# Departing from Thermality of Analogue Hawking Radiation in a Bose-Einstein Condensate

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(Received 5 September 2019; accepted 9 January 2020; published 10 February 2020)

We study the quantum fluctuations in a one-dimensional Bose-Einstein condensate realizing an analogous acoustic black hole. The taking into account of evanescent channels and of zero modes makes it possible to accurately reproduce recent experimental measurements of the density correlation function. We discuss the determination of Hawking temperature and show that in our model the analogous radiation presents some significant departure from thermality.

DOI: 10.1103/PhysRevLett.124.060401

The Hawking effect [1] being of kinematic origin [2] can be transposed to analogue systems, as first proposed by Unruh [3]. Among the various platforms that have been proposed for observing induced or spontaneous analogous Hawking radiation and related phenomena, the ones for which the experimental activity is currently the most intense are surface water waves [4–10], nonlinear light [11–17], excitonic polaritons [18], and Bose-Einstein condensed atomic vapors [19–22].

Because of their low temperature, their intrinsic quantum nature, and the high experimental control achieved in these systems, Bose-Einstein condensates (BECs) seem particularly suitable for studying analogue Hawking effect. Steinhauer and colleagues have undertaken several studies of quasiunidimensional configurations, making it possible to realize analogue black hole horizons in BEC systems, and made claims of observation of Hawking radiation [20–22]. Their results have triggered the interest of the community [23–33] and generated a vivid debate [34,35]. One of the goals of the present Letter is to contribute to this debate, and to partially close it, at least in what concerns density correlations around an analogue black hole horizon. A definite theoretical answer can be obtained thanks to a remark that had been overlooked in previous works: one needs to develop the quasiparticle operator on a complete basis set for properly describing the density fluctuations. This is achieved in the first part of this Letter, and we apply this theoretical approach to the analysis of the experimental results of Ref. [22].

While in general relativity the thermality of the Hawking radiation is constrained by the laws of black hole thermodynamics, no such general principle is expected to hold for analogue systems [2]. It is nonetheless commonly accepted that the spectrum of analogous Hawking radiation only weakly departs from thermality [36–38], and that all relevant features of an analogue system can be understood on the basis of a hydrodynamical, long wavelength description. However, the phenomenology of analogous systems provides mechanisms supporting the impossibility of a perfectly thermal analogue Hawking radiation [39]. In the second part of this Letter we argue that in the BEC case we are considering, it is legitimate to determine a Hawking temperature from the information encoded in the density correlation function, but we show that some features of the radiative process at hand significantly depart from thermality and propose a procedure for confirming our view.

We consider a one-dimensional configuration in which the quantum field $\hat{\Psi}(x,t)$ is a solution of the Gross-Pitaevskii equation

$$i\hbar\partial_t\hat{\Psi} = -\frac{\hbar^2}{2m}\partial_x^2\hat{\Psi} + [g\hat{n} + U(x)]\hat{\Psi}. \tag{1}$$

In this equation $m$ is the mass of the atoms, $\hat{n} = \hat{\Psi}^\dagger\hat{\Psi}$, and the term $g\hat{n}$ describes the effective repulsive atomic interaction ($g > 0$). We have studied several external potentials $U(x)$ making it possible to engineer a sonic horizon, but we only present here the results for a step function: $U(x) = -U_0\Theta(x)$ with $U_0 > 0$. The reason for this choice is twofold: (i) This potential has been realized experimentally in Refs. [21,22]; (ii) from the three configurations analyzed in Ref. [31], this is the one that leads to the signal of quantum nonseparability which is the largest and the most resilient to temperature effects.

In the spirit of Bogoliubov's approach, we write the quantum field as

$$\hat{\Psi}(x,t) = \exp(-i\mu t/\hbar)[\Phi(x) + \hat{\psi}(x,t)], \tag{2}$$

where $\mu$ is the chemical potential. $\Phi(x)$ is a classical field describing the stationary condensate and $\hat{\psi}(x,t)$ accounts for small quantum fluctuations. Although such a separation is not strictly valid in one dimension, it has been argued in Ref. [31] that it constitutes a valid approximation over a large range of one-dimensional densities. In the case we

 

