Prediction of a Paramagnetic Meissner Effect in Voltage-Biased Superconductor–Normal-Metal Bilayers

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Conventional superconductors respond to external magnetic fields by generating diamagnetic screening currents. However, theoretical work has shown that one can engineer systems where the screening current is *paramagnetic*, causing them to *attract* magnetic flux—a prediction that has recently been experimentally verified. In contrast to previous studies, we show that this effect can be realized in simple superconductor-normal-metal structures with no special properties, using only a simple voltage bias to drive the system out of equilibrium. This is of fundamental interest, since it opens up a new avenue of research, and at the same time highlights how one can realize paramagnetic Meissner effects without having odd-frequency states at the Fermi level. Moreover, a voltage-tunable electromagnetic response in such a simple system may be interesting for future device design.

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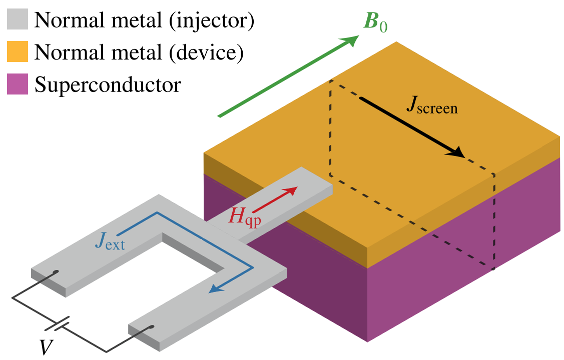
Introduction.—The defining properties of conventional superconductors [1,2] are their perfect conductance of electric currents and the Meissner effect, whereby dissipationless electric currents screen magnetic fields. Both properties arise due to a coherent condensate of electron pairs (Cooper pairs) which exhibits spontaneous symmetry breaking, and it is of fundamental interest to understand both in depth.

In bulk superconductors, the Meissner effect is *diamagnetic*, meaning that the screening currents try to expel magnetic flux. The story is more complicated in *proximity* structures, where superconductors and nonsuperconductors are combined to engineer novel device functionality. Diamagnetic screening in such structures has been investigated [3,4], and several interesting impurity effects have been found [5,6]. A cylindrical geometry can increase the diamagnetic so-called overscreening [7]. At ultralow temperatures, a reentrant effect was observed experimentally [8], and even an overall paramagnetic response in thermal equilibrium [9]. Other systems with unexpected properties are superconductor/ferromagnet (S/F) devices, where Cooper pairs can leak from S to F. The Cooper pairs of a conventional superconductor are singlet even-frequency pairs, i.e., they carry no net spin and respect time-permutation symmetry. Once they leak into F, some of these are converted into triplet odd-frequency pairs, which have fundamentally different properties [10–17]. One example is that odd-frequency pairs can give rise to a *paramagnetic* Meissner effect, where the screening currents *attract* magnetic flux [17–23]. This effect has been predicted for a variety of S/F setups, and has been confirmed experimentally via muon-rotation experiments [24]. It has also been predicted in, e.g., metals with repulsive electron–electron interactions [25] and at the interfaces of d-wave superconductors [26,27]. In these systems, the effect is caused by midgap states which are linked to odd-frequency pairing [17].

We consider a fundamentally different way of realizing the paramagnetic effect: by driving a superconductor-normal-metal (S/N) bilayer out of equilibrium via a voltage bias. Our suggested setup is visualized in Fig. 1, and explained in detail in what follows. The mechanism is again related to odd-frequency superconductivity; we will see that an essential ingredient is large subgap peaks in the normal-metal density of states (DOS), and these appear at energies where odd-frequency pairs dominate [17,28]. However, our setup does not require that these reside precisely at the Fermi level (midgap states). Instead, the Meissner response in our setup is determined by the DOS at a voltage-controlled finite energy. Our predictions can be verified via the same setup as Ref. [24]. The excited distribution decays over the inelastic scattering length, which can be several micrometers at low temperatures [29]. This is the limiting factor for the lateral dimensions of the device.

