Quantum Fluxes at the Inner Horizon of a Spherical Charged Black Hole

Noa Zilberman,* Adam Levi,† and Amos Ori‡
Department of Physics, Technion, Haifa 32000, Israel

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In an ongoing effort to explore quantum effects on the interior geometry of black holes, we explicitly compute the semiclassical flux components $\langle T_{uu}\rangle_{\mathrm{ren}}$ and $\langle T_{vv}\rangle_{\mathrm{ren}}$ (u and v being the standard Eddington coordinates) of the renormalized stress-energy tensor for a minimally coupled massless quantum scalar field, in the vicinity of the inner horizon (IH) of a Reissner-Nordström black hole. These two flux components seem to dominate the effect of backreaction in the IH vicinity, and furthermore, their regularization procedure reveals remarkable simplicity. We consider the Hartle-Hawking and Unruh quantum states, the latter corresponding to an evaporating black hole. In both quantum states, we compute $\langle T_{uu}\rangle_{\mathrm{ren}}$ and $\langle T_{vv}\rangle_{\mathrm{ren}}$ in the IH vicinity for a wide range of Q/M values. We find that both $\langle T_{uu}\rangle_{\mathrm{ren}}$ and $\langle T_{vv}\rangle_{\mathrm{ren}}$ attain finite asymptotic values at the IH. Depending on Q/M, these asymptotic values are found to be either positive or negative (or vanishing in between). Note that having a nonvanishing $\langle T_{vv}\rangle_{\mathrm{ren}}$ at the IH implies the formation of a curvature singularity on its ingoing section, the Cauchy horizon. Motivated by these findings, we also take initial steps in the exploration of the backreaction effect of these semiclassical fluxes on the near-IH geometry.

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Introduction.—The analytically extended Kerr and Reissner-Nordström (RN) metrics, describing, respectively, spinning and spherical charged isolated black holes (BHs), reveal a traversable passage through an inner horizon (IH) to another external universe [1,2].

Consider a traveler intending to access this other universe. To do so, she must pass through the BH interior, and in particular, through the IH. What will she encounter along her way? Is her mission doomed to fail? Does this external universe actually exist? Answering these questions requires understanding how quantum fields change the internal geometry of BHs. The most renowned phenomenon in which quantum effects profoundly transform the classical spacetime picture is the process of BH evaporation due to Hawking radiation [3,4]. In fact, already at the classical level, it was demonstrated that introducing matter (or perturbation) fields on BH backgrounds may affect their regularity. A notable example is the null weak [5] curvature singularity that forms along the Cauchy horizon (CH, the IH ingoing section) in both spinning [6–9] and spherical charged [10–16] BHs. The analogous effect of quantum perturbations is often expected to be significantly stronger [17–19], but this issue remains inconclusive, making it the main motivation for this work.

A theoretical framework that lends itself to this problem is the semiclassical formulation of general relativity, considering matter fields as quantum fields propagating in a classical curved spacetime, obeying the semiclassical Einstein field equation, given (in units $G = c = 1$) by

$$G_{\alpha\beta} = 8\pi\langle T_{\alpha\beta}\rangle_{\mathrm{ren}}. \tag{1}$$

Here, $G_{\alpha\beta}$ is the Einstein tensor, and the source term $\langle T_{\alpha\beta}\rangle_{\mathrm{ren}}$ is the renormalized expectation value of the stress-energy tensor (RSET) associated with the quantum field. Note the emergent requirement for self-consistency: spacetime curvature induces a nontrivial stress energy in the quantum fields which, in turn, deforms the spacetime metric—an effect known as backreaction. A possible way to handle this complexity is to break the problem into steps of increasing order in the mutual effect, initially computing $\langle T_{\alpha\beta}\rangle_{\mathrm{ren}}$ for a fixed, classical background metric. But already at this level, one faces a serious challenge: the computation of the RSET on curved backgrounds.

Recall that, already in flat spacetime, the stress-energy tensor of a quantum field formally diverges, but this is usually handled through the normal-ordering scheme, which is ill defined in curved spacetime. The intricate regularization procedure required in curved spacetime, along with its inevitable numerical implementation, has made this computation a decades-lasting hurdle in the study of semiclassical problems. However, the recently developed pragmatic mode-sum regularization (PMR) method [20–23], rooted in covariant point splitting [24,25], has made this task more accessible. (See, however, earlier works employing other methods, e.g., [26–39]).

The PMR method overcomes the main difficulty in the numerical implementation of point splitting by treating the coincidence limit analytically, through construction of "modewise" counter terms. It has been successfully used in recent years to compute both the vacuum expectation value $\langle\Phi^2\rangle_{\mathrm{ren}}$ and the RSET for a quantum scalar field Φ on various BH exteriors [20–23,40]. On BH interiors, however, only $\langle\Phi^2\rangle_{\mathrm{ren}}$ has been computed in that method so far (initially for Schwarzschild [41], reproducing previous results [32]). Although $\langle\Phi^2\rangle_{\mathrm{ren}}$ is not the quantity most relevant for backreaction, nevertheless, it provides valuable insights for the computation of the more divergent RSET. In particular, in a recent paper [42], $\langle\Phi^2\rangle_{\mathrm{ren}}$ was investigated both numerically and analytically inside RN, with extensive study of the IH vicinity. The RSET trace (for a minimally coupled scalar field) was consequently found to diverge at the IH. This Letter continues previous work, providing novel results for certain key components of the RSET inside a BH—which directly demonstrate the divergence of semiclassical energy-momentum fluxes at the CH. [43].