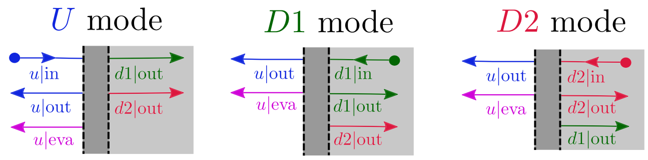


FIG. 1. Sketch of the different channels contributing to the incoming quantum modes $U$, $D1$, and $D2$. In each plot the background BEC propagates from left to right, the white region corresponds to the upstream subsonic flow, the gray one to the interior of the analogous black hole (downstream supersonic flow), and the region of the horizon is represented by the dark gray shaded interface. The Hawking channel and its partner are labeled $u|$out and $d2|$out. The $d1|$out channel is a companion propagating away from the horizon, inside the analogous black hole region. Each mode ($U$, $D1$, and $D2$) is seeded by an ingoing channel ($u|$in, $d1|$in, and $d2|$in) whose group velocity is directed towards the horizon.

consider, $\Phi$ is a solution of the classical Gross-Pitaevskii equation describing a sonic horizon: the $x<0$ profile is half a dark soliton [40], with $\Phi(x\to-\infty)=\sqrt{n_u}\exp(ik_u x)$, where $n_u$ and $V_u=mk_u/\hbar$ ($>0$) are the upstream asymptotic density and velocity, respectively. The downstream ($x>0$) flow of the condensate corresponds to a plane wave: $\Phi(x>0)=\sqrt{n_d}\exp(ik_d x-i\pi/2)$. The asymptotic upstream and downstream sound velocities are $c_{(u,d)}=\sqrt{gn_{(u,d)}/m}$. The analogous black hole configuration corresponds to a flow that is asymptotically upstream subsonic ($V_u<c_u$) and downstream supersonic ($\hbar k_d/m=V_d>c_d$).

We describe the quantum fluctuations on top of this classical field within a linearized approach. The relevant modes are identified by using the asymptotic ingoing (i.e., directed towards the acoustic horizon) and outgoing channels, far from the horizon. As discussed in previous references [41–45] and recalled in [46], the Bogoliubov dispersion relation supports a decomposition of $\hat\psi$ onto three incoming modes that we denote as $U$, $D1$, and $D2$. For instance, the $U$ mode is seeded by an upstream incoming wave that we denote as $u|$in, which propagates towards the horizon with a long wavelength group velocity $V_u+c_u$. It is scattered onto two outgoing transmitted channels (propagating in the analogue black hole away from the horizon), which we denote as $d1|$out and $d2|$out with respective long wavelength group velocities $V_d+c_d$ and $V_d-c_d$ (both positive) and one outgoing reflected channel (propagating away from the horizon, outside of the analogue black hole, with long wavelength group velocity $V_u-c_u<0$). The corresponding three scattering coefficients are denoted as $S_{d1,u}$, $S_{d2,u}$, and $S_{u,u}$. There is also an upstream evanescent wave ($u|$eva) that carries no current, does not contribute to the $S$ matrix, but is important for fulfilling the continuity relations at $x=0$. The situation is schematically depicted in Fig. 1.

The frequency-dependent boson operators associated to the three incoming modes $U$, $D1$, and $D2$ are denoted as $\hat b_U$, $\hat b_{D1}$, and $\hat b_{D2}$; they obey the commutation relations $[\hat b_L(\omega),\hat b^\dagger_{L'}(\omega')]=\delta_{L,L'}\delta(\omega-\omega')$. In addition, Bose-Einstein condensation is associated with a spontaneously broken $U(1)$ symmetry that implies the existence of supplementary zero modes of the linearized version of (1). As discussed in Ref. [50], one is lead to introduce two new operators $\hat{\mathcal{P}}$ and $\hat{\mathcal{Q}}$ accounting for the global phase degree of freedom, and the correct expansion of the quantum fluctuation field reads

$$\begin{aligned}\hat\psi(x,t)={}&-i\Phi(x)\hat{\mathcal{Q}}+iq(x)\hat{\mathcal{P}}+\int_0^\infty\frac{d\omega}{\sqrt{2\pi}}\sum_{L\in\{U,D1\}}\\&\times[u_L(x,\omega)e^{-i\omega t}\hat b_L(\omega)+v_L^*(x,\omega)e^{i\omega t}\hat b_L^\dagger(\omega)]\\&+\int_0^\Omega\frac{d\omega}{\sqrt{2\pi}}[u_{D2}(x,\omega)e^{-i\omega t}\hat b^\dagger_{D2}(\omega)\\&+v^*_{D2}(x,\omega)e^{i\omega t}\hat b_{D2}(\omega)].\end{aligned}\tag{3}$$

