Partial Equilibration of the Anti-Pfaffian Edge due to Majorana Disorder

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We consider electrical and thermal equilibration of the edge modes of the anti-Pfaffian quantum Hall state at $\nu = 5/2$ due to tunneling of the Majorana edge mode to trapped Majorana zero modes in the bulk. Such tunneling breaks translational invariance and allows scattering between Majorana and other edge modes in such a way that there is a parametric difference between the length scales for equilibration of charge and heat transport between integer and Bose mode, on the one hand, and for thermal equilibration of the Majorana edge mode, on the other hand. We discuss a parameter regime in which this mechanism could explain the recent observation of quantized heat transport [M. Banerjee *et al.*, Nature (London) **559**, 205 (2018)].

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Driven in part by the dream of building a quantum computer [1], the goal of observing Majorana fermions in condensed matter has been extremely prominent in the past few years [2–4], with much effort devoted to finding signatures of Majorana zero modes in charge transport. In addition, Majorana edge modes existing at the boundary of a topological state of matter also have a unique signature in heat transport: they contribute one half of the thermal conductance quantum $K_0 = \kappa_0 T = (\pi^2 k_B^2/3h)T$ to the thermal Hall conductance, qualitatively different from integer and Abelian fractional quantum Hall states, whose thermal conductance is quantized in integer multiples of K_0. Recently, a half-integer thermal Hall conductance was indeed observed in the $\nu = 5/2$ quantum Hall state [5], providing evidence for a Majorana edge mode.

The thermal Hall conductance is a universal characteristic of a quantum Hall state, since it is independent of details of the edge structure like disorder and interactions. For this reason, it came as a surprise that the experimental value of approximately $\frac{5}{2}K_0$ differs from the theoretical value $\frac{3}{2}K_0$ for the anti-Pfaffian quantum Hall states, which is expected to be realized on the $\nu = 5/2$ plateau according to exact diagonalization in the absence of disorder [6,7]. Several other possible candidate states do not agree with the experimentally observed thermal Hall conductance either. While there does exist one proposed state, the particle-hole symmetric Pfaffian state [8], which has $\frac{5}{2}K_0$ thermal Hall conductance, various arguments have ruled this out for the experiments of Ref. [5] (see the discussions in Refs. [9,10]).

The ideal topologically protected thermal Hall conductance can only be observed experimentally when all edge channels are in thermal equilibrium with each other, such that their contributions add up to the universal value, assuming no heat dissipates into the bulk [11]. If a sample is shorter than the thermal equilibration length, then deviations from the universal value are expected. In particular, if the Majorana mode of the anti-Pfaffian edge is not equilibrated, a thermal Hall conductance of $\frac{5}{2}K_0$ in agreement with the experimental observation is expected [9,12,13]. (In contrast, since the Pfaffian phase of matter [14] has only copropagating edges, $\frac{7}{2}K_0$ is predicted whether or not there is edge equilibration.) However, under the assumption that scattering processes leading to equilibration between edge modes are due to charge disorder, it is unlikely that charge transport perfectly equilibrates so as to give perfectly quantized electrical conductance [15,16] while at the same time the Majorana mode falls out of thermal equilibrium [9,12,13].

In this Letter, we present a different mechanism for edge equilibration which relies on “Majorana disorder,” i.e., a coupling between the edge Majorana mode and localized Majorana zero modes in the bulk. In the current discussion the disorder acts nonperturbatively to allow for a new type of scattering process mediated by tunneling to Majorana zero modes on trapped quasiparticles in the bulk. We give a detailed calculation of the thermal conductance as a function of temperature in reasonable agreement with experiment. Thus we suggest that the anti-Pfaffian is the only state of matter which is in agreement with the experimental observations of Ref. [5].

Our strategy is to demonstrate the qualitatively new Majorana disorder mechanism, but not necessarily to precisely model the experiment—as there are many parameters of the experiment that are not accurately known anyway. Nonetheless, we will show that for not too unreasonable parameters we can roughly describe the experiment. We will later relax some of our assumptions and suggest that our mechanism may be more general.

