to nine loops from the explicit results in [9]. These empirically found properties will later be converted into bootstrap axioms and used to fully determine our correlator. Some of these empirical observations can be *a posteriori* derived and better understood as discussed in more detail in [15].

Single-valuedness and ladders: Our explicit results in [9] provided the octagon [16] a multilinear combination of ladder integrals:

$$\mathbb{O} = 1 + \sum_{n=1}^{\infty} \sum_{J=n^2}^{\infty} \sum_{\vec{j} \in Z_n^+(J)} g^{2J} \times d_{\vec{j}} \times f_{j_1} \cdots f_{j_n}, \qquad (2)$$

where $Z_n^+(J)$ represents the group of sets of positive integers $\vec{j} \equiv \{j_1 \cdots j_n\}$ which add up to $j_1 + \cdots + j_n = J$. The rational coefficients $d_{\vec{j}}$ are not known in closed form and could be zero for some integer partitions. The basis of conformal ladder integrals is given by [17]

$$f_p = -v \sum_{j=p}^{2p} \frac{j!(p-1)![-\log(z\bar{z})]^{2p-j}}{(j-p)!(2p-j)!} \left[\frac{\mathrm{Li}_j(z) - \mathrm{Li}_j(\bar{z})}{z - \bar{z}}\right]$$

where $v = (1-z)(1-\bar{z})$.

This expansion of $\mathbb{O}$ makes manifest its single-valuedness and its uniform maximal transcendentality at each loop order.

Double-trace OPE channel: Here we consider the OPE expansion in channel 2-3, see Fig. 1. Unlike the other two-channels (1-2 and 2-4) [18], this one receives double-trace contributions already at leading twist $2K$.

This OPE limit corresponds to $z \to 1, \bar{z} \to 1$ (or $v \to 0, u \to 1$). At weak coupling we find the behavior of the octagon in this kinematics to be given by the following (similar truncations have been observed in the study of extremal three-point functions in [19]):

$$\lim_{z,\bar{z}\to 1} \mathbb{O}(z,\bar{z}) = \mathsf{a}(z,\bar{z},g^2) + \mathsf{b}(z,\bar{z},g^2)\log v, \qquad (3)$$

where both functions a and b have a series expansion in the coupling g^2 and the cross ratios $(1-z)$ and $(1-\bar{z})$.

In the limit of large operators where (1) holds up to arbitrary loop order this octagon limit (3) implies that the simplest four-point function has at most a $\log^2 v$ singularity. This type of truncations is expected in the planar limit for OPE channels dominated by double-trace operators hence we dub this channel as the double trace channel [15].

Null-square limit: This limit corresponds to the kinematics where the external operators become lightlike separated: $x_{12}^2, x_{24}^2, x_{34}^2, x_{13}^2 \to 0$ forming a null square. This limit of the four-point function was considered in [20] for smaller operators where a relationship between null correlators and null polygonal Wilson loops was established.

For our simplest four-point function, see (1), the nontrivial part of this null limit is given by the limit of the octagon [21]

$$\lim_{z\to 0, \bar{z}\to\infty} \log \mathbb{O}(z,\bar{z}) = -\tilde{\Gamma}(g)\log^2(z/\bar{z}) + \frac{1}{2}g^2(\log^2(-z) + \log^2(-1/\bar{z})), \qquad (4)$$

where the coefficient $\tilde{\Gamma}$ admits an expansion in the coupling

$$\tilde{\Gamma}(g) = \frac{1}{2}g^2 - \frac{1}{6}\pi^2 g^4 + \frac{8}{45}\pi^4 g^6 - \frac{68}{315}\pi^6 g^8 + \mathcal{O}(g)^{10}.$$

To appreciate better the simplicity of (4) we contrast it against the result for short operators, $K = 2$. For the case $K = 2$ the coefficient $\tilde{\Gamma}$ is replaced by the cusp anomalous dimension Γ_{cusp} which is associated to the energy density of the flux tube between the Wilson lines. It also appears in the anomalous dimension of the large spin leading twist-operator $\mathrm{Tr}(ZD^S Z)$ dominating the light-cone OPE

$$\Delta = S + 2 + \Gamma_{\mathrm{cusp}}(g)\log S + O(1/S).$$

For our simplest correlator the operator(s) dominating the light-cone OPE is of the form $\mathrm{Tr}(Z^{(K/2)} D^S X^{(K/2)})$. Furthermore the limit $K \gg 1$ implies a huge number of nearly degenerate operators at leading twist K. It would be interesting to analyze how these two latter considerations account for the difference between $\tilde{\Gamma}$ and Γ_{cusp}. In particular the latter contains odd zeta numbers while the former only even zeta numbers.