In this expression the $u_L$'s and $v_L$'s are the usual Bogoliubov coefficients (their explicit form is given for instance in Ref. [44]), and the quantization of the $D2$ mode is atypical, as discussed in several references [41,42,51]. The function $q(x)$ is one of the components of the zero eigenmodes; see [46]. Omitting the contribution of the zero mode operators $\hat{\mathcal{P}}$ and $\hat{\mathcal{Q}}$ would correspond to using an incomplete basis set for the expansion of the quantum fluctuations; in other words, their contribution is essential for verifying the correct commutation relation $[\hat\psi(x,t),\hat\psi^\dagger(y,t)]=\delta(x-y)$. The operator $\hat{\mathcal{Q}}$ is associated with the global phase of the condensate. $\hat{\mathcal{P}}$ is the canonical conjugate operator ($[\hat{\mathcal{Q}},\hat{\mathcal{P}}]=i$) that typically appears in the quadratic Hamiltonian $\hat H_{\rm quad}$ describing the dynamics of the quantum fluctuations with a $\hat{\mathcal{P}}^2$ contribution, while $\hat{\mathcal{Q}}$ does not [50,52,53]. This means that the degree of liberty associated with the broken symmetry has no restoring force—as expected on physical grounds—and that the zero excitation quantum state $|{\rm BH}\rangle$ describing the analogous black hole configuration verifies $\hat{\mathcal{P}}|{\rm BH}\rangle=0$ and $\hat b_L(\omega)|{\rm BH}\rangle=0$ for $L\in\{U,D1,D2\}$.

Once the appropriate expansion (3) has been performed, and the correct quantum state $|{\rm BH}\rangle$ has been identified, one can compute the density correlation function,

$$\begin{aligned}G_2(x,y)&=\langle:\hat n(x,t)\hat n(y,t):\rangle-\langle\hat n(x,t)\rangle\langle\hat n(y,t)\rangle\\&\simeq\Phi(x)\Phi^*(y)\langle\hat\psi^\dagger(x,t)\hat\psi(y,t)\rangle\\&\quad+\Phi(x)\Phi(y)\langle\hat\psi^\dagger(x,t)\hat\psi^\dagger(y,t)\rangle+{\rm c.c.}\end{aligned}\tag{4}$$

In this equation, the symbol $:$ denotes normal ordering and the final expression is the Bogoliubov evaluation of $G_2$, encompassing the effects of quantum fluctuations at leading order. At zero temperature, the average $\langle\cdots\rangle$ in Eq. (4) is taken over the state $|{\rm BH}\rangle$. Although this state is thermodynamically unstable and cannot support a thermal

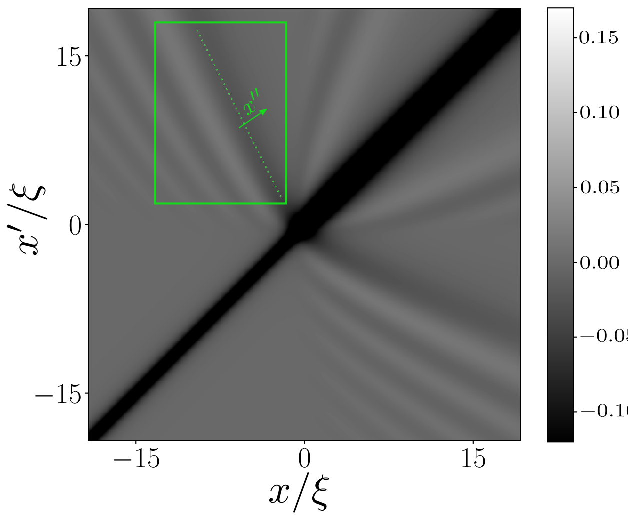


FIG. 2. Intensity plot of the dimensionless correlation function $\xi(n_u n_d)^{-1/2} G_2(x, x')$ for $x$ and $x'$ close to the horizon. The parameter $\xi = \sqrt{\xi_u \xi_d}$ is the geometrical mean of the healing lengths $\xi_u$ and $\xi_d$, where $\xi_{(u/d)} = \hbar(mgn_{(u,d)})^{-1/2}$. The line of anticorrelation in the upper left and lower right quadrants corresponds to the merging close to the horizon of the Hawking-partner $(u|\text{out} - d2|\text{out})$ and Hawking-companion $(u|\text{out} - d1|\text{out})$ correlations. The green rectangle delimits the region where we average $G_2$ for comparison with experimental data (see Fig. 3).

