Measurement-Device-Independent Verification of Quantum Channels

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The capability to reliably transmit and store quantum information is an essential building block for future quantum networks and processors. Gauging the ability of a communication link or quantum memory to preserve quantum correlations is therefore vital for their technological application. Here, we experimentally demonstrate a measurement-device-independent protocol for certifying that an unknown channel acts as an entanglement-preserving channel. Our results show that, even under realistic experimental conditions, including imperfect single-photon sources and the various kinds of noise—in the channel or in detection—where other verification means would fail or become inefficient, the present verification protocol is still capable of affirming the quantum behavior in a faithful manner with minimal trust on the measurement device.

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The ability to transmit and store quantum states and coherently manipulate the timing of photonic signals is a crucial requirement in quantum technologies [1]. Quantum communication links in combination with quantum memories form the quantum channels that offer these capabilities. These quantum channels thus play a pivotal role in enabling full scale quantum networks [2], promising unconditionally secure communication and the prospect of distributed quantum computing.

With the development of such quantum channels, especially quantum memories, comes the challenge of certifying their capabilities [3,4]. In particular, we seek the ability to discern a truly nonclassical channel from a cheap knock-off, such as a channel that simply measures the input state and approximately reprepares it at the output. While the latter could preserve some information about the state, it cannot preserve the exact quantum state nor any previously established correlations, rendering it useless for quantum applications. We denote channels of this sort as entanglement breaking (EB), in contrast to true quantum memories or coherent quantum channels, which preserve entanglement at least to some extent.

Consider an unknown channel that is claimed to be nonclassical, i.e., entanglement preserving. The most straightforward approach to obtain a complete characterization of the channel is a tomographic reconstruction of the channel's process matrix [5]. In practice, however, this approach is too resource intensive for all but low-dimensional channels, and further requires precise control and trust in all parts of the experiment. In most cases, such trust is undesirable or cannot be guaranteed at all. One way to overcome this is by using the correlations of entangled quantum systems, where a violation of a Bell inequality certifies the presence of entanglement even when the measurements are performed by untrusted black-box devices. Consequently, when considering an ideal scenario, Bell-test-based protocols allow for the verification of quantum channels in a so-called *device-independent* (DI) way, that is without requiring any trust in the experimental devices [6] even provide guarantees on the quality of the channel via self-testing [7]. On the flip side, these approaches cannot capture all nonclassical channels [8] and are subject to challenging loopholes [9–11] that make them very fragile to losses and experimentally difficult to implement. Moreover, in practice we rarely face the situation where nothing can be trusted such that a fully device-independent approach is necessary. Instead, while we might face an untrusted measurement device, we typically have access to a trust-worthy state preparation device allowing us to generate well-defined quantum states of our choice: a scenario commonly referred to as measurement-device-independent (MDI).

MDI schemes were first proposed in the context of quantum key distribution [12], and then applied to entanglement verification [13] for spatially separated subsystems within the framework of semiquantum games [14,15], providing several advantages over Bell tests [16]. These methods have since been extended to quantum steering [17] and to the analysis of entanglement structure [18] and its quantification [19–21], which has been demonstrated experimentally [22–24]. However, these schemes were solely focused on probing correlations within quantum states. More recently, it was shown that MDI approaches and semiquantum games can be repurposed to test the quantum properties of a channel, e.g., in situations where one party wants to test another party's ability to maintain the quantum properties of a system over time [11], such as in a quantum memory.

Here we demonstrate experimentally that MDI verification of quantum channels is a simple technique that is highly robust to experimental imperfections and viable with current technology. We study the performance and success probability of the method for channels suffering from depolarizing and dephasing noise, taking into account all experimental imperfections (such as imperfect state preparation when multiple copies of the input state are prepared), a problem that so far has not received sufficient attention. Under all conditions achievable with current technology, we find that the MDI approach outperforms Bell-test-based techniques in terms of resource requirements as well as noise resilience without the need for extra assumptions such as fair sampling. This method can thus certify a much wider range of channels and remains practical under realistic experimental conditions.

