

Ab initio Determination of the Phase Diagram of CO₂ at High Pressures and TemperaturesBeatriz H. Cogollo-Olivo^{1,*}, Sananda Biswas,² Sandro Scandolo,³ and Javier A. Montoya⁴¹Universidad de Cartagena, Doctorado en Ciencias Físicas, 130001 Cartagena de Indias, Colombia²Institut für Theoretische Physik, Goethe-Universität Frankfurt, 60438 Frankfurt am Main, Germany³The Abdus Salam International Centre for Theoretical Physics (ICTP), Strada Costiera 11, 34151 Trieste, Italy⁴Universidad de Cartagena, Instituto de Matemáticas Aplicadas, 130001 Cartagena de Indias, Colombia (Received 25 October 2019; accepted 10 February 2020; published 2 March 2020)

The experimental study of the CO₂ phase diagram is hampered by strong kinetic effects leading to wide regions of metastability and to large uncertainties in the location of some phase boundaries. Here, we determine CO₂'s thermodynamic phase boundaries by means of *ab initio* calculations of the Gibbs free energy of several solid phases of CO₂ up to 50 Gigapascals. Temperature effects are included in the quasiharmonic approximation. Contrary to previous suggestions, we find that the boundary between molecular forms and the nonmolecular phase V has, indeed, a positive slope and starts at 21.5 GPa at $T = 0$ K. A triple point between phase IV, V, and the liquid phase is found at 35 GPa and 1600 K, indicating a broader region of stability for the nonmolecular form than previously thought. The experimentally determined boundary line between CO₂-II and CO₂-IV phases is reproduced by our calculations, indicating that kinetic effects do not play a major role in that particular transition. Our results also show that CO₂-III is stabilized at high temperature and its stability region coincides with the P - T conditions where phase VII has been reported experimentally; instead, phase II is the most stable molecular phase at low temperatures, extending its region of stability to every P - T condition where phase III is reported experimentally.

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Widely studied during the past years, carbon dioxide (CO₂) is a fascinating system that, despite its simple molecular form at ambient conditions, exhibits a rich polymorphism at high pressures and temperatures, with up to seven crystalline structures reported experimentally so far, in addition to an amorphous form (see Fig. 1). At room temperature the molecular gas transforms into a liquid at 7.5 MPa which then solidifies at 0.5 GPa into CO₂-I, a molecular crystal with space group Pa $\bar{3}$ [1,2]. By further increasing pressure at ambient temperature, CO₂-I transforms to the orthorhombic phase III (*Cmca* space group) above 10 GPa, with a minimal volume change [3]. A recent theoretical study has provided insights into the microscopic mechanism of the Pa $\bar{3}$ -to-*Cmca* transition [4]. Heating compressed CO₂-III above ~470 K [5,6] leads to the transformation into phase II. However, this transition is not reversible: CO₂-II can be recovered at ambient temperature while pressurized, suggesting that CO₂-III is a kinetic product of the compression of CO₂-I and not a stable phase [5,7]. With the exception of a recent theoretical study [8], all previous theoretical work confirms that CO₂-II is more stable than CO₂-III at ambient temperature and below. Initially described as a structure with carbon in an unconventional sixfold coordination, phase II was interpreted as an intermediate state between the molecular and the extended solid form of CO₂ [9], however, subsequent studies disproved the existence of such an

intermediate bonding state and identified the structure of phase II as composed of undistorted molecules, with space group $P4_2/mnm$ [10]. CO₂-II transforms into CO₂-IV when it is heated in the 500–720 K range, depending on