We hereby consider a spherically symmetric charged BH, whose geometry is described by the RN metric

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega^2,$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$, and $f(r) \equiv 1\text{–}2M/r + Q^2/r^2$ with mass M and charge Q. We consider a nonextremal BH, with $0 < Q/M < 1$. The event horizon (EH) and the IH are located at $r = r_+$ and $r = r_-$, respectively, with $r_\pm \equiv M \pm \sqrt{M^2 - Q^2}$. For later use, we define the two surface gravity parameters, $\kappa_\pm = (r_+ - r_-)/2r_\pm^2$.

Upon this background, we introduce an (uncharged) minimally coupled massless scalar quantum field $\Phi(x)$, obeying the (covariant) d'Alembertian equation, $\Box\Phi = 0$. We decompose the field into modes, which, owing to the metric symmetries, may be separated into $e^{-i\omega t}$, spherical harmonics $Y_{lm}(\theta, \varphi)$, and a function of r [44]. The latter is encoded in the radial function $\psi_{\omega l}(r)$, satisfying

$$\frac{d^2\psi_{\omega l}}{dr_*^2} + [\omega^2 - V_l(r)]\psi_{\omega l} = 0, \tag{2}$$

with the effective potential

$$V_l(r) = f(r)\left[\frac{l(l+1)}{r^2} + \frac{df/dr}{r}\right]. \tag{3}$$

r_* is the standard tortoise coordinate defined through $dr/dr_* = f(r)$, varying from $r_* \to -\infty$ at the EH to $r_* \to \infty$ at the IH.

In the BH interior, $f(r) < 0$, meaning the coordinate r is now timelike. Then, assuming a free incoming wave at the EH, Eq. (2) is endowed with the initial condition

$$\psi_{\omega l} \cong e^{-i\omega r_*}, \qquad r_* \to -\infty. \tag{4}$$

We consider our field in two quantum states: the Hartle-Hawking (HH) state [45,46], corresponding to a BH in thermal equilibrium, and the more physically realistic Unruh state [47], describing an evaporating BH.

We introduce the null Eddington coordinates inside the BH, $u = r_* - t$ and $v = r_* + t$. The flux components of the RSET, $\langle T_{uu}\rangle_{\mathrm{ren}}$ and $\langle T_{vv}\rangle_{\mathrm{ren}}$, are of particular interest [48]. The reason is threefold. First and foremost, as we shall see, it is these components that seem to be the most significant for backreaction near the CH, with a remarkable accumulating effect on the form of the metric (as opposed to minor local distortions associated with other RSET components). In addition, note that, although the classical RN background contains a nonzero stress-energy tensor (of the sourceless electromagnetic field), its T_{uu} and T_{vv} components vanish identically, leaving quantum contributions to prevail. Finally, their regularization procedure turns out to be especially manageable. Accordingly, aiming for the "heart" of the RSET in the context of backreaction, this work focuses on the flux components $\langle T_{uu}\rangle_{\mathrm{ren}}$ and $\langle T_{vv}\rangle_{\mathrm{ren}}$ in the IH vicinity.

In the next section, we implement the PMR θ-splitting variant [21,49] to obtain expressions for the renormalized semiclassical flux components in both quantum states, revealing notable simplicity when taking the IH limit. Then, we provide numerical results for various Q/M values, noting various issues that arise. Finally, we present a preliminary analysis of backreaction and implications to the fate of our traveler.

Developing the near-IH flux expressions.—In what follows, indices U and H correspond to the Unruh and HH states, respectively. As mentioned, we shall only consider the two flux components $\langle T_{uu}\rangle_{\mathrm{ren}}$ and $\langle T_{vv}\rangle_{\mathrm{ren}}$, and for their uniform treatment, we introduce the symbol y, representing either u or v.

The basic PMR expression for the trace-reversed RSET is given in Eq. (2.6) of Ref. [23]. In the case of interest (i.e., the flux components $\langle T_{yy}\rangle_{\mathrm{ren}}$ evaluated at $r \to r_-$ using θ-splitting), two remarkable simplifications occur: (i) the PMR counterterm $\tilde{L}_{yy}(x, x')$ vanishes [49,50]; and (ii) since $g_{yy} = 0$, T_{yy} coincides with its trace-reversed counterpart. The expression then simplifies to

$$\langle T_{yy}\rangle_{\mathrm{ren}}(x) = \frac{1}{2}\lim_{x'\to x} G^{(1)}(x, x')_{,yy'}, \tag{5}$$

where $G^{(1)}(x, x') = \langle\{\Phi(x), \Phi(x')\}\rangle$, and $\{p(x), q(x')\}$ denotes $p(x)q(x') + p(x')q(x)$. We can also express $G^{(1)}$ as

$$G^{(1)}(x, x') = \hbar\sum_{l,m}\int_0^\infty d\omega\, E_{\omega lm}(x, x'), \tag{6}$$

where the mode contributions $E_{\omega lm}(x, x')$ inside a RN BH, in the HH state, are given by

$$E^H_{\omega lm}(x, x') = \coth\tilde{\omega}[J^R + J^L + (\cosh\tilde{\omega})^{-1}J^{RL}],$$