A related idea was discussed in Refs. [30,31], where they suggested that a microwave-irradiated superconductor might become paramagnetic. However, they concluded that the paramagnetic state would be unstable, and could therefore not be realized. In contrast, our system avoids this instability by realizing the paramagnetic effect in a proximity system instead of a bulk system. Moreover, voltage control may be more desirable than microwave control for potential applications.

A similar setup to ours was investigated in Ref. [32], where they calculated the Meissner response of S/N



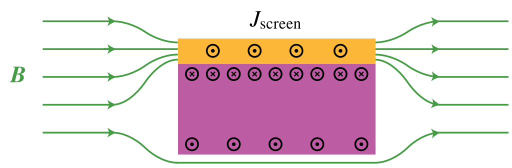


FIG. 1. Top figure: Suggested experimental setup. The left end features a quasiparticle injector (gray). The voltage source forces an electric current J_{ext} through a normal-metal wire, causing an excess of electrons and holes to accumulate in the middle of the wire. This drives a diffusion H_{qp} of excess quasiparticles onto an adjacent normal-metal film (yellow), thus driving it out of equilibrium. Note that no charge is injected into the yellow device by the voltage source: the gray wire connects two reservoirs at $\pm V/2$, so its midpoint has zero net charge accumulation. The diffusion H_{qp} thus consists of an equal number of electrons and holes. This film is also proximitized by a conventional superconductor underneath (purple), causing Andreev bound states to form there. The combination results in a paramagnetic effect, whereby an external magnetic field $\boldsymbol{B}_0$ is *enhanced* by the screening currents in the normal metal. Since whether the film reacts dia- or paramagnetically depends on the voltage, the device can be tuned between these Meissner responses *in situ*. Bottom figure: cross-sectional view of the device during operation, showing how the magnetic field $\boldsymbol{B}$ is deformed by the screening currents J_{screen}. The normal metal can have a paramagnetic response, whereby it attracts magnetic flux. The superconductor underneath remains diamagnetic, and therefore expels magnetic flux.

structures driven out of equilibrium using a voltage-controlled quasiparticle injector. However, they did *not* find a paramagnetic response, and the main reason appears to be their parameters. We analytically predict the effect for clean materials, thick superconductors, and low temperatures. In contrast, Ref. [32] considered dirty materials, thin superconductors, and high temperatures. This suppresses the subgap peaks in the DOS which are essential for a paramagnetic response.

Motivation.—Consider a superconducting system that is exposed to a weak magnetic field $\boldsymbol{B} = \nabla \times \boldsymbol{A}$, which we describe via a vector potential $\boldsymbol{A}$. For concreteness, let us consider a geometry where a thin film at $0 < z < d$ is subjected to a magnetic field $\boldsymbol{B} \sim \boldsymbol{e}_y$, which we describe via the vector potential $\boldsymbol{A} = A(z)\boldsymbol{e}_x$. This is identical to the experimental geometry employed in Ref. [24]. In the clean and nonlocal limits, the linear-response screening current is given by [3,33]

$$\boldsymbol{J} = -K\langle \boldsymbol{A} \rangle_z = -\frac{K}{d}\int_0^d \mathrm{d}z' \boldsymbol{A}(z'). \qquad (1)$$

Here, we have introduced the screening kernel

$$K = \frac{1}{3}e^2 v_F^2 \left[N_F - \int_{-\infty}^{+\infty} \mathrm{d}\varepsilon N(\varepsilon) \left(-\frac{\partial f}{\partial \varepsilon} \right) \right], \qquad (2)$$

where ε is the quasiparticle energy, $f(\varepsilon)$ the distribution function, $N(\varepsilon)$ the DOS, N_F the Fermi-level DOS in the nonsuperconducting state, v_F the Fermi velocity, and e the electron charge. Note that $\boldsymbol{J}$ is not a function of position z in the nonlocal limit. These equations are derived in standard textbooks on superconductivity [1,34], and have previously been used to, e.g., predict paramagnetic effects in materials with repulsive electron interactions [25], d-wave superconductors [26], and microwave-irradiated superconductors [30,31]. We provide a simple and compact derivation of this equation within the quasiclassical formalism in the Supplemental Material [35].

