Absence of Evidence for the Ultimate Regime in Two-Dimensional Rayleigh-Bénard Convection

Zhu *et al.* [1] report direct numerical simulations of turbulent thermal convection in two dimensions with planar no-slip isothermal walls and Rayleigh numbers (Ra) ranging from 10^8 to 10^{14}. For the Nusselt number (Nu) the authors report a scaling of $\mathrm{Nu} \sim \mathrm{Ra}^{0.35}$ for the four data points with $10^{13} \leq \mathrm{Ra} \leq 10^{14}$. They also decomposed Nu into contributions from "plume-ejecting" (Nu_e) and "plume-impacting" (Nu_i) regions of the spatial domain reporting $\mathrm{Nu}_e \sim \mathrm{Ra}^{0.38}$ for those four data points, interpreting this as evidence of a so-called "ultimate" regime of thermal convection characterized by *bulk* heat transport scaling $\mathrm{Nu} \sim \mathrm{Ra}^{1/2}$ modulo logarithmic corrections [2].

Although hypotheses concerning the nature of boundary layers [2] constitute one ingredient of this system, the fundamental characterization of the state of convection is the asymptotic Nu-Ra relation [2–4]. Zhu *et al.* [1] drew an arbitrary line through the final four heat flux data [$10^{13} \leq \mathrm{Ra} \leq 10^{14}$]. When we perform an objective least-squares power law fit to these data we find $\mathrm{Nu} = 0.035 \times \mathrm{Ra}^{0.332}$ with an empirical exponent that is *indistinguishable* from $1/3$, the so-called "classical" scaling exponent [5–7].

Moreover, the data from $\mathrm{Ra} = 10^8$ to 10^{13} are extremely well described by extrapolation of a previous fit, $\mathrm{Nu} = 0.138 \times \mathrm{Ra}^{2/7}$, from high resolution simulations for $10^7 \leq \mathrm{Ra} \leq 10^{10}$ [8]. Indeed, the power law fit of those 5 decades of their data yields the scaling exponent 0.289, indistinguishable (less than 1.2%) from $2/7$.

Compare Fig. 1 here to Fig. 1 of Ref. [1]. The clear deviation of the full dataset from pure scaling, combined with the limited range of Ra (one decade) and the small size of the dataset (just four points) over which the classical $1/3$ scaling appears, precludes definitive extrapolation to asymptotically large Ra. Nevertheless the 2D heat transport results reported by Zhu *et al.* are reminiscent of previous 3D simulations [9,10] and experiments [11–13] consistent with crossovers from $\mathrm{Nu} \sim \mathrm{Ra}^{2/7}$ to $\mathrm{Nu} \sim \mathrm{Ra}^{1/3}$ for various Rayleigh numbers between 2×10^9 and 10^{11}.

In summary, while Zhu *et al.* [1] do not report any detailed statistical analysis of their data, we have shown that when dividing their data into different segments and forcing a power law fit for the last decade as they have proposed, the reduced dataset is consistent with $\mathrm{Nu} \sim \mathrm{Ra}^{1/3}$—in quantitative agreement with the classical theories [5–7]—*not* the $\mathrm{Ra}^{0.35}$ scaling reported by Zhu *et al.* [1]. Moreover, the remaining data up to $\mathrm{Ra} = 10^{13}$ are consistent with $\mathrm{Nu} \sim \mathrm{Ra}^{2/7}$ scaling in qualitative accord with other reported 2D [8] and 3D results [9–13].

Therefore the claim by Zhu *et al.* [1] that their data suggest that two-dimensional convective turbulence reaches the ultimate regime characterized by bulk heat transport scaling $\mathrm{Nu} \sim \mathrm{Ra}^{1/2}$ with logarithmic corrections [2] is not, in fact, supported by their data.

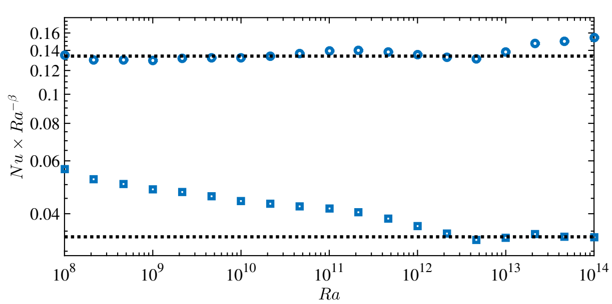

FIG. 1. Compensated plots of $\mathrm{Nu} \times \mathrm{Ra}^{-\beta}$ vs Ra. Circles (top) are data of Zhu *et al.* [1] with $\beta = 2/7$ while squares (bottom) are the same data with $\beta = 1/3$.

This work was supported by National Science Foundation Grants No. DMS-1515161 and No. DMS-1813003, Vetenskapsrådet No. 638-2013-9243, and All Souls College, Oxford.

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Received 18 April 2018; published 16 December 2019
DOI: 10.1103/PhysRevLett.123.259401

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