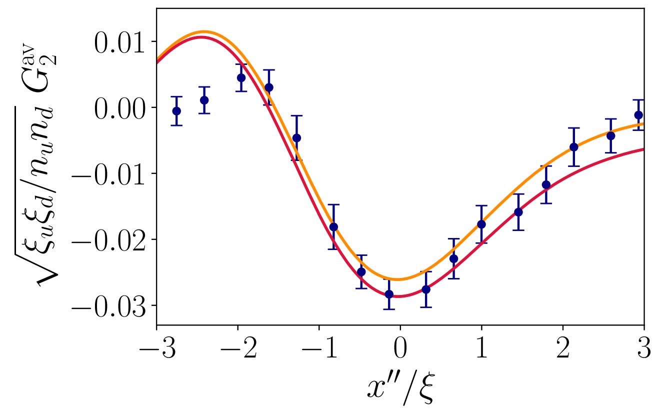


FIG. 3. Red solid line: zero temperature density correlation function $G_2^{\rm av}(x, x')$ plotted as a function of $x''$. The blue dots with error bars are the results of Ref. [22]. The orange solid line is the finite temperature result for $k_B T = 0.2gn_u$, i.e., $T \simeq 1.9T_H$.

distribution, finite temperature effects can still be included as explained for instance in Refs. [31,41,42].

In 2008 a collaboration between teams from Bologna and Trento [54,55] pointed out that, in the presence of a horizon, $G_2$ should exhibit nonlocal features resulting from correlations between the different outgoing channels, in particular, between the Hawking quantum and its partner ($u|\text{out} - d2|\text{out}$ correlation in our terminology). The importance of this remark lies in the fact that, due to the weak Hawking temperature $T_H$ (at best one fourth of the chemical potential [44]), the direct Hawking radiation is expected to be hidden by thermal fluctuations, whereas density correlations should survive temperature effects in typical settings [42]. This idea has been used to analyze the Hawking signal in Ref. [22], where a stationary correlation pattern was measured in the vicinity of the horizon. In this region, it is important for a theoretical treatment to account for the position dependence of the background density and to include the contribution of the evanescent channels in the expansion (3). We also checked that it is essential to take into account the contribution of the zero modes to obtain a sensible global description of the quantum fluctuations. The corresponding two-dimensional plot of the density correlation pattern is represented in Fig. 2. $G_2$ has been computed at zero temperature, for $V_d/c_d = 2.90$, which imposes $V_u/c_u = 0.59$ [44,46]. This value is chosen to reproduce the experimental configuration studied in Ref. [22]. The dotted line in the upper left quadrant of Fig. 2 marks the anticorrelation curve that results from the Hawking-partner $(u|\text{out} - d2|\text{out})$ and Hawking-companion $(u|\text{out} - d1|\text{out})$ correlations. We find that these two correlation lines, which separate at large distance from the horizon [42,44,55], merge close to the horizon, as also observed experimentally.

A precise comparison of our results with experiment can be achieved by following the procedure used in Ref. [22], which consists in averaging $G_2$ over the region inside the green rectangle represented in Fig. 2. One defines a local coordinate $x''$ that is orthogonal to the locus of the minima of $G_2$, and one plots the averaged $G_2$ (denoted as $G_2^{\rm av}$) as a function of the variable $x''$. This is done in Fig. 3. We insist that the good agreement between our approach and the experimental results can only be achieved through a correct description of the quantum fluctuations—Eq. (3)—including the contribution of zero modes and evanescent channels.