[cf. Eq. (4.3) in [44]] where

$$J^R = \{f^R_{\omega lm}(x), f^{R*}_{\omega lm}(x')\}, \qquad J^L = \{f^L_{\omega lm}(x), f^{L*}_{\omega lm}(x')\},$$

and

$$J^{RL} = 2\Re[\rho^{\mathrm{up}}_{\omega l}\{f^R_{\omega lm}(x), f^{L*}_{(-\omega)lm}(x')\}].$$

Here, $\tilde{\omega} \equiv \pi\omega/\kappa_+$, the star denotes complex conjugation, and $\Re$ marks the real part. Hereafter, $\rho^{\mathrm{up}}_{\omega l}$ ($\tau^{\mathrm{up}}_{\omega l}$) represents the reflection (transmission) coefficient for the "up" modes outside the BH [44]. The mode functions $f^{R,L}_{\omega lm}(x)$ are given by

$$f^{R,L}_{\omega lm}(x) = \frac{1}{r\sqrt{4\pi|\omega|}} Y_{lm}(\theta,\varphi)\tilde{f}^{R,L}_{\omega l}(t,r),$$

where $\tilde{f}^R_{\omega l} = e^{-i\omega t}\psi_{\omega l}(r)$ and $\tilde{f}^L_{\omega l} = e^{i\omega t}\psi_{\omega l}(r)$, and $\psi_{\omega l}(r)$ is the aforementioned radial function solving Eq. (2) with the initial condition (4). (For more details, see [44].)

A similar expression exists for the Unruh-state counterpart, $E^U_{\omega lm}$. In what follows, we shall describe the analysis for the HH state solely. For the Unruh state, the analysis is similar, and we shall merely quote final results below (with the more detailed derivation deferred to [50]). Note that, due to time-inversion symmetry of the HH state (unlike the Unruh state), $\langle T_{uu}\rangle^H_{\mathrm{ren}} = \langle T_{vv}\rangle^H_{\mathrm{ren}}$ everywhere.

We are interested in the asymptotic behavior at the IH, where the effective potential $V_l(r)$ vanishes like $f \propto r - r_-$. Hence, the radial equation, Eq. (2) for $\psi_{\omega l}$ admits the general asymptotic solution $A_{\omega l}e^{i\omega r_*} + B_{\omega l}e^{-i\omega r_*}$ (with constant coefficients $A_{\omega l}, B_{\omega l}$), which, in turn, implies

$$\tilde{f}^R_{\omega l} \cong A_{\omega l}e^{i\omega u} + B_{\omega l}e^{-i\omega v}, \qquad \tilde{f}^L_{\omega l} \cong A_{\omega l}e^{i\omega v} + B_{\omega l}e^{-i\omega u}. \tag{7}$$

Equations (5), (6) yield

$$\langle T_{yy}\rangle^H_{\mathrm{ren}}(x) = \frac{\hbar}{2}\lim_{x'\to x}\sum_{l,m}\int_0^\infty d\omega\, E^H_{\omega lm}(x,x')_{,yy'}.$$

It is interesting to inspect $E^H_{\omega lm}(x,x')_{,yy'}$ within the near-IH approximation (7). Consider, for example, the contribution coming from the J^R term. Focusing for concreteness on $y = u$, we readily see that the $\partial_{uu'}$ operator annihilates the terms depending on v in Eq. (7). Also, $r_{,u} = f/2 \propto r - r_-$ vanishes at $r \to r_-$, altogether yielding at the limit $(u', v', \varphi') \to (u, v, \varphi)$ (corresponding to θ splitting) and $r \to r_-$

$$J^R_{,uu'} \to \{Y_{lm}(\theta,\varphi), Y^*_{lm}(\theta',\varphi)\}|A_{\omega l}|^2. \tag{8}$$

Remarkably, although J^R itself does contain terms like $\propto e^{i\omega(v+u)} = e^{2i\omega r_*}$ at the IH limit, $J^R_{,uu'}$ is free of such oscillatory terms—and is, in fact, entirely independent of r_* (and t). This simplification occurs for all three "J" terms in the expression for $E^H_{\omega lm}(x,x')_{,uu'}$. Combining their contributions and summing over m, one readily obtains at the IH

$$\langle T_{uu}\rangle^H_{\mathrm{ren}} = \hbar\lim_{\delta\theta\to 0}\sum_{l=0}^\infty \frac{2l+1}{8\pi}P_l(\cos\delta\theta)F^H_l, \tag{9}$$

where $\delta\theta \equiv \theta' - \theta$, and $F^H_l \equiv \int_0^\infty d\omega\,\hat{E}^H_{\omega l}$ where

$$\hat{E}^H_{\omega l} = \frac{\omega\coth\tilde{\omega}}{\pi r_-^2}[|A_{\omega l}|^2 + \cosh^{-1}\tilde{\omega}\Re(\rho^{\mathrm{up}}_{\omega l}A_{\omega l}B_{\omega l})], \tag{10}$$

(see fuller derivation in [50]).