For a translationally invariant edge, the edge modes of the anti-Pfaffian comprise three (downstream) integer quantum Hall edge modes, an upstream (reverse-running) bosonic

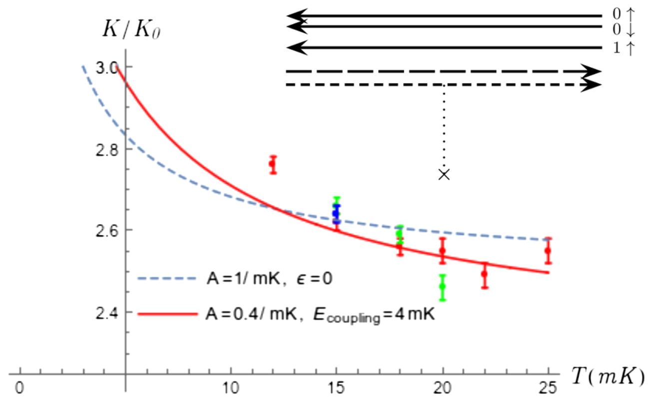

FIG. 1. Thermal conductance as a function of temperature. Points are experimental data from Ref. [5]. Red, green, and blue points are $\nu = 2.50, 2.49, 2.51$, respectively. The dashed curve is the $E_{\mathrm{coupling}} \to 0$ limit while keeping finite $A = 1/\mathrm{mK}$. The solid curve is Eq. (8) given in the text with $A = 0.4/\mathrm{mK}$ and $E_{\mathrm{coupling}} = 4$ mK. For both curves the only scattering mechanism considered is the Majorana disorder, showing the robustness of the mechanism to detailed parameters. Inset: Proposed model of anti-Pfaffian edge. Three integer edge modes (solid) flow downstream, $0\uparrow$, $0\downarrow$, and $1\uparrow$. A Bose edge mode (long dashes) and a Majorana edge mode (short dashes) flow upstream. A trapped Majorana zero mode (marked x) is coupled (dots) to the Majorana edge mode.

edge mode, and an upstream (reverse-running) Majorana edge mode [17,18] (see Fig. 1, inset). As emphasized in Ref. [12], one typically expects a momentum mismatch between different edge modes, so that tunneling of an electron between edge modes requires a change in momentum. Previous discussions have assumed that such a momentum change is provided by charge disorder [9,12,13]. For the moment, we will assume that charge disorder is weak such that translational symmetry breaking can be neglected (i.e., we assume a clean edge). While this seems like a rather strong assumption, we will later discuss how essentially the same physics can apply even in the presence of charge disorder.

We thus start by considering a translationally invariant edge potential. Without disorder one might expect neither electrical nor thermal equilibration between edge modes. However, in the bulk there should be trapped quasiparticles or quasiholes near the edge—each one harboring a Majorana zero mode. In the absence of disorder these particles will form some sort of Wigner crystal (or glass) minimizing their energies with the background potential as well as minimizing their interaction energies with each other. Let us assume that some of these quasiparticle locations are not too far from the edge. We also assume that the Coulomb energy is large enough so that the trapped quasiparticles do not change their positions.

Generically, there will be coupling of the trapped Majorana zero mode to the Majorana edge mode, as shown in Fig. 1, inset. Such coupling of the edge to a trapped Majorana has been analyzed in a number of different contexts before [19–23]. The result of such a coupling is to produce an energy dependent scattering phase shift to the edge Majorana of the following form,

$$e^{i\varphi(E)} = \frac{E + iE_{\mathrm{coupling}}}{E - iE_{\mathrm{coupling}}}, \tag{1}$$

where E_{coupling} is the (temperature independent) strength of the coupling between the trapped Majorana mode and the edge (see Supplemental Material, Sec. III, for derivation [24]). This is analogous to the phase shift of an electronic level coupled to a continuum [25]—except that here E must be positive since we have Majoranas.