It has been noticed by Steinhauer [56] that the determination of $G_2(x, x')$ in the upper left (or lower right) quadrant of the $(x, x')$ plane makes it possible to evaluate the Hawking temperature thanks to the relation

$$S_{u,d2}(\omega) S^*_{d2,d2}(\omega) = \langle \hat{c}_U(\omega) \hat{c}_{D2}(\omega) \rangle = \frac{\mathcal{S}_0^{-1}}{\sqrt{n_u n_d L_u L_d}} \int_{-L_u}^{0} dx \int_0^{L_d} dx' e^{-i(k_H x + k_P x')} G_2(x, x'). \tag{5}$$

In this expression $S$ is the matrix that describes the scattering of the different channels onto each other, and $\mathcal{S}_0(\omega) = (u_{k_H} + v_{k_H})(u_{k_P} + v_{k_P})$ is the static structure factor, where the $u_k$'s and the $v_k$'s are the standard Bogoliubov amplitudes of excitations of momentum $k$ (see, e.g., Refs. [57,58]). The $\hat{c}_L$'s are outgoing modes related to the incoming ones by the $S$ matrix [42]

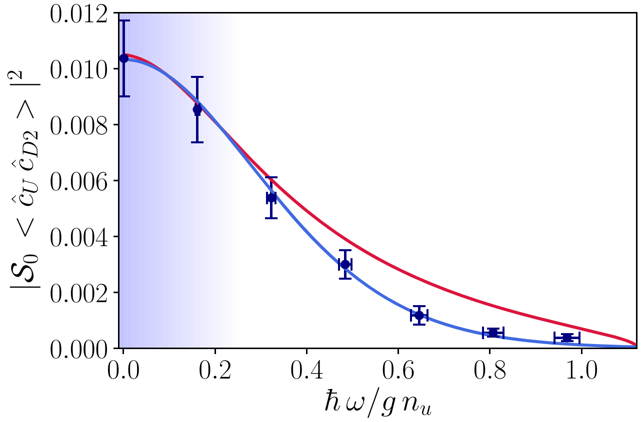


FIG. 4. Hawking-partner correlation signal represented as a function of the dimensionless energy. The red solid curve is the theoretical result from Eq. (5). The dots with error bars are from Ref. [22]. They are obtained after processing the experimental result for $G_2$ by means of the Fourier transform (5). The blue region corresponds to a domain where the ratio of Hawking and partner wave vectors is equal to its long wavelength value within a 10% accuracy. The blue solid curve is the theoretical result obtained by neglecting dispersive effects in Eq. (5) and discarding the contribution of the companion $d1|\text{out}$ channel (see the text).

$$\begin{pmatrix} \hat{c}_U \\ \hat{c}_{D1} \\ \hat{c}_{D2}^\dagger \end{pmatrix} = \begin{pmatrix} S_{u,u} & S_{u,d1} & S_{u,d2} \\ S_{d1,u} & S_{d1,d1} & S_{d1,d2} \\ S_{d2,u} & S_{d2,d1} & S_{d2,d2} \end{pmatrix} \begin{pmatrix} \hat{b}_U \\ \hat{b}_{D1} \\ \hat{b}_{D2}^\dagger \end{pmatrix}. \quad (6)$$

The Fourier transform of $G_2$ in Eq. (5) is performed at fixed $\omega$, for wave vectors $k_H(\omega)$ and $k_P(\omega)$, which are the momenta relative to the condensate of a Hawking quantum and its partner ($u|\text{out}$ and $d2|\text{out}$ channels in our terminology) having an energy $\hbar\omega$ in the laboratory frame. The integration region $[-L_u, 0] \times [0, L_d]$ lies in the upper left quadrant of Fig. 2, and should be adapted for each value of $\omega$ in such a way that [31,59]

$$\frac{L_u}{|V_{g,H}(\omega)|} = \frac{L_d}{V_{g,P}(\omega)}, \quad (7)$$

where $V_{g,H}(\omega)$ $[V_{g,P}(\omega)]$ is the group velocity of a Hawking quantum (of a partner) of energy $\hbar\omega$. We have checked that once the prescription (7) is fulfilled, formula (5) is very well verified [46]. It is then intriguing to observe that, while theory and experiment both agree on the value of $G_2$ in real space (Fig. 3), they do not for the correlation $\langle \hat{c}_U(\omega)\hat{c}_{D2}(\omega)\rangle$: as can bee seen in Fig. 4, the agreement is restricted to the low energy regime. This is the bluish region in the figure, which corresponds to a domain where the ratio $k_H(\omega)/k_P(\omega)$ is equal to its long wavelength value $(c_u - V_u)/(c_d - V_d)$ with an error less that 10%.