Many well-known results for Meissner effects can be seen directly from Eqs. (1) and (2). In equilibrium, the distribution has a Fermi-Dirac form, which at low temperatures reduces to a step function $f(\varepsilon) \approx \theta(-\varepsilon)$. Substituted into Eq. (2), this produces the simplified equation $K \sim N_F - N(0)$. For a BCS superconductor, there is a gap around the Fermi level $\varepsilon = 0$, and $N(0) = 0$ causes $K > 0$. This produces a diamagnetic response. On the other hand, in systems with odd-frequency pairing, one can have a zero-energy peak in the DOS, and $N(0) > N_F$ causes $K < 0$. This produces a paramagnetic response.

We are interested in a new way to realize the paramagnetic Meissner effect: by manipulating the distribution $f(\varepsilon)$ instead of the DOS $N(\varepsilon)$. Before we discuss its exact physical origin, let us just assume that one can induce a two-step Fermi-Dirac-like distribution, which at low temperatures reduces to

$$f(\varepsilon) \approx [\theta(+\Omega - \varepsilon) + \theta(-\Omega - \varepsilon)]/2. \qquad (3)$$

We note that the effect of Ω is essentially to excite electrons in the range $0 < \varepsilon < \Omega$ and holes in the range $-\Omega < \varepsilon < 0$, resulting in an excited energy mode or increased effective temperature. Substituting the above into Eq. (2), and using the electron-hole symmetry of the DOS $N(+\varepsilon) = N(-\varepsilon)$, we get

$$K = \frac{1}{3}e^2 v_F^2 [N_F - N(\Omega)]. \qquad (4)$$

In other words, if we can tune Ω, it is now sufficient that $N(\varepsilon) > N_F$ at *some* energy ε for us to realize a paramagnetic state. For example, consider the DOS of a BCS superconductor,

$$N_S(\varepsilon) = N_F \frac{|\varepsilon|}{\sqrt{\varepsilon^2 - \Delta_0^2}} \theta(|\varepsilon| - \Delta_0). \quad (5)$$

Clearly, the step function indicates that $N_S(\Omega) = 0$ within the gap $|\Omega| < \Delta_0$, resulting in a purely diamagnetic response there. However, if we can increase its value to $|\Omega| > \Delta_0$, suddenly we find that $N_S(\Omega) \gg N_F$ due to the BCS coherence peaks, resulting in a strong paramagnetic response instead. It would therefore be interesting if we could find a system where Ω could be tuned *in situ*, making it possible to actively toggle between diamagnetic and paramagnetic Meissner responses. Note that we use Δ to denote a general superconducting gap, while Δ_0 denotes the gap of a bulk superconductor at zero temperature. In Eq. (5), we therefore have $\Delta = \Delta_0$, but in general proximity systems we can get a position-dependent gap satisfying $0 \leq \Delta(z) \leq \Delta_0$.

Model system.—One way to realize the distribution in Eq. (3) is to voltage bias a normal-metal wire. At low temperatures, the distributions at the two ends of the voltage source are just $f_\pm(\varepsilon) \approx \theta(\pm eV/2 - \varepsilon)$, which we use as our boundary conditions. If the wire is short compared to the inelastic scattering length of the material, which diverges at low temperatures [43], the Boltzmann equation for the distribution reduces to a Laplace equation $\nabla^2 f = 0$ [36,37]. Near the center of the wire, the solution is just $f = (f_+ + f_-)/2$. In other words, this allows us to realize Eq. (3), where $\Omega = eV/2$ is a voltage-tunable control parameter. This result is robust to the presence of superconductivity and for resistive interfaces [38,39].