The key here is to realize that at energies high compared to the coupling energy, the Majorana edge mode is undisturbed by its coupling to the trapped mode (φ is close to zero). However, at low energies compared to the coupling energy, the Majorana mode is maximally phase shifted by an angle of π. In particular, for an edge Majorana with wave vector k, such that $E = v_0 k \ll E_{\mathrm{coupling}}$, with v_0 the Majorana mode velocity, the wave function takes the form e^{ikx} for $x < x_0$ and $-e^{ikx}$ for $x > 0$. This function has Fourier modes $\sim e^{i(q-k)x_0}/(q-k)$, allowing overlap of this Majorana edge mode with other edge modes even with substantial momentum mismatch. Thus, we should expect there should be scattering into the Majorana edge mode at energies less than E_{coupling} but not at energies much greater than E_{coupling}.

Suppose further that the coupling energy happens to be somewhat smaller than the temperature. In this case we have a mechanism by which scattering of charge occurs only when the energy of the Majorana is sufficiently low, thus keeping the heat from being transferred to the Majorana mode—potentially achieving charge equilibration without thermal equilibration.

Let us now be more precise about the details of the scattering model we solve. We consider scattering to a single integer mode ($1\uparrow$ in Fig. 1) which we write using fermionic fields $\{\psi(x), \psi^\dagger(x')\} = \delta(x - x')$. The Majorana edge mode is ξ_0, and we will use a convenient representation [17,18,26] of the Bose mode in terms of two Majorana operators ξ_1 and ξ_2. These Majorana fields are self-conjugate $\xi_\alpha^\dagger = \xi_\alpha$ and have fermionic anticommutations $\{\xi_\alpha(x), \xi_\beta(x')\} = \delta_{\alpha\beta}\delta(x - x')$. The Hamiltonian of the edges is then given by

$$H_0 = i\int dx\left[v_i\psi^\dagger(x)\partial_x\psi(x) + \sum_{\alpha=0,1,2}\frac{v_\alpha}{2}\xi_\alpha(x)\partial_x\xi_\alpha(x)\right], \tag{2}$$

where $v_i < 0$ is the integer mode velocity, $v_0 > 0$ is the Majorana ξ_0 velocity, and $v_1 = v_2 = v_b > 0$ is the Bose velocity. In the presence of large disorder scattering, Refs. [17,18] consider a fixed point where $v_0 = v_1 = v_2$. However, here we are assuming low disorder limit and generally we expect that the Majorana velocity v_0 is somewhat less [27] than the Bose or integer velocities v_b and v_i. On the right-hand side we assume a reservoir at temperature T and voltage 0; on the left we assume reservoir with temperature $T + \Delta T$ and voltage V.

In addition, we add an interaction induced scattering term to allow an electron to scatter from the integer to the fractional edges. This is of the form

$$H_1 = \alpha \int dx e^{ipx} \psi^\dagger(x)\xi_0(x)\xi_1(x)\xi_2(x) + \text{H.c.}, \quad (3)$$

where α is a coupling constant with dimensions of velocity which should be roughly on the order of the edge mode velocity (to be detailed below and in the Supplemental Material [24]), and p is the wave vector mismatch between the integer and fractional modes (assumed to be on the order of the inverse magnetic length). Here the electron in the fractional edges is made of a product of the three Majoranas. In the absence of additional disorder, due to the wave vector mismatch p, there can be no scattering at low voltage and low temperature difference between the edge modes.

Finally, we add the single Majorana impurity γ_{qp} zero mode ($\gamma_{qp}^2 = 1$ and $\{\gamma_{qp}, \xi_j(x)\} = 0$), via the Hamiltonian

$$H_2 = i\lambda\gamma_{qp}\xi_0(x_0), \quad (4)$$

where x_0 is the position of the coupling, and λ is the coupling constant. If we start by ignoring the Bose and integer mode, it is easy to show that the phase shift to the ξ_0 mode due to the coupling H_2 is given by Eq. (1), where $E_{\text{coupling}} = \lambda^2/v_0$ (see Supplemental Material, Sec. III, for detailed derivation [24]).

