Steering Heat Engines: A Truly Quantum Maxwell Demon

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We address the question of verifying the quantumness of thermal machines. A Szilárd engine is truly quantum if its work output cannot be described by a local hidden state model, i.e., an objective local statistical ensemble. Quantumness in this scenario is revealed by a steering-type inequality which bounds the classically extractable work. A quantum Maxwell demon can violate that inequality by exploiting quantum correlations between the work medium and the thermal environment. While for a classical Szilárd engine an objective description of the medium always exists, any such description can be ruled out by a steering task in a truly quantum case.

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Introduction.—Experimental progress has led to unprecedented possibilities of preparation, control, and measurement of small quantum systems, where quantum and thermal fluctuations have to be considered on equal footing. In particular, fundamental concepts of thermodynamics have been revisited from a quantum point of view. This has led to a quantum interpretation of thermal states [1–4], the development of quantum fluctuation theorems [5–15] and the concepts of quantum heat engines [16–24]. One of the key points in these investigations is the question what is fundamentally quantum about these extensions. For instance, whether and how is a quantum heat engine qualitatively and quantitatively different from its classical counterpart? Is quantumness useful in thermodynamics? Influences of quantum features like coherence [25,26], discord [27], and entanglement [28,29] on the efficiency of quantum engines have been reported, which show that the answer can be positive under suitable conditions. However, other investigations show that quantumness can even be a hindrance for efficient thermal machines, which can be regarded as classical supremacy in such situations [30–32].

In this Letter we want to address quantumness of thermal machines from a different perspective. We consider a heat engine truly quantum if its work output cannot be explained by a local hidden state (LHS) model, i.e., by a local statistical model. Even though the issue of hidden classicality is fundamental to quantum information, it only rarely appears in the context of quantum thermodynamics [33]. In this Letter we give a verifiable criterion for the quantumness of thermodynamical systems, indicating the lack of a classical statistical description. Most remarkably, the classicality sets an upper bound on the extractable work for certain scenarios.

Quantum Szilárd engine.—The prototypical example we want to study is a quantum modification of the Szilárd engine [34,35]. The classical version consists of a single atom in a box which is in contact with a thermal bath. In equilibrium the atom is in a Gibbs state, a statistical mixture of different phase space points. For work extraction, the demon has knowledge about the microstate of the system.

So far, quantum versions of this heat engine have been investigated using different underlying systems [19,21, 36–38]. In these examples the demon performs quantum measurements on the work medium, acquiring information about local properties of the heat engine only. Here, we want to exploit the fact that such a local thermal state may arise naturally from a global entangled state of the work medium and its environment, as supported, for instance, by the eigenstate thermalization hypothesis [1–4,39]. In contrast to previous proposals, the demon obtains her information from measurements on the environment rather than the work medium [40,41]. A truly quantum Szilárd engine can be revealed by deriving local work extraction bounds which cannot be violated by any local statistical ensemble description, that is a LHS model. These bounds do neither rely on the knowledge about the shared system-environment state, nor on any assumptions about the properties of the environment (semi-device independent).

Work medium.—Let us assume that the work medium is a finite quantum system $\mathcal{S}$ with Hamiltonian $H_\mathcal{S}$. Its Gibbs state reads

$$\rho_\mathcal{S}^{\mathrm{Gibbs}} = \sum_i \frac{e^{-\beta E_i}}{Z} |i\rangle\langle i| = \sum_i p_i |i\rangle\langle i|, \qquad (1)$$

where $\beta = 1/k_B T$, E_i is the energy of the ith energy eigenstate $|i\rangle$ and $Z = \sum_i e^{-\beta E_i}$. Locally, the Gibbs state can be seen as a statistical mixture of the energy eigenstates but it can equally be decomposed into infinitely many other ensembles $\mathcal{D} = \{p_k; \rho_k\}$ with $\sum_k p_k \rho_k = \rho_\mathcal{S}^{\mathrm{Gibbs}}$, $p_k \geq 0$ and $\sum_k p_k = 1$. Any such decomposition can be given by an extension to a bipartite state $\rho_{\mathcal{S}\mathcal{E}}$ of system and environment, with $\rho_{\mathcal{S}}^{\text{Gibbs}} = \text{Tr}_{\mathcal{E}}\{\rho_{\mathcal{SE}}\}$, and a local POVM $\{M_k\}$ on $\mathcal{E}$, such that $p_k = \text{Tr}\{(\mathbb{1} \otimes M_k)\rho_{\mathcal{SE}}\}$ and $\rho_k = \text{Tr}_{\mathcal{E}}\{(\mathbb{1} \otimes M_k)\rho_{\mathcal{SE}}\}$.