The sequence F^H_l appearing in Eq. (9) approaches a nonvanishing constant $\beta \equiv F^H_{l\to\infty}$. One can show [50], analytically, that $\beta = (\kappa_-^2 - \kappa_+^2)/24\pi r_-^2$. Taking the $\delta\theta \to 0$ limit (using the methods of Ref. [21]; see, also, [50]), we obtain the final result

$$\langle T^-_{uu}\rangle^H_{\mathrm{ren}} = \langle T^-_{vv}\rangle^H_{\mathrm{ren}} = \hbar\sum_{l=0}^\infty\frac{2l+1}{8\pi}(F^H_l - \beta). \tag{11}$$

Here, the upper "−" index indicates the IH limit.

The analogous Unruh-state expression is [50]

$$\langle T^-_{yy}\rangle^U_{\mathrm{ren}} = \langle T^-_{yy}\rangle^H_{\mathrm{ren}} + \hbar\sum_{l=0}^\infty\frac{2l+1}{8\pi}\Delta F^U_{l(yy)}, \tag{12}$$

where $\Delta F^U_{l(yy)} \equiv \int_0^\infty d\omega\,\Delta\hat{E}^U_{\omega l(yy)}$ and

$$\Delta\hat{E}^U_{\omega l(yy)} = \frac{\omega}{2\pi r_-^2}(1 - \coth\tilde{\omega})|\tau^{\mathrm{up}}_{\omega l}|^2(|A_{\omega l}|^2 + \delta^v_y). \tag{13}$$

Note that the two Unruh-state flux components are not independent: From energy-momentum conservation, $4\pi r^2(\langle T_{uu}(x)\rangle^U_{\mathrm{ren}} - \langle T_{vv}(x)\rangle^U_{\mathrm{ren}})$ is constant (it is actually the Hawking outflux; see [50]).

Numerical results.—Recalling the Wronskian relation $|\tau^{\mathrm{up}}_{\omega l}|^2 = 1 - |\rho^{\mathrm{up}}_{\omega l}|^2$, the final expressions (11), (12) for the near-IH fluxes in both quantum states reveal simple dependence on $A_{\omega l}, B_{\omega l}$ and $\rho^{\mathrm{up}}_{\omega l}$. We numerically compute $A_{\omega l}$ and $B_{\omega l}$ by integrating the radial Eq. (2) from r_+ to r_- (and $\rho^{\mathrm{up}}_{\omega l}$ likewise, by solving the radial equation outside the BH). Then, we compute the three flux quantities $\langle T^-_{yy}\rangle_{\mathrm{ren}}$ (that is, $\langle T^-_{yy}\rangle^H_{\mathrm{ren}}, \langle T^-_{uu}\rangle^U_{\mathrm{ren}}$, and $\langle T^-_{vv}\rangle^U_{\mathrm{ren}}$) at the IH, as prescribed in Eqs. (11), (12). For further numerical details, see [50]. We find exponential convergence of both the integral over ω (entailed in $F^H_l, \Delta F^U_l$) and the sum over l, for all three quantities $\langle T^-_{yy}\rangle_{\mathrm{ren}}$, as they attain well-defined finite values. Note that a finite nonvanishing $\langle T^-_{vv}\rangle_{\mathrm{ren}}$ implies a curvature singularity at the CH, since transforming to a regular Kruskal-like coordinate $V = -e^{-\kappa_- v}$ yields $\langle T^-_{VV}\rangle_{\mathrm{ren}} \propto e^{2\kappa_- v} \to \infty$.

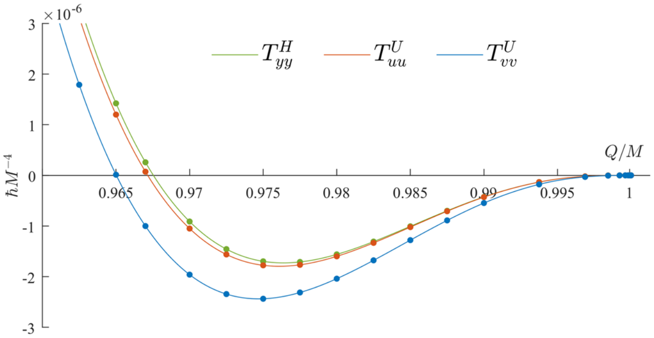

FIG. 1. $\langle T^-_{yy}\rangle_{\rm ren}$ (namely $\langle T^-_{uu}\rangle^U_{\rm ren}, \langle T^-_{vv}\rangle^U_{\rm ren}$, and $\langle T^-_{uu}\rangle^H_{\rm ren} = \langle T^-_{vv}\rangle^H_{\rm ren}$) as a function of Q/M. The points correspond to the numerical data, while the solid curve is interpolated.