We now calculate the tunneling current between the integer and fractional edges. See Supplemental Material, Sec. I, for details [24]. We use Fermi's golden rule to describe the tunneling of an electron between edges. The complexity comes from the fact that the electron is fractionalized between Bose and Majorana modes. The tunneling current through the impurity is [28]

$$\begin{aligned} J^\alpha \sim \int dx \int dx' \int dE X^\alpha \\ \times [e^{ip(x-x')} G^L_<(E, x', x) G^R_>(E + eV, x', x) \\ - e^{-ip(x-x')} G^L_>(E, x', x) G^R_<(E + eV, x' x)], \end{aligned} \quad (5)$$

where $\alpha = e$ or E (for charge current or energy current), $X^e = -e$ and $X^E = E$ with p the momentum mismatch between the right- and left-moving edges, and V the voltage difference. On the left-moving integer edge, $G^L_{>,<}(E, x', x) \sim e^{\pm i(E/v_i)(x-x')} n_F(\mp E)$, where $n_F(E) = 1/(1 + e^{\beta E})$ denotes the Fermi distribution, and $\beta = 1/k_B T$. The right-moving electron Green's function can be expressed as a convolution of Bose and Majorana Green's functions $G^R_{>,<}(E,x,x') \sim \int dE' G^b_{>,<}(E-E',x',x) G^\xi_{>,<}(E',x',x)$. Here, $G^b_{>,<}(E,x,x') \sim \mp E n_B(\mp E) e^{\mp i(E/v_b)(x-x')}$. The Majorana Green's function in the absence of the impurity is $G^{\xi,0}_{>,<}(E, x, x') \sim n_F(\mp E) e^{\mp i(E/v_0)(x-x')}$. In the presence of an impurity, the Majorana Green's function is given by $G^\xi_{>,<}(E, x, x') = G^{\xi 0}_{>,<}(E, x, x') F(E, x, x')$ with a phase shift $F(E, x, x')$ from the impurity at position x_0 given by $F(E, x, x') = e^{i\varphi(E)}$ for $x > x_0 > x'$ or $F(E, x, x') = e^{-i\varphi(E)}$ for $x < x_0 < x'$ and $F(E, x, x') = 1$ otherwise, where $\varphi(E)$ is given by Eq. (1).

Evaluating the tunneling current Eq. (5) using the above Green's functions (see Supplemental Material, Sec. I [24]), we obtain results in line with the expectations described earlier. We can easily examine the limit of very weak coupling E_{coupling} with the assumption that the wave vector mismatch p between the Bose mode and the integer mode is larger than T/v_0. In this limit the electrical conductance from the integer to fractional (combination of Bose and Majorana) modes is given by

$$G = \frac{\pi|\alpha|^2 E_{\text{coupling}} T}{8|v_i| v_b^2 v_0 p^2} G_0, \quad (6)$$

with $G_0 = e^2/h$. The thermal conductance in this limit is more complicated since the three edge modes can have three different temperatures. We find the corresponding thermal conductances to be

$$\begin{aligned} K^{ib} &= (k_B/e)^2(\pi^2/2)TG, \\ K^{im} &= \epsilon K^{ib}, \\ K^{bm} &= 2\epsilon K^{ib}, \\ \epsilon &= [32/(9\pi^3)] E_{\text{coupling}}/T \approx 0.1 E_{\text{coupling}}/T, \end{aligned} \quad (7)$$

where i, b, and m indicate the integer, Bose, and Majorana edge modes. (For example, the thermal current between the integer and Bose mode is K^{ib} times the temperature difference between these two modes.) There are no thermoelectric couplings due to the particle-hole symmetry of the model [29], and the influence of Joule heating on edge temperature and shot noise [30] is neglected due to the leading order expansion in the tunnel coupling α.

Assuming the coupling E_{coupling} is sufficiently smaller than T, the parameter ϵ will be small and the thermal conductance into the Majorana mode will be much less than that into the Bose mode. Thus one should have a regime where there is electrical equilibration, and the Bose mode is fully thermally equilibrated, but the Majorana mode is not. Inclusion of Coulomb interaction between the integer and Bose mode may change the linear temperature dependence in Eq. (6) to $T^{2\Delta-3}$ (via a change in the scaling dimension of the tunneling operator [18]). In the absence of intermode Coulomb coupling, $\Delta = 2$, and for sufficiently strong

Coulomb coupling, $\Delta < 3/2$ causes a phase transition to a random fixed point with perfect equilibration in the low temperature limit [18]. Since in the experiment [5] equilibration gets worse with lower temperature, we conclude $\Delta > 3/2$ and the result Eq. (6) for $\Delta = 2$ is representative for the nonrandom fixed point. In addition, crucially, K^{ib} will still be given by Eq. (7) up to order unity constant, and K^{im} and K^{bm} will still be suppressed a factor of ϵ.