If the center of such a wire is now connected to a different material, the wire functions as a quasiparticle injector. Essentially, the electrons and holes that are excited in the normal-metal wire diffuse into the adjacent material, thus inducing the distribution $f = (f_+ + f_-)/2$ there as well. This is just one way to excite a distribution like in Eq. (3). Other alternatives that may be experimentally relevant include applying the voltage bias directly to the other material via tunneling contacts [38], or using microwaves to excite the quasiparticles [30,31]. We also note that Eq. (3) has previously been shown to induce other interesting effects in superconducting systems [6,38,39,44–54], including a superconducting transistor [6,44–47], and a loophole in the Chandrasekhar-Clogston limit [38].

If we could simply connect the quasiparticle injector to a BCS superconductor, the combination of Eqs. (4) and (5) should have a paramagnetic response for voltages $eV/2 > \Delta_0$. Unfortunately, for such large voltages, the superconducting state becomes energetically unfavourable [38,39,53,54], a phenomenon that is intimately related to Chandrasekhar-Clogston physics [38,54–56]. The solution is to consider S/N proximity systems, where we can produce peaks with $N(\varepsilon) > N_F$ at subgap energies $\varepsilon < \Delta_0/2$. Note that these peaks correspond to energies where odd-frequency pairing dominates [17,28]. In this way, we can induce a paramagnetic response in N, while S remains diamagnetic and stable. Figure 1 visualizes the experimental setup suggested based on the arguments above.

We take S to lie in $-\infty < z < 0$, and N to lie in $0 < z < d$. We emphasize that the interface at $z = d$ borders to vacuum. The system is assumed to be infinite and translation invariant in the xy plane and the effect of the quasiparticle injector has been included by having a two-step distribution function in the N layer. Furthermore, to make analytical progress, we assume that there is a negligible inverse proximity effect so that $\Delta(z) \approx \Delta_0 \theta(-z)$, that the S/N interface at $z = 0$ is completely transparent, that the normal-metal/vacuum interface at $z = d$ is specularly reflecting, and that the materials are clean. In these limits, the DOS in S is just given by Eq. (5). Assuming $\Delta \approx \Delta_0$ in S even outside of equilibrium should be reasonable: at low temperatures, this is known to hold for voltages $eV/2$ up to ∼70% of the bulk gap Δ_0 [39], and we are only interested in subgap voltages here. In N, the DOS has Andreev bound states below the gap, which for $\varepsilon \ll \Delta_0$ produces the DOS

$$N_N(\varepsilon) = N_F(\varepsilon/2\varepsilon_A)\psi_1(\lfloor \varepsilon/2\varepsilon_A + 1/2 \rfloor + 1/2), \quad (6)$$

where the Andreev energy $\varepsilon_A = \pi v_F/4d$ and ψ_1 is the trigamma function (see Fig. 2). Technically, Eq. (6) is only valid for $\varepsilon \geq 0$, but the negative-energy solution follows trivially from the symmetry $N_N(+\varepsilon) = N_N(-\varepsilon)$. We provide a complete derivation of this result within the quasiclassical formalism in the Supplemental Material [35]. This result was originally derived via the Bogoliubov–de Gennes formalism in Ref. [40]; their results are identical to ours in the limit $\varepsilon \ll \Delta_0$ if we use the series representation of the polygamma function. It is worth noting that for $\varepsilon < \varepsilon_A$, the result is just linear: $N_N(\varepsilon) = N_F(\pi^2/4)(\varepsilon/\varepsilon_A)$.