We now consider a typical experimental scenario, where a client (Alice) wishes to test a potentially dishonest quantum memory (provided by Bob) before deployment in a quantum network. Alice is assumed to possess a trusted preparation device, which is a scenario that naturally lends itself to the use of semiquantum games. Here, Alice repeatedly asks Bob a set of randomized "questions" by sending him quantum states and gets back a classical answer from Bob in every round. Bob is then asked to maximize a payoff function chosen to witness whatever quantum property Bob claims to possess. Since the questions are nonorthogonal, Bob merely knows the set of possible questions, but cannot know which question is asked in each round and thus cannot cheat. This method thus allows Alice to witness whether Bob possesses the claimed quantum property without having to trust him.

In each round, Alice sends successively two questions with a time delay between them, which forces Bob to store the first question until the second one arrives. In our notation, the first question is a state chosen at random from a finite set $\{\xi_x\}$ indexed by x, while the second question is chosen from a finite set $\{\psi_y\}$ indexed by y; both questions are sampled with uniform probability. After receiving the second question, Bob returns a classical answer b back to Alice. Bob is asked to maximize a prearranged payoff function $\omega(b,x,y)$ using the quantum channel $\mathcal{N}$ at his disposal. In analogy with Bell scenarios, we write the payoff then achieved $W=\sum_{bxy}\omega(b,x,y)P(b|xy)$. The combination of the coefficients $\omega(b,x,y)$ with the sets $\{\xi_x\}$ and $\{\psi_y\}$ is a temporal semiquantum game [11]. Every such game has an upper bound W_{EB} on the payoff achievable when Bob has only an entanglement-breaking channel at its disposal.

In our experiment, the sets $\{\xi_x\}$ and $\{\psi_y\}$ are identical and composed of symmetric informationally complete single-qubit quantum states which form a nonorthogonal basis of the Hilbert space with constant pairwise overlap of $1/3$ [25]. As shown in the Supplemental Material [26], other set of states can be chosen for the protocol with some implementation benefits. Nonorthogonality ensure that, although Bob knows the set of possible questions, he cannot with certainty know which questions are being asked in each round of the game. We write $x, y=0, 1, 2, 3$ the indices of these two successive questions ξ_x, ψ_y sent to Bob, while Bob sends back a classical answer $b=0, 1$. The property we are testing is a claim made by Bob that he possesses a nonentanglement-breaking channel corresponding to a Choi matrix Φ [28]. Accordingly, this Choi matrix is entangled, a fact that can be tested by an entanglement witness [29] F such that $\mathrm{tr}[F\Phi]>0$ while $\mathrm{tr}[F\Phi_{\mathrm{EB}}]\leq 0$ for Choi matrices of entanglement-breaking channels. Our payoff coefficients $\omega(b,x,y)$ are chosen so that $\omega(b=1,x,y)=0$, while $\omega(b=0,x,y)$ provides a decomposition of that witness $F=\sum_{xy}\omega(0,x,y)(\xi_x^\top\otimes\psi_y^\top)$. This ensures [11] that $W_{\mathrm{EB}}=0$ while $W>0$ when Bob actually implements the channel he claims to possess and projects the joint two-photon state onto a singlet state $|\Psi^-\rangle$. In each round, Bob announces his result b to Alice, who computes the payoff using her knowledge of the prepared questions. We studied the effects of an imperfect quantum channel by simulating additional depolarizing noise $\mathcal{N}_{\mathrm{P}}$ or dephasing noise $\mathcal{N}_\phi$, defined as

$$\mathcal{N}_{\mathrm{P}}(\rho)=(1-p)\rho+p\mathbb{1}/2,$$
$$\mathcal{N}_\phi(\rho)=\left(1-\frac{p}{2}\right)\rho+\frac{p}{2}\sigma_3\rho\sigma_3, \tag{1}$$

where $\mathbb{1}$ and σ_3 are the identity and Pauli Z operator, respectively, and $0\leq p\leq 1$ is the noise strength. In both cases, the optimal payoff coefficients are found to be

$$\omega(b=0,x,y)=\begin{cases}-5/8 & \text{if } x=y\\ 1/8 & \text{otherwise}\end{cases}$$
$$\omega(b=1,x,y)=0, \tag{2}$$

where $b=0$ corresponds to a successful projection of the joint state onto the singlet state and $b=1$ to any other measurement outcome.