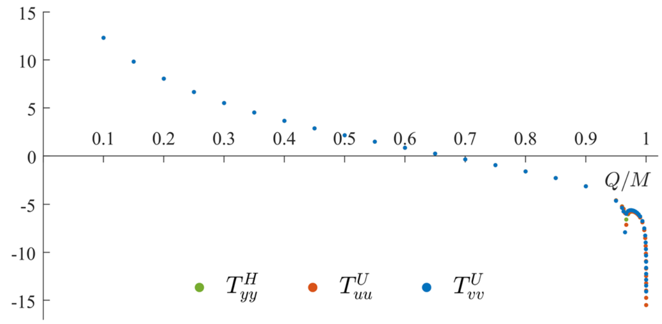

FIG. 2. $\log_{10}|\langle T^-_{yy}\rangle_{\rm ren}/\hbar M^{-4}|$ for a wider Q/M range. The steep drop at ~ 0.97 corresponds to the fluxes changing sign. Note that, in most Q/M values, the three quantities are indistinguishable here.

Remarkably, the three quantities $\langle T^-_{yy}\rangle_{\rm ren}$ may be either positive or negative, depending on Q/M. We find that, sufficiently close to extremality, all three flux components become negative, whereas further away from extremality, they are all positive. Whether the diverging $\langle T_{VV}\rangle$ is positive or negative is crucial for the nature of tidal deformation (contraction vs expansion), a point expanded, hereafter. Figure 1 displays the three flux quantities $\langle T^-_{yy}\rangle_{\rm ren}$ in the range $0.96 < Q/M < 1$, exhibiting the transition from positive to negative values at around $Q/M \sim 0.97$. More precisely, the sign change occurs at Q/M values of $q^U_v \cong 0.9650, q^U_u \cong 0.9671$, and $q^H_y \cong 0.9675$ for $\langle T^-_{vv}\rangle^U_{\rm ren}, \langle T^-_{uu}\rangle^U_{\rm ren}$, and $\langle T^-_{yy}\rangle^H_{\rm ren}$, respectively.

Figure 2 displays the three flux quantities in a wider range $0.1 \le Q/M < 1$. Note the very rapid increase in the fluxes as Q/M decreases. This is, perhaps, not too surprising, since a decrease in Q/M implies an (even faster) decrease in r_-/M and, correspondingly, an increasing curvature at the IH.

Another notable feature is the decay of the fluxes as $Q/M \to 1$. Remarkably, in the near-extremal domain (characterized by $|Q/M - 1| \ll 1$), the flux computation lends itself to analytical treatment (which we defer to a future paper [54]), leading to excellent agreement with the numerical data illustrated on the rightmost part of Fig. 1.

Backreaction near the CH.—The semiclassical backreaction, being of order $\propto \hbar/M^2 = (m_p/M)^2$ (where m_p denotes the Planck mass), is basically an extremely weak effect for macroscopic BHs. For instance, for astrophysical BHs, it is typically $< 10^{-75}$. However, these effects accumulate along the EH, causing its area to drastically shrink upon evaporation. Likewise, as we shall shortly see, semiclassical effects may also accumulate near the CH (and in addition, they become singular there). Thus, semiclassical backreaction is presumably negligible—and hence, the actual backreacted geometry should be well approximated by the original RN metric—as long as (i) the BH hasn't had the chance yet to significantly evaporate (that is, the v interval since the BH formation is much smaller than the evaporation timescale), and (ii) we are not too close to the CH.

To address backreaction, we write the general spherically symmetric metric in double-null coordinates as $-e^\sigma du dv + r^2 d\Omega^2$. The two unknown metric functions, $r(u,v)$ and $\sigma(u,v)$, are to be determined from the semiclassical Einstein equation, Eq. (1). This system contains constraint equations, which are two independent ordinary differential equations (ODEs) (one along each null direction) that involve the flux components $\langle T^-_{yy}\rangle_{\rm ren}$ only; and evolution equations, which are two coupled partial differential equations involving $\langle T_{uv}\rangle_{\rm ren}$ and $\langle T_{\theta\theta}\rangle_{\rm ren}$. Our analysis will mainly rely on the two constraint equations, which we write uniformly as

$$r_{,yy} - r_{,y}\sigma_{,y} = -4\pi r\langle T_{yy}\rangle_{\rm ren}. \tag{14}$$

Now, to proceed, we shall restrict the analysis to the weak-backreaction domain, in which $r, \sigma_{,y}$, and $\langle T_{yy}\rangle_{\rm ren}$ (but not necessarily $r_{,y}$) are still well approximated by their original RN background values. [55] Correspondingly, in what follows, we consider the RN-background RSET and explore its backreaction effect via the semiclassical Einstein equation.

Furthermore, we shall focus on the near-CH portion of this weak-backreaction domain [56]. In this region, we may replace the right hand side of Eq. (14) by the constant $-4\pi r_-\langle T^-_{yy}\rangle_{\rm ren}$, and $\sigma_{,y}$ by $-\kappa_-$ (its near-CH value in RN). We obtain a trivial linear ODE for $r_{,y}$, which is easily solved. After an exponentially decaying term ($\propto e^\sigma$) is dropped, we are left with

$$r_{,y} \cong -4\pi(r_-/\kappa_-)\langle T^-_{yy}\rangle_{\rm ren}. \tag{15}$$

This result expresses a small but steady asymptotic drift of $r(u,v)$ in both null directions. In the long run (i.e., at sufficiently large u and/or v), this drift would result in a major deviation of r from its RN value—which would eventually lead us away from the weak-backreaction domain.