Let us discuss this discrepancy in some detail. The interest of Eq. (5) lies in the fact that the scattering matrix coefficient $S_{u,d2}$ is the equivalent of the Hawking $\beta$ parameter: its squared modulus is expected to behave as a Bose thermal distribution $n_{T_H}(\omega)$ with an effective temperature $T_H$, the Hawking temperature [1]. In an analogous system such as ours, because of dispersive effects, this equivalence is only valid in the long wavelength limit, typically in the blue region of Fig. 4. This suggests a possible manner to reconcile theory and experiment: we assume that the ratio $k_H(\omega)/k_P(\omega)$ is $\omega$ independent and equal to its low energy value, $(c_u - V_u)/(c_d - V_d)$ (this value is denoted as $\tan\theta$ in Refs. [21,22]). We also assume that, in the scattering process schematically illustrated in Fig. 1 for the $D2$ mode, the companion $d1|\text{out}$ channel plays a negligible role, so that the $|S_{d1,d2}|^2$ term can be omitted in the normalization condition $|S_{d2,d2}|^2 = 1 + |S_{u,d2}|^2 + |S_{d1,d2}|^2$ of the $S$ matrix (see, e.g., Ref. [42]). Then one obtains

$$|S_{u,d2}|^2 |S_{d2,d2}|^2 \simeq n_{T_H}(\omega)[1 + n_{T_H}(\omega)]. \quad (8)$$

Using the experimental values from Ref. [22] for $V_\alpha$ and $c_\alpha$ ($\alpha \in \{u, d\}$) and for the Hawking temperature $T_H$ leads, within approximation (8), to the blue curve of Fig. 4 that agrees with the results published in Ref. [22] (blue dots with error bars). It is important to note that this procedure is self-consistent in the following sense: If one performs numerically the Fourier transform (5) over a domain that, instead of fulfilling the relation (7), verifies the $\omega$-independent condition $L_u/|V_u - c_u| = L_d/(V_d - c_d)$, appropriate in a nondispersive, long wavelength approximation, one obtains a result (not shown for legibility, but see [46]) close to a thermal spectrum, i.e., to the blue curve in Fig. 4. Although this procedure is self-consistent, it is not fully correct, as can be checked by the fact that the resulting value of $\langle \hat{c}_U(\omega)\hat{c}_{D2}(\omega)\rangle$ only agrees with the exact one (red curve in Fig. 4) in the long wavelength limit. Stated differently: this procedure leads to the erroneous conclusion that the radiation is fully thermal. However, since all approaches coincide in the long wavelength regime (blue colored region of Fig. 4), they all lead to the correct determination of the Hawking temperature. For a flow with $V_d/c_d = 2.9$, our theoretical treatment yields $k_B T_H/(g n_u) = 0.106$, whereas the experimental value reported for this quantity in Ref. [22] is 0.124 (corresponding to a Hawking temperature $T_H = 0.35$ nK).

In conclusion, our work sheds a new light on the study of quantum correlations around an analogous black hole horizon, and on the corresponding Hawking temperature. From a theoretical point of view, we argue that the contribution of zero modes is essential for constructing a complete basis set necessary to obtain an accurate description of the quantum fluctuations. This claim is supported by the excellent agreement we obtain when comparing our results with recent experimental ones. On the experimental side, we substantiate the determination of the Hawking temperature presented in Ref. [22], although we find that the Hawking spectrum is not thermal for all wavelengths. We identify a natural but unfounded procedure for analyzing the information encoded in $G_2(x, x')$ that leads to the opposite conclusion; we show that, within our approach, an alternative analysis of the correlation pattern accurately accounts for nonhydrodynamical effects. It would thus be interesting to reanalyze the data published in Ref. [22] to investigate if the windowing (7) we propose for Eq. (5) modifies the experimental conclusion for the Hawking-partner correlation signal and confirms the departure from thermality we predict.

We acknowledge fruitful discussions with I. Carusotto, M. Lewenstein, and J. Steinhauer, whom we also thank for providing us with experimental data.

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