The experiment was implemented with the setup shown in Fig. 1(a). Pairs of 1550 nm single photons are generated via degenerate parametric down-conversion (PDC) in a custom-poled potassium titanyl phosphate (KTP) crystal [30,31] pumped by 775 nm, 1.6 ps pulses with 80 MHz repetition rate, and 75 mW of average pump power, focused to a beam waist of 350 μm. After being separated from the pump with a dichroic mirror (DM), one photon of each pair is loosely filtered, transmitted on a polarizing beam-splitter

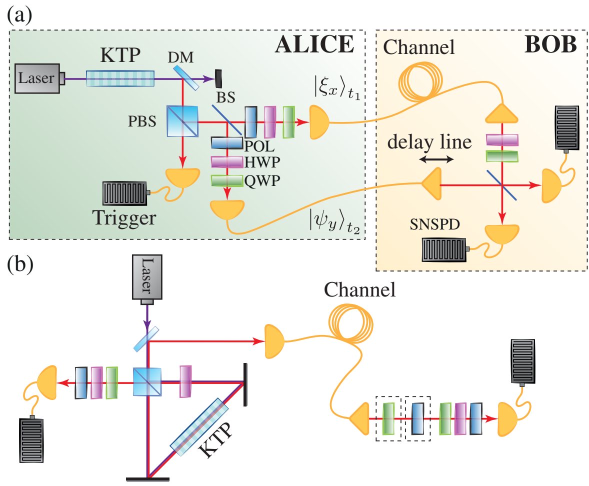

FIG. 1. Experimental setup. (a) MDI entanglement witness setup: the left-green (right-yellow) shading indicates the trusted (untrusted) parts of the experiment. (b) Untrusted channel verification setup via Bell test.

(PBS), and detected by a superconducting nanowire single-photon detector (SNSPD, 80% nominal quantum efficiency, ~200 Hz dark counts), providing the heralding signal for its twin photon. We benchmark the source by sending the downconverted photons directly to the SNSPDs, bypassing Bob's part of the setup. We measure a brightness of 140 kHz detected coincident counts and 65% heralding efficiency.

Alice encodes a probe state ξ_x in the heralded photon using a sequence of a polarizer (POL), half-wave (HWP), and quarter-wave plate (QWP). This probe state represents the first question in our semiquantum game, which Bob receives at time t_1, and is asked to process in his alleged quantum channel. Bob's quantum channel is a 15 m single-mode fiber emulating a quantum memory with fixed storage time of ~75 ns: this value exceeds the SNSPD's reset time (~50 ns), the minimum time interval required by Alice's source to herald a second photon. An HWP and a QWP are used to implement a noisy channel with variable noise-strength p by applying a combination of Pauli operators according to Eq. (1) for a measurement time proportional to p. At a later time t_2, Alice prepares in the same way a second probe state ψ_y—corresponding to the second question in the game—and sends it to Bob, who is asked to perform a joint measurement on the two states via two-photon interference on a beam splitter (BS) and broadcast the outcome. Bob uses a tunable delay line to synchronize the two photons' arrival time at the BS: only a coincidence click event of the detectors after the BS corresponds to $b = 0$ in Eq. (2) (i.e., a successful projection on the singlet state), while any other event corresponds to $b = 1$.

Unlike the MDI protocol described above, a Bell-test approach, i.e., a fully device-independent verification of a quantum channel requires Alice to prepare entangled quantum state. We produce entangled pairs of photons ($99.34^{+0.01}_{-0.09}\%$ purity and $99.62^{+0.01}_{-0.04}\%$ fidelity with the Ψ^- state) via PDC in Sagnac interferometric scheme [32], as shown in Fig. 1(b). Alice then sends one photon of the entangled pair to Bob, who sends it through his channel. An additional set of HWP, QWP, and POL is used to introduce controllable dephasing and depolarizing noise. After the stored photon has been retrieved, a Clauser-Horne-Shimony-Holt Bell test [33] is performed on the joint system, and a violation of the inequality guarantees the genuine quantumness of the channel.

Figure 2(a) shows the result of our MDI channel verification for dephasing and depolarizing noise compared to a Bell-test approach with fair sampling assumption (i.e., neglecting the losses in the untrusted part of the setup). We show that, even in this idealized scenario, the MDI