Let us assume that heat is not flowing into the Majorana mode. If we further assume that the $1\uparrow$ integer mode does not mix with the other integer edge modes, then this mode along with the Bose mode form a system of two counterpropagating edges similar to the case of Refs. [15,16]. In such cases thermal equilibration is diffusive, and the system may not fully equilibrate. This physics is certainly seen in experiment [31] at $\nu = 2/3$, and, as pointed out in Ref. [13], is likely also occurring in experiment [5] at $\nu = 2 + 2/3$ with a similar assumption that the outer two integer edge modes are not mixing with the other modes. Since the conductances are dropping proportional to T at low temperature, we should expect that equilibration should be particularly bad at low temperature. Should the Bose mode go out of thermal equilibrium with the integer mode, the measured thermal conductance should rise [9], which is precisely what is observed in experiment.

The conductances and thermal conductances calculated so far are conductances between edge modes through a single scattering center. The electrical conductance between edge modes per unit length is given by $\tilde{G} = n_{imp}G$, where n_{imp} is the number of scatterers per unit length. We can define a characteristic charge equilibration length $\ell_e^b = G_0/\tilde{G}$. To determine the total electrical conductance along the edge we use the relationship between current and chemical potential being given by $j_\alpha = G_\alpha \delta\mu_\alpha$, with $G_i = G_0$ and $G_b = G_0/2$. We then include scattering between the two edges via $\partial_x j_{i,b} = \pm\tilde{G}(\delta\mu_i - \delta\mu_b)$. The solutions of these equations show us that corrections to the quantized electrical conductance will be order e^{-L/ℓ_e^b} with L the length of the edge. (See Supplemental Material, Sec. IV [24].) Since the electrical conductance is well quantized, we must assume that $L/\ell_e^b \gg 1$.

Similarly to the electrical case, the thermal conductances per unit length between edge modes $\alpha, \beta \in \{i, b, m\}$ are given by $\tilde{K}^{\alpha\beta} = n_{imp}K^{\alpha\beta}$ giving a characteristic thermal length for equilibrating the Bose and integer modes given by $\ell_q^b = K_0/\tilde{K}^{ib} = 2l_e^b/3$, with $K_0 = (\pi^2/3)Tk_B^2/h$. The thermal current along an edge is given by $J_\alpha = c_\alpha K_0 \delta T_\alpha$, where $\alpha = \{i, b, m\}$ and $c_\alpha = (-1, 1, 1/2)$ is the signed central charge of the different edge modes. We then include scattering between edges via $\partial_x J^\alpha = -\sum_\beta \tilde{K}^{\alpha\beta}\delta T_\beta$, with $\tilde{K}^{\alpha\alpha}$ defined to give energy conservation $\sum_\beta \tilde{K}^{\alpha\beta} = 0$. Because we have counterpropagating modes [15,16,31,32], as in the case of $\nu = 2/3$, corrections to the measured quantized thermal conductance will be algebraic. The solution of this system of equations (detailed in Supplemental Material, Sec. IV. B [24]) gives us the net thermal conductance of the edge (including $2K_0$ from the lowest Landau level edges),

$$K/K_0 = 2.5 + \frac{2}{1 + AT} - \epsilon C(AT), \tag{8}$$

where $A = L/(l_q^b T)$ is a temperature independent constant and where $\epsilon = [32/(9\pi^3)](E_{\text{coupling}}/T)$ is the above-discussed small parameter which we can approximate as zero if the Majorana mode is decoupled from the integer and Bose modes. For $x \gg 1$, we have $C(x) \approx x$. We expect the thermal equilibration length for the Majorana mode to scale as l_q^B/ϵ, which can be much longer than the length of the sample.