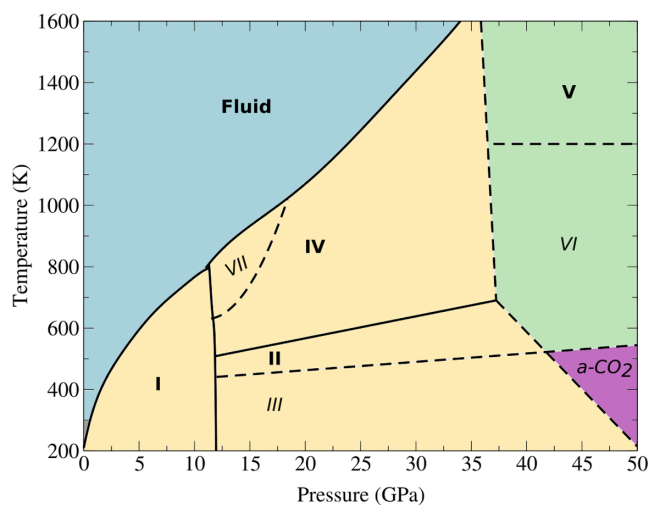


FIG. 1. CO₂ phase diagram adapted from Ref. [15]. Yellow, green, blue, and purple areas correspond to the molecular, non-molecular, fluid, and amorphous forms of CO₂, respectively. Solid lines correspond to thermodynamic phase boundaries, while dashed lines are kinetic boundaries. Names in bold and italic indicate thermodynamic and metastable phases, respectively.

pressure [5,7]. Phase IV, similar to phase II, was also initially interpreted as an intermediate bonding state [11]; here again, this interpretation was disproved by showing experimentally that CO₂-IV is still composed of well defined linear molecules and that its crystalline structure is rhombohedral with space group $R\bar{3}c$ [12]. At higher temperatures, an intermediate phase between CO₂-I and CO₂-IV was observed by heating CO₂-I to ~ 950 K and compressing it up to 20 GPa. The crystal structure of this molecular high-temperature stable phase (CO₂-VII) belongs to space group $Cmca$. [13]. Despite the fact that CO₂-VII and CO₂-III have the same space group, some differences in their Raman spectra and in their lattice parameters suggested that their structures might be quantitatively and qualitatively different [13]. However, a recent theoretical study has shown that CO₂-III and CO₂-VII belong to the same configurational energy minimum and that CO₂-III is a low temperature metastable manifestation of CO₂-VII [14].

The nonmolecular CO₂-V phase was first synthesized by laser heating CO₂-III above 40 GPa and 1800 K [16], and its crystalline structure has now been determined as a fully tetrahedral partially collapsed cristobalitelike structure, with space group $I\bar{4}2d$ [17,18]. By compressing CO₂-II to 50 GPa at 530 and 650 K, another nonmolecular form of carbon dioxide (CO₂-VI) was obtained [11]. Its vibrational spectra is consistent with those of metastable layered tetrahedral structures, as shown in [19]. In addition to the molecular and nonmolecular phases, an amorphous form of carbon dioxide (a-CO₂) was observed upon compressing CO₂-III in the pressure range from 40 to 48 GPa at room temperature [7]. The microscopic structure of a-CO₂ has been explained as a frustrated mixture of threefold and fourfold coordinated carbon atoms, in an intermediate metastable form towards fully tetrahedral coordination [20]. Finally, CO₂-V has been reported to dissociate into elemental carbon (diamond) and oxygen ($\epsilon - O_2$) at pressures between 30 and 80 GPa, and temperatures above 1700 K, [21,22]. However, more recent theoretical [23] and experimental [24] works have not observed a transition from the nonmolecular CO₂-V phase into a dissociated state. Moreover, Dziubek *et al.* [24] confirmed that CO₂-V is the only stable phase among all known nonmolecular forms of carbon dioxide, as already proposed by a previous experimental work [25] as well as by theoretical structural searches in the previous decade. The fate of CO₂ at high pressures has important implications for the Earth's global carbon budget [26]. CO₂ degassing in the upper mantle affects melting beneath oceanic ridges [27] and carbonate decomposition [28] may have implications for plume formation in the lower mantle [29]. Therefore, a precise assessment of the transition lines between CO₂ phases, can offer a better understanding of the dynamics of CO₂ within the context of the deep carbon cycle. The goal of defining a complete thermodynamic