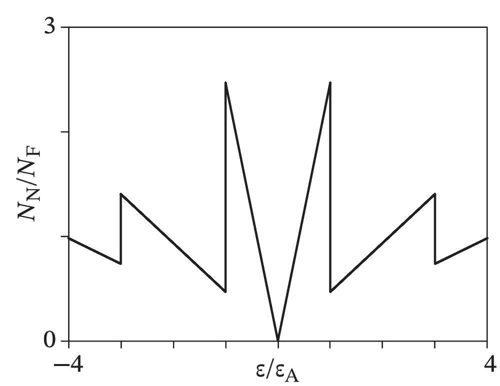


FIG. 2. Density of states in the normal metal. The energy is normalized to the Andreev energy, which for, e.g., $d = 3\xi$ would be $\varepsilon_A \approx \Delta_0/4$. Note that the peaks where $N > N_F$ correspond to energies where odd-frequency Cooper pairs dominate in the normal metal [17,28].

Let us now consider the screening kernels in this proximity system using Eq. (4) with $\Omega = eV/2$ and the densities of states derived above. In S, we have already established that $N_S(\Omega) = 0$ yields a purely diamagnetic response. This is usually described via the magnetic penetration depth $\lambda = 1/\sqrt{K_S}$,

$$K_S = \frac{1}{\lambda^2} = \frac{1}{3} e^2 v_F^2 N_F. \tag{7}$$

Equation (6) gives a more interesting expression,

$$K_N = \frac{1}{\lambda^2}[1 - (V/2V_A)\psi_1(\lfloor V/2V_A + 1/2 \rfloor + 1/2)], \tag{8}$$

where we reused the penetration depth λ defined in S, and introduced the Andreev voltage $V_A = 2\varepsilon_A/e = \pi v_F/2ed$.

Now that we have an expression for the screening kernels, we can solve the Maxwell equation $-\nabla^2 \boldsymbol{A} = \boldsymbol{J}$ together with the screening equation $\boldsymbol{J} = -K\langle \boldsymbol{A}\rangle_z$. As boundary conditions, we have $\nabla \times \boldsymbol{A}(d) = \boldsymbol{B}_0$ at the vacuum boundary, and $\boldsymbol{A}(-\infty) = 0$ deep inside S. We have considered a geometry where we can write $\boldsymbol{A} = A(z)\boldsymbol{e}_x$, which means that the applied magnetic field $\boldsymbol{B}_0 = A'(d)\boldsymbol{e}_y$. As an approximation, one might also set $\boldsymbol{A}(0) \approx 0$ to make analytical progress, meaning that S is assumed to perfectly screen fields near its interface. Thus, the equations for the gauge field inside N can be written

$$\begin{aligned} A''(z) &= K_N \langle A\rangle_z, \\ A'(d) &= B_0, \\ A(0) &= 0. \end{aligned} \tag{9}$$

The solution to the differential equation is $A(z) = az^2 + bz + c$ with $a = K_N\langle A\rangle_z/2$. The boundary conditions then provide the constraints $b = B_0 - 2ad$ and $c = 0$. Together, these yield

$$A(z) = K_N\langle A\rangle_z (z^2/2 - zd) + B_0 z. \tag{10}$$

We can then calculate the average $\langle A\rangle_z$. Using the moments $\langle z\rangle_z = d/2$ and $\langle z^2\rangle_z = d^2/3$, and solving for $\langle A\rangle_z$, we find

$$\langle A\rangle_z = \frac{B_0 d/2}{1 + K_N d^2/3}. \tag{11}$$

We now go back to Eq. (10) to calculate the magnetic field

$$B(z) = A'(z) = B_0 + K_N\langle A\rangle_z (z - d). \tag{12}$$

Substituting Eq. (11) into this result, we obtain an analytical result for the magnetic field inside N:

$$B(z) = B_0\left(1 + \frac{K_N(z-d)(d/2)}{1 + K_N d^2/3}\right). \tag{13}$$

The net magnetic field change $\Delta B = B(0) - B(d)$ induced by the screening currents can then be calculated as

$$\frac{\Delta B}{B_0} = -\frac{K_N d^2/2}{1 + K_N d^2/3}. \tag{14}$$

To obtain the final results, we just have to substitute in Eq. (8)

$$\frac{\Delta B}{B_0} = -\frac{\rho/2}{\lambda^2/d^2 + \rho/3},$$
$$\rho(V) = 1 - (V/2V_A)\psi_1(\lfloor V/2V_A + 1/2\rfloor + 1/2). \tag{15}$$