In (4) the exponents of $\log(-z)$ and $\log(-1/\bar{z})$ truncate at degree two while for the case $K=2$ there is an extra complicated function of the cross ratios determined in [22] which accounts for the backreaction of the flux-tube on the heavy particle that propagates along the null square, see [20].

We expect these differences can be explained following an analysis similar to [22,23] including the nontrivial R charge and large $K \gg 1$ limit of our simplest correlator [15]. It would also be interesting to see if $\tilde{\Gamma}$ satisfies a linear integral equation as is the case for $\Gamma_{\rm cusp}$ [24,25].

Bootstrapping the octagon.—We now postulate that the analytic properties described in the previous section are valid at all loops and can be used to define a bootstrap problem by means of three axioms. (i) Ladder integrals: these span the family of functions that appear in the loop corrections of the correlator. They appear in multilinear combinations with uniform maximal transcendentality at any loop order. (ii) Steinmann relations: the octagon satisfy these relations which establish the vanishing of its double discontinuity

$$\mathrm{Disc}_1\mathrm{Disc}_1\mathbb{O}(z,\bar{z})=0, \tag{5}$$

where Disc_1 denotes the discontinuity after performing the analytic continuation $(1-z)\to(1-z)e^{i\pi}$ and $(1-\bar{z})\to(1-\bar{z})e^{i\pi}$. This condition guarantees the truncation to $\log v$ in the OPE expansion $z\to 1, \bar{z}\to 1$ at weak coupling. (iii) Light-cone asymptotics: in the null-square limit $z\to 0$ and $z\to\infty$ we demand a simple asymptotics of the logarithm of the octagon:

$$\lim_{z\to 0,\bar{z}\to\infty}\log\mathbb{O}(z,\bar{z}) = a_{0,0}+a_{1,0}\log(-z)+a_{0,1}\log(-1/\bar{z}) + a_{1,1}\log(-z)\log(-1/\bar{z}) + a_{2,0}\log^2(-z)+a_{0,2}\log^2(-1/\bar{z}), \tag{6}$$

where the relevant condition is the absence of higher logs and we do not impose any conditions on the coefficients $a_{i,j}$.

In the following sections we show how to resolve these three conditions to determine the octagon and the simplest four-point function at any loop order.

A Steinmann basis of ladder integrals: The vanishing of the double discontinuity (ii) motivates the search for a basis of functions that satisfy this property. Here we combine (i) and (ii) to look for this basis of functions in the space of ladder integrals. We start with an ansatz of the form

$$\mathcal{S}_i^{(m,n)} = \sum_{k_1+\cdots+k_n=m} d^{(i)}_{k_1,\ldots,k_n} f_{k_1}\cdots f_{k_n} \tag{7}$$

With this ansatz we are assuming an organization of our Steinmann basis into families $\mathcal{S}^{(m,n)}$ whose elements have uniform transcendentality of order m and are constructed with n ladders. We are provisionally using the subindex i to label the different elements $\mathcal{S}_i^{(m,n)}$ on each family.

In order to find our basis we simply need to take into account the discontinuities of the ladders:

$$\mathrm{Disc}_1 f^{(n)}(z,\bar{z}) \sim 2\pi i[\log(z\bar{z})]^{n-1}\log\left(\frac{z}{\bar{z}}\right)$$

$$\mathrm{Disc}_1\mathrm{Disc}_1 f^{(n)}(z,\bar{z})=0,$$

then imposing the Steinmann relations

$$\mathrm{Disc}_1\mathrm{Disc}_1\mathcal{S}_i^{(m,n)}=0, \tag{8}$$

we solve for the coefficients d in the ansatz (7).

This exercise was performed in [13] where some solutions to (8) were presented and identified with fishnet Feynman integrals. Here we will provide all solutions but without a Feynman integral interpretation.