phase diagram for this basic molecular system has been elusive to experimentalists due to the unique and incredibly strong kinetic limitations and the metastability that are present in CO₂'s molecular and extended forms, this, in addition to diverse interpretations of the experimental diffraction data coming from very small samples in diamond anvil cell (DAC) experiments. It is, then, problematic that, after many decades of research, there is no final thermodynamic phase diagram for CO₂ in a range of P - T conditions that have been accessible in the lab by diamond anvil cell experiments since the end of the previous century. In this sense, by avoiding uncertainties coming from kinetic limitations and metastability, our Letter presents a well motivated purely *ab initio* density functional theory (DFT)-based determination of the complete phase diagram of CO₂ in an ample pressure and temperature range.

The currently accepted phase diagram including all the mentioned forms of solid CO₂ along with the region where it becomes a fluid is shown in Fig. 1.

In this particular system, strong kinetic effects hinder the experimental determination of the phase boundaries, while the small size of the samples in high-pressure experiments makes structure determinations quite difficult. As a consequence, several questions remain open regarding the nature and location of the phase boundaries and the stability of the phases reported in Fig. 1. In the molecular portion of the phase diagram, open questions include the relative stability of CO₂-II and CO₂-III at low temperature, and the nature of CO₂-VII, in particular its structural relationship with CO₂-III. At higher pressures, one of the fundamental questions is the location of the phase boundary between molecular and nonmolecular phases. Santoro *et al.*, for example, proposed a phase diagram where the boundary between molecular and nonmolecular phases at room temperature is located at 20 GPa, roughly half-way between the lowest pressure of quenching and the pressure of synthesis for this phase [30,31]. Moreover, the kinetic boundary between CO₂-III and the a-CO₂ nonmolecular structure, i.e., the P - T region where the transformation occurs upon compression, has a negative slope [32], while basic thermodynamic considerations suggest that the slope of the true phase boundary should be positive [31]. Theoretical determinations of the molecular-nonmolecular boundary at zero temperature, based on *ab initio* electronic structure methods, predict transition pressures in the range between 18 and 21 GPa when going from both CO₂-II and CO₂-III to the nonmolecular forms [28,30,33].

In this Letter, we extend the theoretical determination of the phase diagram of CO₂ to finite temperatures for all stable phases except CO₂-I. Phase boundaries between the molecular phases II, III, and IV, and the nonmolecular phase V are calculated based on an *ab initio* approach; for the determination of free energies, the vibrational contributions are treated in the quasiharmonic approximation (QHA).

Ab initio electronic structure calculations were carried out using DFT and the projector augmented wave method, as implemented in the Quantum ESPRESSO suite [34,35] with a kinetic energy cutoff of 200 Ry for the plane-wave basis set. The generalized gradient approximation was employed for the exchange-correlation energy and implemented using the Perdew-Burke-Ernzerhof functional [36]. Our system is well represented by this approach as shown by previously calculated intramolecular C = O bond lengths of CO₂-II at different levels of theory, which are generally consistent with our values [14,37,38]. The Monkhorst-Pack method [39] was used to generate the *k* point grids for sampling the Brillouin zone. Variable-cell optimization of all structural parameters was performed for the four phases in the range of pressures between 10 and 70 GPa. Density functional perturbation theory within the linear response scheme [40] was used to calculate phonon frequencies at zero temperature. The zero-point energy and the finite-temperature contributions to the Helmholtz free energy were computed in the QHA [41,42], which is valid where harmonic effects dominate the material's properties. It is commonly accepted as a criterion that increasingly relevant contributions coming from anharmonicities are expected to appear at $T \sim 1.2\Theta_D$ and above [43,44]. Even more, the range of temperatures in which QHA remains valid is significantly expanded under high pressure [43–45]. For this particular study, the calculated Θ_D takes values between 3485 K and 3500 K for the three molecular forms considered at the lowest pressure in our work (10 GPa), which are clearly higher than the highest temperature registered in this study, i.e., 1600 K. Thus, the temperature region under consideration in this work spans approximately from 0.05 to 0.45 Θ_D , assuring us of the validity of the quasi-harmonic approximation below the melting curve of CO₂ for all solid forms. For the construction of the pressure-temperature phase diagram, the Helmholtz free energy at different temperatures was fitted to a third order Birch-Murnaghan equation of state (EOS). Finally, the Gibbs free energy was calculated as