This provides us with a simple analytical result for the linear response ΔB of a clean proximitized metal to an applied field B_0. The result is expressed in terms of the Andreev voltage $V_A = \pi v_F/2ed$. This can be put into more familiar terms by introducing the superconducting coherence length $\xi = v_F/\Delta_0$; for instance, an N of length $d = 3\xi$ would yield $V_A \approx \Delta_0/2e$. This magnetic shift as a function of voltage is shown in Fig. 3.

Discussion.—Our main result is Eq. (15), which provides a simple analytical solution for the magnetic field shift ΔB that occurs for a given external magnetic field B_0 and voltage V. These predictions can be tested using a muon-rotation experiment to directly probe the local magnetic field at different points inside the device, using a similar setup as in Ref. [24].

The striking results are shown in Fig. 3, where we see that for sufficiently thick normal metals, ΔB appears to diverge as the voltage V approaches the Andreev voltage V_A. Since we considered a completely clean material at zero temperature, there is an abrupt transition between paramagnetism and diamagnetism as the voltage is increased beyond the Andreev voltage. In realistic systems, such sharp features are

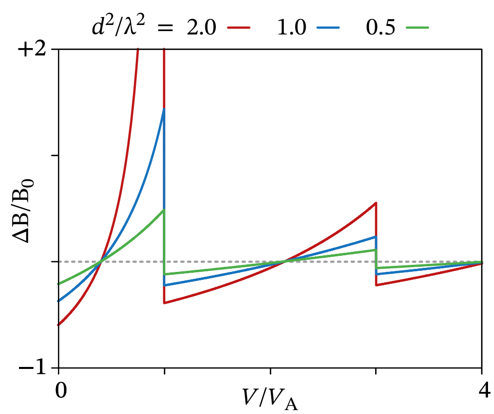


FIG. 3. Magnetic shift in the normal metal as a function of the applied voltage. Different curves correspond to different thicknesses d of the normal metal. Note that when $d > \lambda$, the paramagnetic effect produces a field $B_0 + \Delta B$ many times larger than the applied field B_0.

smeared by finite temperature, elastic scattering, and inelastic scattering. We provide some numerical results and discussion showing these effects of nonideal systems in the Supplemental Material [35].

The small ratio of the injector-N contact area to the full area of the N film allows one to ignore the injection process itself when considering the N/S bilayer. Since we assume a mean free path much larger than the size of the N film, the injected electrons spread over the whole film. We have also assumed perfect transparency at the S/N interface. Since a finite interface resistance can be expected to dampen the resonance peaks in the DOS of N, we would expect the paramagnetic Meissner effect to become weaker for opaque interfaces. We also note that in regions where $\Delta B \gg B_0$, a linear-response calculation is not technically valid anymore, and a full nonlinear-response calculation is warranted if one requires quantitatively rigorous results. Nevertheless, we would expect our results to remain qualitatively valid in such systems, and investigating this rigorously would be an interesting avenue for further research. For instance, to determine whether a spontaneous magnetic flux can appear or not would require a nonlinear-response calculation [25]. Another interesting proposition for further research would be to investigate whether a paramagnetic Meissner effect can be induced in dirty systems as well. While no such effect was detected in Ref. [32], they focused on high temperatures and thin superconductors, while the opposite limit may be the relevant one.

Conclusion.—Using a linear-response calculation, we have demonstrated how nonequilibrium effects can give rise to a paramagnetic Meissner response. Moreover, we have provided a specific experimental proposal where the magnetic response can be controlled *in situ* via an applied voltage. In addition to being relevant to the fundamental study of the Meissner effect and odd-frequency superconductivity, our results demonstrate a way to control the interaction between superconducting structures and magnetic fields via nonequilibrium effects which may be relevant to future superconducting device design.

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