We solved equation (8) finding the coefficients $d^{(i)}_{k_1\cdots k_n}$ for all $m\le 26$, $n\le 5$ and observe the following properties of the solutions: for $m<n^2$ there are no solutions, for $m=n^2$ and $m=n^2+1$ there is only one solution, and all solutions we found admit determinant representations.

This experience allows us to propose a Steinmann basis of ladders in the form of determinants. In short, the elements of our Steinmann basis can be identified with the minors of the infinite dimensional matrix

$$\begin{pmatrix} f_1 & f_2 & f_3 & \cdots \\ f_2 & f_3 & \cdots & \cdots \\ f_3 & \cdots & \cdots & \cdots \\ \vdots & \cdots & \cdots & \cdots \end{pmatrix},$$

more specifically we label these minors as

$$M_{i_1,i_2,\ldots,i_n} = \begin{vmatrix} f_{i_1} & f_{i_2-1} & \cdots & f_{i_n-n+1} \\ f_{i_1+1} & f_{i_2} & \cdots & f_{i_n-n+2} \\ \vdots & \vdots & \ddots & \vdots \\ f_{i_1+n-1} & f_{i_2+n-2} & \cdots & f_{i_n} \end{vmatrix}, \tag{9}$$

where the subindexes on $M_{i_1,i_2,\ldots,i_n}$ correspond to the elements on the diagonal and the subindexes on the first row of the matrix must satisfy

$$0<i_1<i_2-1<\cdots<i_n-n+1.$$

Using these minors we define our Steinmann basis of ladders as:

$$S_{k_1\cdots k_n} = \left[\prod_{o=1}^{n} p_{k_o}\right] M_{k_1\cdots k_n}, \tag{10}$$

where the rescaling $p_k = \{1/[k!(k-1)!]\}$ is just performed for later convenience. The families $\mathcal{S}^{(m,n)}$ are spanned as follows

$$S_{k_1,\ldots,k_n} \in \mathcal{S}^{(m,n)} \quad \text{if } k_1 + \cdots + k_n = m.$$

Lastly, considering the property of maximal transcedentality and assuming our Steinmann basis $\mathcal{S}^{(m,n)}$ is complete (we have checked this basis is complete for all $m \leq 26$, $n \leq 5$ and assume it is also the case for arbitrary m, n), we build an ansatz for each loop order of a function $\mathbb{O}$ satisfying (i) and (ii)

$$\mathbb{O} = 1 + \sum_{n=1}^{\infty} \sum_{m=n^2}^{\infty} (g^2)^m \sum_{S \in \mathcal{S}^{(m,n)}} c_{k_1\cdots k_n} S_{k_1\cdots k_n}. \qquad (11)$$

Fixing all coefficients with light-cone asymptotics: In order to fix the coefficients $c_{k_1,\ldots,k_n}$ in the ansatz we impose the third analytic property (iii). This condition of exponentiation in the null-square limit allows us to relate coefficients of high loop orders to the ones at lower loops. To take this limit in our ansatz we simply need to consider the light-cone limit of the ladders

$$\lim_{z\to 0,\bar{z}\to\infty} f_j(z,\bar{z}) = \sum_{m=0}^{j} \sum_{n=0}^{j} b_{m,n}^{(j)} \log^m(-z) \log^n(-1/\bar{z}), \qquad (12)$$

where $b_{m,n} = 0$ if $m+n$ is odd or otherwise:

$$b_{m,n}^{(j)} = \frac{j!(j-1)!(2-2^{m+n-2j+2})(2j-m-n)!}{(-1)m!n!(j-m)!(j-n)!} \zeta_{2j-m-n}.$$

Notice that the light-cone ladder in (12) is manifestly symmetric under the exchange of cross ratios $z \leftrightarrow -1/\bar{z}$ and our ansatz of ladders directly inherits this feature.