From Eq. (15), it becomes clear that this remarkable accumulative effect is dictated solely by the flux components, namely, it is independent of the other RSET components.

To discuss the physical implications of this result, let us assume our infalling traveler moves towards the IH ingoing section and approaches the near-IH domain where the semiclassical drift is present. Now, we shall consider the effect of the drift in the v direction [57]. We emphasize that, although the near-CH drift in r is very "slow" in terms of v (i.e., $r_{,v} \ll 1$), it actually happens at an exceedingly fast rate for our infalling traveler—which (in the fiducial RN geometry) would arrive the CH at a finite proper time [58]. The nature of this physical effect may crucially depend on the sign of $\langle T_{vv}^{-}\rangle_{\rm ren}$—and hence, on the value of Q/M. For $Q/M < q_v^{U,H}$, $\langle T_{vv}^{-}\rangle_{\rm ren} > 0$, and correspondingly, our traveler will undergo sudden contraction. However, for $Q/M > q_v^{U,H}$, $\langle T_{vv}^{-}\rangle_{\rm ren}$ is negative—implying an abrupt expansion.

This analysis still needs to be extended to the domain of strong backreaction, which actually entails two types of extensions: (i) to the domain of very late time (i.e., very large v), in which significant evaporation has already occurred [59], and (ii) to the region very close to the CH.

Discussion.—Motivated by long-standing expectations that semiclassical effects may drastically influence the interior geometry of spinning or charged BHs, this work focused on the RSET flux components (for a minimally coupled massless scalar field), in the IH vicinity, on a fixed RN background. We presented novel results for the flux components in the Unruh and HH states for various Q/M values. Both flux components $\langle T_{uu}\rangle_{\rm ren}$ and $\langle T_{vv}\rangle_{\rm ren}$—in both quantum states—exhibit finite asymptotic values at the IH. Recall, however, that a nonvanishing finite $\langle T_{vv}\rangle_{\rm ren}$ implies unbounded curvature (and unbounded tidal force) at the CH ($v \to \infty$), because the corresponding Kruskal-like component $\langle T_{VV}\rangle_{\rm ren}$ then diverges as $e^{2\kappa_- v}$.

Hiscock [18] previously demonstrated that, in the Unruh state in a Kerr-Newman BH, either $\langle T_{uu}^{-}\rangle_{\rm ren}$ or $\langle T_{vv}^{-}\rangle_{\rm ren}$ (or possibly both) are nonvanishing—indicating that the corresponding Kruskal fluxes diverge on at least one of the two IH sections. Still, this result left the semiclassical CH singularity inconclusive: Note that it is exclusively the ingoing section of the IH which maintains the causal and physical role of a CH in an astrophysical BH. [60] Our results show that both $\langle T_{uu}^{-}\rangle_{\rm ren}$ and $\langle T_{vv}^{-}\rangle_{\rm ren}$ are generically nonvanishing—demonstrating for the first time the divergence of the Kruskal flux component $\langle T_{VV}\rangle_{\rm ren} \propto e^{2\kappa_- v}$ at the CH.

It is also worth comparing the semiclassical RSET divergence $\propto e^{2\kappa_- v}$ found here with its classical counterpart. Classical perturbations are known to give rise to curvature divergence at the CH, typically like $v^{-n}e^{2\kappa_- v}$ (with n a positive integer depending on the type of perturbation) [10,12,61]. In this sense, the aforementioned semiclassical divergence at the CH is stronger than the one associated with classical perturbations.

Our numerical results indicate that all flux components change their signs at around $Q/M \sim 0.97$, being negative for larger Q/M and positive (and typically much larger) for smaller Q/M values. The sign may have crucial implications to the nature of the tidal effect: catastrophic contraction (for $\langle T_{vv}^{-}\rangle_{\rm ren} > 0$) vs expansion (for $\langle T_{vv}^{-}\rangle_{\rm ren} < 0$).

We also made initial steps towards analyzing the semiclassical backreaction effects of the fluxes on the near-CH geometry (in both the Unruh and HH states). The result expressed in Eq. (15) hints for drastic deformation of the area coordinate r on approaching the CH. However, the analysis provided here was rather preliminary. It should be extended, as mentioned, beyond the domain of weak backreaction. In particular, this picture may change in the next iteration, in which the RSET is reevaluated with respect to the backreacted geometry.

Other obvious extensions are in order. First, it would be worthwhile to generalize the analysis to all RSET components and, also, to the entire interior domain $r_- < r < r_+$. More importantly, this investigation should be extended from the scalar to the more realistic electromagnetic quantum field—and in addition, from the spherical RN background to the astrophysically much more relevant background of a spinning BH.

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*noazilber@campus.technion.ac.il
†leviadam@gmail.com
‡amos@physics.technion.ac.il

[1] B. Carter, Complete analytic extension of the symmetry axis of Kerr's solution of Einstein's equations, Phys. Rev. **141**, 1242 (1966).