$$G(P, T) = F[V(P, T), T] + PV(P, T), \quad (1)$$

Room-temperature equations of state obtained with the above approximations are compared with experimental data for phases CO₂-II, CO₂-III, CO₂-IV, and CO₂-V in Fig. 2. The agreement is good and confirms the validity of the approach. Phase boundaries constructed based on the calculated Gibbs free energies are shown in Fig. 3. It is worth mentioning that although most phases in this study are molecular, the van der Waals approximation was not used since, in simulations under very high pressure, the dispersion function becomes constant at distances much shorter than the standard van der Waals radii, resulting in not affecting the valence geometries or energies [46,47].

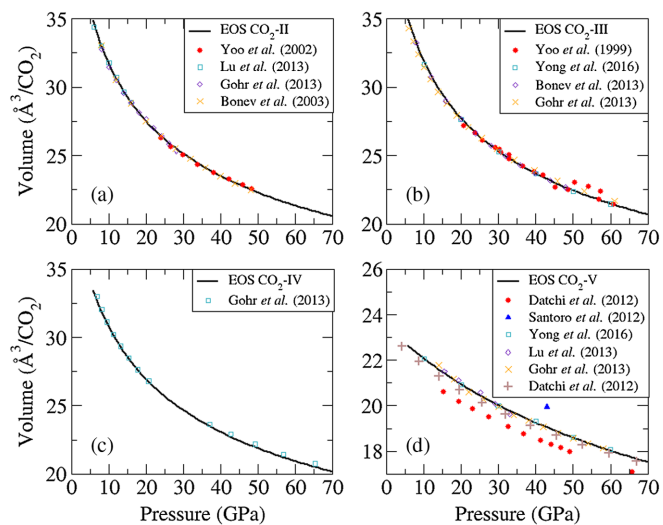


FIG. 2. Pressure-volume relation of phases (a) II, (b) III, (c) IV, and (d) V of CO₂ at room temperature are shown in black solid lines. For each case, reported values from experimental (red circles) and theoretical (blue squares, purple diamonds, and yellow crosses) studies for the different phases are displayed as well.

We begin our discussion with an analysis of the molecular solid region of the phase diagram. This region is indicated in yellow in Fig. 1; it contains the molecular phases I, II, III, and IV, and its upper bound in pressure coincides with the experimentally reported transitions to the nonmolecular phases. Since phase I as well as its boundaries with the other phases are well known and constrained, we focus specifically on phases II, III, and IV, at pressures higher than 12 GPa. According to the enthalpy-pressure relations, with and without the zero-point energy contribution, at $T = 0$ K CO₂-II is the most stable molecular phase in the pressure range considered, until the transition to CO₂-V. This indicates that the orthorhombic *Cmca* structure (phase III) obtained experimentally from the compression of phase I is, indeed, only metastable at low temperatures. Notice that this remains true even after the inclusion of zero-point contributions, in agreement with previous reports [5,48]. At variance with our results as well as with previous theoretical work, a recent theoretical study [8] proposes a transition boundary between phases II and III in which CO₂-III is stable up to ~ 570 K at 19 GPa. This is at odds with experimental observations where the kinetic transition from CO₂-III to CO₂-II occurs at much lower temperatures [5]. Instead, our calculations show that CO₂-III becomes more stable than CO₂-II at higher temperatures [solid green line with stars in Fig. 3(a)]. The transition temperature between CO₂-III and CO₂-II has a strong pressure dependence and reaches values in excess of 1000 K close to the boundary with the nonmolecular phase V, with respect to its value close to CO₂-I. Comparing the free energies of CO₂-II and CO₂-IV, we find that the boundary between phases II and IV [solid brown line with down triangles in Fig. 3(a)] agrees quite well with