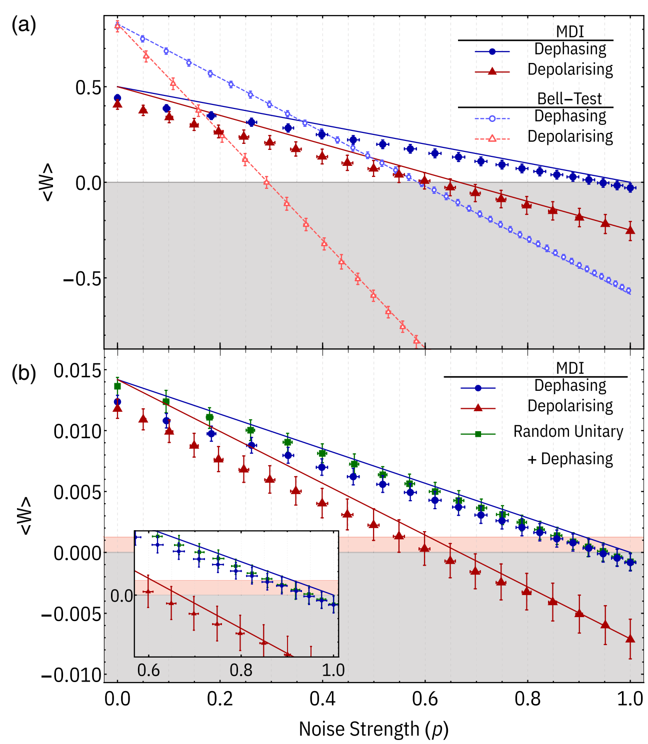

FIG. 2. Experimental results. MDI entanglement witness measures (a) with and (b) without fair sampling assumption. Lines and points represent the theoretical prediction and the experimental data, respectively. The gray-shaded area corresponds to entanglement-breaking channels, while the red-shaded area represents the actual threshold for entanglement-breaking channels, taking into account Bob's optimal cheating strategy. The discrepancy between theory prediction and data points is mainly due to imperfect two-photon interference on Bob's BS (we estimate a Bell-state measurement fidelity $\lesssim 95\%$). Note that the theoretical lines are computed for illustration purposes only using the full knowledge of the setup and therefore would not be available to Alice in the actual implementation of the protocol. The error bars in the data points represent 3σ statistical confidence regions obtained via Monte Carlo resampling ($n = 10^5$) assuming a Poissonian photon-counting distribution.

approach outperforms the Bell test as it can certify quantumness for larger noise strength than the Bell test (where the magnitude of noise each experimental approach can tolerate is computed from the intersection of the average payoff with the EB threshold). In theory, the quantum nature of the depolarizing channel can be witnessed up to a noise level of $p < 2/3$, while the Bell-based tests can only certify the channel up to $p \sim 0.29$. Surprisingly, the quantumness of a dephasing channel can be certified for any finite amount of noise, while a Bell-test approach can at best verify the channel up to $p \lesssim 0.58$. Crucially, under ideal experimental conditions—i.e., no loss and perfect single photon sources—the best strategy Bob can use to convince Alice that he is in possession of a genuine quantum channel is to truthfully reveal the result of the joint measurement. Any other tactic would not maximize the payoff function [11].

In order to guarantee device independence, the Bell-test approach would require very high detection efficiencies that are at best at the limit of current technical capabilities. The MDI approach, on the other hand, is less demanding in terms of experimental requirements. The effects of losses and imperfections on our protocol are twofold. First, lost photons lead to a decreased payoff, and second, Bob can exploit imperfections to cheat. Studying the latter possibility in some more detail, we note that most state-of-the-art photon sources suffer from a small probability of emitting multiple photons at a time, which Bob can exploit to extract information about the questions sent by Alice. Bob could then use this information to artificially inflate the payoff function by the following strategy: whenever he gets no more than one photon in each question, he announces an unsuccessful projection on the singlet state (i.e., $b = 1$). If he gets more than one photon in one of the questions and at least one in the other one, he can gather information on the question itself and perform a conveniently chosen local operation and classical communication (LOCC) positive-operator valued measure (POVM) to furtively inflate his payoff. In the Supplemental Material [26], we derive the maximal payoff that Bob could achieve with an EB channel using these strategies and the known characteristics of Alice's photon source. By using this new threshold in our protocol, we can reliably, and without further assumptions, certify whether a channel is quantum, even if Bob is actively trying to cheat. Figure 3 shows the theoretical trade-off between losses, noise, and protocol success for depolarizing noise at a fixed performance of the trusted source (given by the ratio of multiphoton emissions: the red area above the threshold represents the parameters space's region where secure certification of the channel's quantumness can be achieved).