[2] J. C. Graves and D. R. Brill, Oscillatory character of Reissner-Nordström metric for an ideal charged wormhole, Phys. Rev. **120**, 1507 (1960).

[3] S. W. Hawking, Black hole explosions?, Nature (London) **248**, 30 (1974).

[4] S. W. Hawking, Particle creation by black holes, Commun. Math. Phys. **43**, 199 (1975).

[5] F. J. Tipler, Singularities in conformally flat spacetimes, Phys. Lett. **64A**, 8 (1977).

[6] A. Ori, Structure of the Singularity Inside a Realistic Rotating Black Hole, Phys. Rev. Lett. **68**, 2117 (1992).

[7] A. Ori, Oscillatory Null Singularity Inside Realistic Spinning Black Holes, Phys. Rev. Lett. **83**, 5423 (1999).

[8] P. R. Brady, S. Droz, and S. M. Morsnik, Late-time singularity inside nonspherical black holes, Phys. Rev. D **58**, 084034 (1998).

[9] M. Dafermos and J. Luk, The interior of dynamical vacuum black holes I: The C^0-stability of the Kerr Cauchy horizon, arXiv:1710.01722.

[10] W. A. Hiscock, Evolution of the interior of a charged black hole, Phys. Lett. **83A**, 110 (1981).
[11] E. Poisson and W. Israel, Internal structure of black holes, Phys. Rev. D **41**, 1796 (1990).
[12] A. Ori, Inner Structure of a Charged Black Hole: An Exact Mass-Inflation Solution, Phys. Rev. Lett. **67**, 789 (1991).
[13] P. R. Brady and J. D. Smith, Black Hole Singularities: A Numerical Approach, Phys. Rev. Lett. **75**, 1256 (1995).
[14] S. Hod and T. Piran, Mass Inflation in Dynamical Gravitational Collapse of a Charged Scalar Field, Phys. Rev. Lett. **81**, 1554 (1998).
[15] L. M. Burko, Structure of the Black Hole's Cauchy-Horizon Singularity, Phys. Rev. Lett. **79**, 4958 (1997).
[16] M. Dafermos, The interior of charged black holes and the problem of uniqueness in general relativity, Commun. Pure Appl. Math. **58**, 445 (2005).
[17] N. D. Birrell and P. C. W. Davies, On falling through a black hole into another universe, Nature (London) **272**, 35 (1978).
[18] W. A. Hiscock, Quantum-mechanical instability of the Kerr-Newman black-hole interior, Phys. Rev. D **21**, 2057 (1980).
[19] A. C. Ottewill and E. Winstanley, Renormalized stress tensor in Kerr space-time: General results, Phys. Rev. D **62**, 084018 (2000).
[20] A. Levi and A. Ori, Pragmatic mode-sum regularization method for semiclassical black-hole spacetimes, Phys. Rev. D **91**, 104028 (2015).
[21] A. Levi and A. Ori, Mode-sum regularization of $\langle\phi^2\rangle$ in the angular-splitting method, Phys. Rev. D **94**, 044054 (2016).
[22] A. Levi and A. Ori, Versatile Method for Renormalized Stress-Energy Computation in Black-Hole Spacetimes, Phys. Rev. Lett. **117**, 231101 (2016).
[23] A. Levi, Renormalized stress-energy tensor for stationary black holes, Phys. Rev. D **95**, 025007 (2017).
[24] S. M. Christensen, Vacuum expectation value of the stress tensor in an arbitrary curved background: The covariant point separation method, Phys. Rev. D **14**, 2490 (1976).
[25] S. M. Christensen, Regularization, renormalization, and covariant geodesic point separation, Phys. Rev. D **17**, 946 (1978).
[26] P. Candelas, Vacuum polarization in Schwarzschild spacetime, Phys. Rev. D **21**, 2185 (1980).
[27] V. P. Frolov, Vacuum polarization near the event horizon of a charged rotating black hole, Phys. Rev. D **26**, 954 (1982).
[28] M. S. Fawcett, The energy-momentum tensor near a black hole, Commun. Math. Phys. **89**, 103 (1983).
[29] P. Candelas and K. W. Howard, Vacuum $\langle\phi^2\rangle$ in Schwarzschild spacetime, Phys. Rev. D **29**, 1618 (1984).
[30] K. W. Howard and P. Candelas, Quantum Stress Tensor in Schwarzschild Space-Time, Phys. Rev. Lett. **53**, 403 (1984).
[31] K. W. Howard, Vacuum $\langle T^\nu_\mu\rangle$ in Schwarzschild spacetime, Phys. Rev. D **30**, 2532 (1984).
[32] P. Candelas and B. P. Jensen, Feynman Green function inside a Schwarzschild black hole, Phys. Rev. D **33**, 1596 (1986).
[33] P. R. Anderson, $\langle\phi^2\rangle$ for massive fields in Schwarzschild spacetime, Phys. Rev. D **39**, 3785 (1989).
[34] B. P. Jensen and A. C. Ottewill, Renormalized electromagnetic stress tensor in Schwarzschild spacetime, Phys. Rev. D **39**, 1130 (1989).
[35] P. R. Anderson, A method to compute $\langle\phi^2\rangle$ in asymptotically flat, static, spherically symmetric spacetimes, Phys. Rev. D **41**, 1152 (1990).
[36] B. P. Jensen, J. G. McLaughlin, and A. C. Ottewill, Anisotropy of the quantum thermal state in Schwarzschild space-time, Phys. Rev. D **45**, 3002 (1992).
[37] A. C. Ottewill and P. Taylor, Renormalized vacuum polarization and stress tensor on the horizon of a Schwarzschild black hole threaded by a cosmic string, Classical Quantum Gravity **28**, 015007 (2011).
[38] P. R. Anderson, W. A. Hiscock, and D. A. Samuel, Stress-energy tensor of quantized scalar fields in static spherically symmetric spacetimes, Phys. Rev. D **51**, 4337 (1995).
[39] P. Taylor, Regular quantum states on the Cauchy horizon of a charged black hole, Classical Quantum Gravity **37**, 045004 (2020).
[40] A. Levi, E. Eilon, A. Ori, and M. van de Meent, Renormalized Stress-Energy Tensor of an Evaporating Spinning Black Hole, Phys. Rev. Lett. **118**, 141102 (2017).
[41] A. Lanir, A. Levi, and A. Ori, Mode-sum renormalization of $\langle\hat{\Phi}^2\rangle$ for a quantum scalar field inside a Schwarzschild black hole, Phys. Rev. D **98**, 084017 (2018).
[42] A. Lanir, A. Ori, N. Zilberman, O. Sela, A. Maline, and A. Levi, Analysis of quantum effects inside spherical charged black holes, Phys. Rev. D **99**, 061502(R) (2019).
[43] See, also, Ref. [39]; but note that the unusual quantum state constructed there does not allow investigating the anticipated semiclassical CH divergency.
[44] A. Lanir, A. Levi, A. Ori, and O. Sela, Two-point function of a quantum scalar field in the interior region of a Reissner-Nordstrom black hole, Phys. Rev. D **97**, 024033 (2018).
[45] J. B. Hartle and S. W. Hawking, Path-integral derivation of black-hole radiance, Phys. Rev. D **13**, 2188 (1976).
[46] W. Israel, Thermo-field dynamics of black holes, Phys. Lett. **57A**, 107 (1976).
[47] W. G. Unruh, Notes on black-hole evaporation, Phys. Rev. D **14**, 870 (1976).
[48] Note that the fluxes do not contribute to the RSET trace, which was shown to diverge in Ref. [42].
[49] A. Levi, Stress-energy tensor mode-sum regularization in spherically symmetric backgrounds (to be published).
[50] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.124.171302 for demonstrating the vanishing of the PMR counterterms $\tilde{L}_{yy}$ at the IH (Section I), derivation of $\hat{E}^H_{\omega l}$ as it appears in Eq. (10) (Section II), derivation of the "plateau value" $\beta \equiv F^H_{l\to\infty}$ (Section III), an analogous treatment for the Unruh state flux expressions (Section IV), some basic numerical parameters (Section V), and the flux quantities plotted against r away from the IH, revealing a clear approach to the corresponding IH values presented in the Letter (Section VI). The Supplemental Material includes Refs. [51–53].
[51] O. Sela, Quantum effects near the Cauchy horizon of a Reissner-Nordstrom black hole, Phys. Rev. D **98**, 024025 (2018).
[52] B. S. DeWitt, Quantum field theory in curved spacetime, Phys. Rept. C **19**, 295 (1975).
[53] S. M. Christensen and S. A. Fulling, Trace anomalies and the Hawking effect, Phys. Rev. D **15**, 2088 (1977).