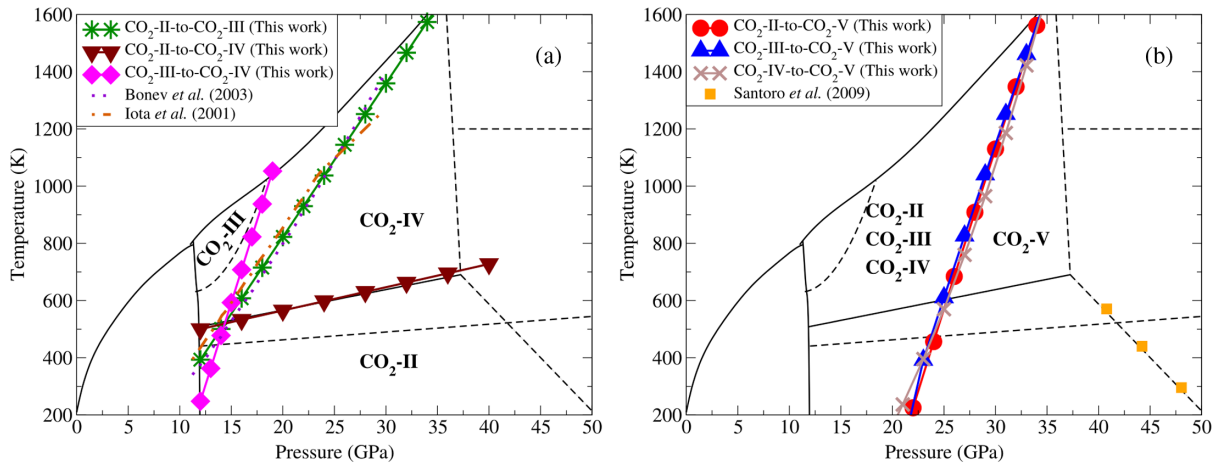


FIG. 3. (a) Phase boundaries between CO₂-II and CO₂-III (green, stars), CO₂-II and CO₂-IV (brown, down triangles), and CO₂-III and CO₂-IV (magenta, diamonds). Suggested boundaries reported by Iota *et al.* [5] (experimental) and Bonev *et al.* [48] (theoretical), are shown in orange dotted-dashed line and violet dotted line, respectively. (b) Phase boundaries between molecular phases CO₂-II (red, circles), CO₂-III (blue, up triangles), and CO₂-IV (gray, crosses), and the nonmolecular phase CO₂-V. Proposed limits of the kinetic region from experimental data from Ref. [32] (orange squares) are also included.

experimental data [12]. The weak pressure dependence of the II-IV boundary reduces the region of stability of phase II with respect to the starred green line in Fig. 3(a), by confining it toward lower temperatures. Finally, we find that the boundary between CO₂-III and CO₂-IV [solid magenta line with diamonds in Fig. 3(a)] is almost vertical, which restricts the domain of stability of phase III to a narrow window of pressure and to temperatures above 400 K. We summarize the results of the free-energy calculations for the three molecular phases II, III, and IV in Fig. 4. Phases I and II emerge as the only stable molecular phases of CO₂ from zero to ambient temperature. Phases III and IV are both stabilized by temperature, and phases II, III, and IV coexist at a triple point located at 15 GPa and 500 K.