In Fig. 1 we show example results of this theory compared against experimental data from Ref. [5]. The two curves have values of A fit to the data given a fixed value of $E_{\text{coupling}} = 0$ or 4 mK, showing that the curve shape is relatively independent of E_{coupling}. For all plotted values of T we have $L/l_q^b = (2/3)(L/l_q^c)$ substantially greater than 1. Thus the measured electrical conductivity will be well quantized.

One possible concern with our model is that the coupling H_2 between the isolated quasiparticle and the edge is assumed to occur at one point x_0. The fact that it is a point coupling is responsible for the appearance of arbitrarily large Fourier modes being active. More realistically, the coupling will be smeared out somewhat. The tunneling from a Majorana impurity to the edge should be exponential with some decay length ζ. If the impurity is a perpendicular distance R from the edge, then the smearing of the coupling along the edge should be roughly $\sim\exp(-\sqrt{R^2 + x^2}/\zeta) \approx e^{-R/\zeta}e^{-x^2/(2R\zeta)}$, with x the distance along the edge, giving a smearing over a length scale on order $w \approx \sqrt{R\zeta}$, preventing the above-described scattering mechanism from being effective if the wave vector mismatch is $p \gtrsim w^{-1}$. We can use an estimate of $\zeta \approx 1.15\ell_B$ from prior numerical work [24,33], so that we also have $E_{\text{coupling}} \approx 1\text{K}e^{-R/\zeta}$. Given that we want E_{coupling} in the mK range, we estimate $R \approx 6\ell_B$, thus bounding $p \lesssim 0.3/\ell_B$. See Supplemental Material, Sec. III. A, for more details [24].

We now relax our prior assumption that there is no charge disorder along the edge. In the presence of charge disorder, if the disorder wavelength is not as large as the momentum mismatch p of the edges, then scattering can not occur due to this disorder wave vector alone. However, one can consider a situation where scattering can occur if the Majorana impurity mechanism provides some of the momentum and the disorder provides the remainder. A detailed calculation of this more complicated mechanism is beyond the scope of this work, but we expect that very similar physics will result.

We now turn to the physical parameters which will give us this desired value of $A = (3\pi/16)|\alpha|^2 E_{\text{coupling}} n_{imp} L / (|v_i| v_b^2 v_0 p^2) \approx 0.4/\text{mK}$ used in Fig. 1 (we need A to be not too much less than 0.4/mK so that the electrical conductivity is well quantized at experimental temperatures). Let us assume the following reasonable parameters: velocity $v_i = v_b = 10^6$ cm/ sec for the integer and Bose modes and $v_0 = 10^5$ cm/ sec for the Majorana edge mode. The coupling constant α also has dimensions of velocity and should be roughly on the same scale. In the Supplemental Material we detail why an estimate of this parameter should be given by $\alpha^2 = \pi^2 v_b \sqrt{v_b v_0}$ [24]. We take $p = 0.1/\ell_B$, and in the experiment $\ell_B = 16$ nm and $L = 150$ μm. Finally, we choose E_{coupling} to be 4 mK $\ll T$ as given in Fig. 1. In order to have $A \approx 0.4/\text{mK}$ this would require one impurity every 120 nm $\approx 8\ell_B$. Note in addition that A scales as the inverse square of both p and the velocities, so that a small reduction in either would allow a much lower density of impurities. We emphasize that there have been no detailed simulations of the anti-Pfaffian edge, and it is possible that the edge potential is strongly screened by the outer edge modes resulting in edge velocities being somewhat smaller than in outer edge modes.

To summarize, we have provided a detailed mechanism that potentially explains the observation of the $K/K_0 \approx 2.5$ from Ref. [5], by showing how the Majorana edge mode can remain out of thermal equilibrium, despite the fact that all of the edge modes are in electrical equilibrium. Further, we show how the same mechanism can roughly explain the temperature dependence of the experimental data.

In compliance with EPSRC policy framework on research data, this publication is theoretical work that does not require supporting research data.

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