Taking into account both losses and multiphoton emissions according to our experimental parameters—overall heralding efficiency of ~17% and multiphoton contribution of ~0.25%—puts us in a regime far beyond where a fully DI approach would apply due to its sensitivity to loss. On the other hand, the MDI protocol reveals itself to be significantly more robust to such experimental imperfections, being still capable of certifying the nature of a quantum channel, as we show in Fig. 2(b).

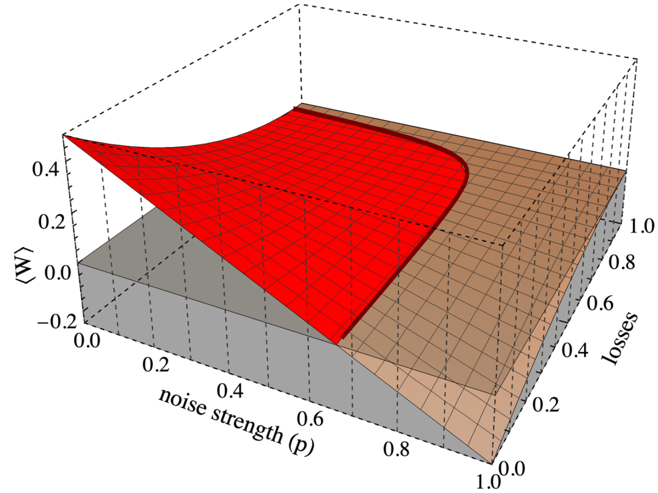

FIG. 3. MDI channel verification tradeoffs. Shown is the ideal expectation value $\langle W \rangle$ of an MDI entanglement witness for an identity channel with depolarizing noise of strength p and different amounts of loss. The multiphoton contribution is fixed at ~0.25%, as in our experimental setup, and results in a decreased size of the parameter region where certification of quantum behavior is possible.

So far, we have discussed MDI certification of a noisy identity channel. In practice, an imperfect channel might also apply some unknown unitary rotation to the stored qubit. In this case, a nominally entanglement-preserving channel might appear to be EB due to the wrong choice of witness or payoff. In order to verify such channels, Alice uses a modified protocol, where she splits the answers obtained from Bob into two sets. The first set is used to reconstruct the channel's process matrix via quantum process tomography and then computing the corresponding entanglement witness (as discussed, e.g., in Ref. [29] or in the Supplemental Material [26]). Alice can then perform the standard MDI verification with the adapted witness on the second dataset. Since only the second stage of this extended protocol is MDI, Bob could attempt to cheat in the first stage. However, all this would achieve is that Alice computes a suboptimal witness, which inevitably lowers the achievable payoff in the second stage. Bob's best strategy to have his channel certified is thus to broadcast the true outcome of the joint measurement he performs while Alice performs the process tomography of the channel, so that she can build the optimal witness for the channel at hand.

Experimentally, the unknown unitary rotation was implemented by means of an HWP and a QWP at the output of the channel, and the MDI protocol was performed in the context of certifying its quantumness in the presence of dephasing noise. The results are shown in Fig. 2(b) with losses and multiphoton corrections taken into account. We note that this protocol behaves as the standard MDI protocol for the identity channel, confirming the suitability of using a MDI approach in more complex scenarios where nontrivial noisy channels are involved.

Discussion.—We have provided a readily accessible MDI recipe for verifying a quantum channel with sustained performance in the presence of noise and loss much beyond the capabilities of fully DI methods, with the minimum possible set of assumption on the device under examination. With minimal demands on the trusted side (i.e., a single photon source), this method is ideally suited as a dependable benchmark for quantum memories and more general quantum channels. With the future vision of large scale quantum networks, this type of verification protocol can be a powerful tool for a security-conscious user of the network, who does not necessarily trust the third party operating the network. A natural extension of this work would be probing different properties of a quantum memory simultaneously. Fidelity, storage time and recall properties could be tested by changing the timing between the probe photons. On the theory side, the protocol could be extended to quantify the quantum nature of the channel instead of verifying it as has been done for the MDI quantification of entanglement [19–21]; in this direction, one would need to use a capacity that quantifies specifically the quantum part of the channel: the negativity of the channel Choi state, or the quantum relative entropies [34] could be used for that purpose.

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Note added in the proof.—Recently, we became aware of this Letter we became aware of similar work that has been recently carried out [35]. The work in this Letter implements a similar protocol, but in a different regime of encoding.

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