Our findings are in agreement with simulations by Bonev *et al.* [48] which suggested that the *Cmca* phase is a temperature stabilized form [48]. Because the structure of phase IV was not known at the time, Bonev *et al.* proposed a wider region of stability for CO₂-III. Interestingly, as can be seen in Fig. 3(a), the *P-T* region of stability of CO₂-III obtained from our calculations has a large overlap with the region of stability reported for the so-called phase VII of CO₂ [13]. A recent theoretical work has shown that phases III and VII have, in fact, the same crystal structure (space group *Cmca*) [14]. Therefore, we confirm that phase III is thermodynamically stable in the *P-T* region where phase VII has been reported. Thus, the observation of phase III outside this region (e.g., at ambient conditions, as a result of the compression of phase I) must be attributed to kinetic effects.

Now, we turn to the boundary between the molecular phases and nonmolecular phase V [Fig. 3(b)]. We find that, at zero temperature, the phase boundary between CO₂-II

and CO₂-V is located at 21.5 GPa. The transition between (metastable) CO₂-III and CO₂-V would, instead, take place at 20.8 GPa in the absence of kinetic effects. This is in good agreement with previous theoretical works [28,30,33]. Phase boundaries between molecular phases and CO₂-V are rather insensitive to the choice of the molecular structure and they all have a positive slope, as already suggested [31]. Considering that nonmolecular phases are denser than molecular ones, a positive slope implies a decrease of entropy in going from molecular to nonmolecular. This is not unexpected, given the stiffness of

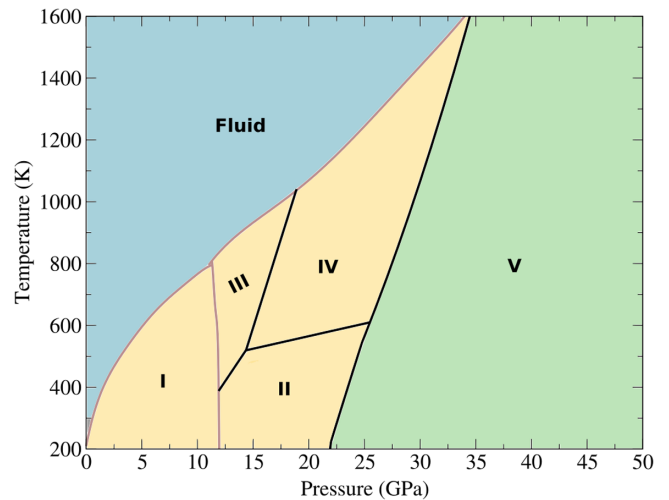


FIG. 4. Theoretical phase diagram for carbon dioxide at high pressure and temperature. Our calculated phase boundaries are shown with solid black lines, while previously reported thermodynamic boundaries are shown in gray. Yellow, green, and blue regions correspond to molecular, nonmolecular, and fluid forms of CO₂.

the nonmolecular structure when compared with the molecular ones. Using the experimentally determined melting line, our calculations show a triple point between phases IV and V, and the liquid phase at 35 GPa and 1600 K. Therefore, the calculations suggest that molecular CO₂ could be stable up to pressures as high as 35 GPa, at high temperature.

In summary, we have presented finite-temperature theoretical calculations in the quasiharmonic approximation for various molecular and nonmolecular solid forms of CO₂. The calculations aimed at resolving experimental uncertainties and inconsistencies due to kinetic effects and metastability. We find that the boundary between the molecular phases and phase V has a positive slope, and starts at 21.5 GPa at $T = 0$ K. We also find that the phase diagram shows a triple point between phases IV, V, and the liquid phase at 35 GPa and 1600 K. This indicates that the nonmolecular phase V has a broader region of stability than previously reported. We were able to reproduce the known thermodynamic boundary line between CO₂-II and CO₂-IV, confirming that kinetic effects are not relevant in that transition. Finally, it was shown that phase II is the most stable molecular phase at low temperatures, extending its region of stability to every P - T condition where phase III has been reported experimentally. However, our results also show that CO₂-III is, instead, stabilized at high temperature and its stability region coincides with the P - T conditions where phase VII has been reported experimentally, implying that phase III and phase VII are, indeed, the same.

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