We then enforce the condition of truncation of the exponents of $\log(z)$ and $\log(-1/\bar{z})$ up to degree two. This provides a set of equations which can be easily solved at each loop order. Up to four loops the solution is:

$$c_2 = -2c_1^2, \qquad c_3 = 6c_1^3, \qquad c_4 = -20c_1^4, \qquad c_{1,3} = c_1^4.$$

In principle, this set of equations could have left some coefficients undetermined or it could have been an overdetermined system with no solution. Remarkably, going to higher loop orders we find that by imposing (iii) all coefficients c in the ansatz (11) are fixed in terms of the single one-loop coefficient c_1. This latter can be associated to the definition of the coupling g^2 and in order to match with the conventions in the literature we set it to $c_1 = 1$. We have performed this exercise up to 24 loops and consider it gives strong support of our conjecture that properties (i), (ii), and (iii) uniquely define the octagon $\mathbb{O}$ and with that our simplest correlator (1) at arbitrary loop order.

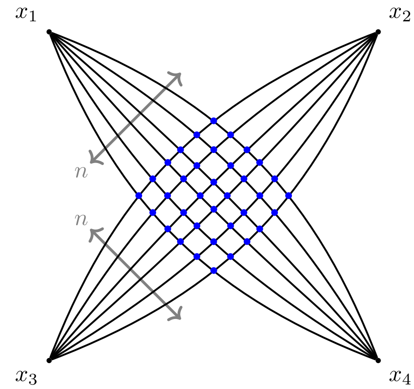

FIG. 2. Fishnet identified with $\mathcal{S}_{1,3\ldots2n-1}$

Furthermore, we have been able to identify the analytic form of an infinite family of coefficients:

$$c_{\underbrace{1,3\ldots2n-1}_{n},2n+1+m} = \binom{2m+4n}{m}, \quad \text{with} \quad m \geq 0.$$

In particular the coefficients $c_{1,3,5\ldots2n-1} = 1$ of the noteworthy elements of our basis $\mathcal{S}_{1,3,5\ldots2n-1}$ which have been identified in [13] as the fishnet Feynman integrals, see Fig. 2.

It is interesting to ask whether other elements of the Steinmann basis of ladders or perhaps linear combinations of them can be identified with other families of Feynman integrals. Finding such identification could be the guiding principle to find the closed form of all coefficients of our (possibly rotated) Steinmann basis. Then all would be set to attempt a resummation and get access to the finite or strong coupling limit. This is a question we hope to address in the future.

Conclusion.—In this short Letter, we have bootstrapped, for the first time in a unitary 4D planar gauge theory, a four-point correlator at all loops in the 't Hooft coupling. This is a correlator of four long protected operators and we call it the simplest due to the simplicity of the analytic properties that define it. These properties, see (i) and (ii), constrain the space of functions of the loop corrections to a reduced Steinmann basis of ladders with determinant representations. The coefficients on this basis are then fully determined by imposing a simple exponentiation in the light-cone limit, see (iii).

An interesting next step is to consider other kinematical limits of our results. We will be reporting our findings in [15], as well as a more thorough study of the analytic properties presented here and their physical implications.

It would also be interesting to find other higher-point correlation functions that satisfy a version of Steinmann relations. If they exist, finding a basis similar to (10) or the Steinmann functions that appear in the context of the S matrix [11,12], would be of relevance to find the loop corrections of these correlators. A natural candidate would be the six-point correlation function proposed in [9], see Fig. 17 therein.

We also consider important to understand which bootstrap conditions we should include to address the case of generic R-charge polarizations and ultimately operators with arbitrary or short scaling dimensions. At weak coupling there is a vast list of results, obtained using bootstrap ideas, for the integrands of these correlators [26–31]. It would be nice to be able to go from the integrand to explicit functions as the ones presented in this Letter.

Recently, bootstrap methods in Mellin space [32,33] and the analytic conformal bootstrap [34–37] have proved fruitful at strong coupling. It would be worthy exploring if these methods can be complemented with bootstrap ideas similar to the ones presented here to get more results starting in the regime of long operators.

Finally, it would be also interesting to see if the remarkable analytic properties of the simplest correlator also appear in observables of the nonunitary Fishnet theory [38] for which exact correlators have recently been computed [39].

I thank T. Bargheer, B. Basso, T. Fleury, V. Goncalves, S. Komatsu, and P. Vieira for enlightening discussions and comments on the Letter. Research at Perimeter Institute is supported in part by the Government of Canada through the Department of Innovation, Science and Economic Development Canada and by the Province of Ontario through the Ministry of Colleges and Universities.

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