[54] N. Zilberman *et al.*, Quantum fluxes at the inner horizon of a nearly extremal spherical charged black hole (to be published).
[55] By its very definition, this weak-backreaction domain must satisfy restrictions (i),(ii) mentioned above.
[56] This is the region in which $e^{-\kappa_-(u+v)}$ is already $\ll 1$ (hence, in the RN background $r, \sigma_{,y}$ and $\langle T_{yy}\rangle_{\rm ren}$ are well approximated by their near-CH values); and correspondingly, the drift effect in r is already present—but still hasn't accumulated much.
[57] Both $\langle T^-_{uu}\rangle_{\rm ren}$ and $\langle T^-_{vv}\rangle_{\rm ren}$ are associated with a drift effect at the CH vicinity, but as mentioned, only $\langle T^-_{vv}\rangle_{\rm ren}$ induces a singular effect there. The effect of $\langle T^-_{uu}\rangle_{\rm ren}$ may be associated with a steady drift of r along the CH.
[58] In particular, recall that $dr/d\tau \propto dv/d\tau \propto e^{\kappa_- v}$.
[59] Obviously, this extension is needed in Unruh state only.
[60] In particular, a semiclassical divergence that occurs at the outgoing IH section of the Kerr-Newman background is not expected to realize in a realistic BH produced by gravitational collapse.
[61] A. Ori, Evolution of linear gravitational and electromagnetic perturbations inside a Kerr black hole, Phys. Rev. D **61**, 024001 (1999).