To investigate the difference of a classical and a quantum Szilárd engine we introduce Alice and Bob. Alice is a demon who can prepare many copies of the global state $\rho_{\mathcal{SE}}$. While she can perform measurements on $\mathcal{E}$, she does not act directly on $\mathcal{S}$ since this would in general disturb the local thermodynamical situation [42]. Bob has access only to the system $\mathcal{S}$ and would like to extract work from its Gibbs state. As shown in Ref. [40] any nonproduct joint state $\rho_{\mathcal{SE}} \neq \rho_{\mathcal{S}} \otimes \rho_{\mathcal{E}}$ allows Bob to extract work from $\mathcal{S}$ if Alice performs suitable measurements on $\mathcal{E}$ and communicates classically with him.

Work extraction scenarios.—Bob wants to extract work from a certain decomposition $\mathcal{D} = \{p_k; \rho_k\}$ of his local Gibbs state. We can describe this work extraction as a transfer of energy due to a suitable coupling between the work medium $\mathcal{S}$ and a work storage system $\mathcal{W}$ with Hamiltonians $H_{\mathcal{S}}$ and $H_{\mathcal{W}}$, respectively [5,7,43,44]. The total Hamiltonian is given by $H = H_{\mathcal{S}} \otimes \mathbb{1} + \mathbb{1} \otimes H_{\mathcal{W}}$. Following the reasoning of Ref. [7] the coupling of $\mathcal{S}$ and $\mathcal{W}$ has to be a unitary transformation that conserves the total energy independently of the initial state of the work storage $\rho_{\mathcal{W}}$. Further constraints ensuring that only work and no heat is transferred are presented in the Supplemental Material [45]. For each state ρ_k in $\mathcal{D}$ Bob can find a suitable unitary $\mathcal{U}_k$ which transfers a non-negative amount of energy from $\mathcal{S}$ to $\mathcal{W}$. The average work extraction associated with this process is given by

$$\Delta W_k = \text{Tr}\{\mathbb{1} \otimes H_{\mathcal{W}}(\mathcal{U}_k \rho_k \otimes \rho_{\mathcal{W}} \mathcal{U}_k^\dagger - \rho_k \otimes \rho_{\mathcal{W}})\}, \qquad (2)$$

where $\rho_{\mathcal{W}}$ is the initial state of the work storage. Because of energy conservation, the change of the inner energy in $\mathcal{S}$ is $\Delta E_k = -\Delta W_k$ and $\Delta W_k \leq \text{Tr}\{H_{\mathcal{S}}\rho_k\}$. The average work Bob can extract from $\mathcal{D}$ by using the set of unitaries $U = \{\mathcal{U}_k\}$ is then given by $\bar{W} = \sum_k p_k \Delta W_k$.

Special cases are pure state decompositions $\mathcal{D}^{\text{pur}} = \{p_k; |\phi_k\rangle\langle\phi_k|\}$. If Bob knows the pure state $|\phi_k\rangle$ of his system, the largest possible amount of work can only be extracted if he performs a suitable *local* unitary operation $\mathcal{U}_k^{\mathcal{S}}$ on $\mathcal{S}$ which acts as $\mathcal{U}_k^{\mathcal{S}}|\phi_k\rangle = |0\rangle$, where $|0\rangle$ is the ground state of the local Hamiltonian $H_{\mathcal{S}}$ whose energy we set to $E_0 = 0$ [52]. As shown in Ref. [7], such a local unitary $\mathcal{U}_k^{\mathcal{S}}$ can indeed always be implemented by an energy conserving global coupling $\mathcal{U}_k$ between $\mathcal{S}$ and $\mathcal{W}$, but requires the work storage to be initialized in a pure state $\rho_{\mathcal{W}}$ which is a coherent superposition of energy eigenstates states of $H_{\mathcal{W}}$. Thus, energy measurements on $\mathcal{W}$ will in general yield probabilistic outcomes [53]. However, Bob is only interested in the average work output under the given $\mathcal{U}_k$, which is, due to the energy conservation, given by the negative energy change in $\mathcal{S}$, that is $\Delta W_k = -\Delta E_k = \langle\phi_k|H_{\mathcal{S}}|\phi_k\rangle - \langle 0|H_{\mathcal{S}}|0\rangle = \langle\phi_k|H_{\mathcal{S}}|\phi_k\rangle$. Accordingly, if Alice can provide the pure state decomposition $\mathcal{D}^{\text{pur}}$, Bob can extract on average $\bar{W}^{\text{pur}} = \sum_k p_k \langle\phi_k|H_{\mathcal{S}}|\phi_k\rangle = \langle H_{\mathcal{S}}\rangle_{\rho_{\mathcal{S}}^{\text{Gibbs}}}$ which is, not surprisingly, the inner energy of the work medium. Thus, equivalently to the classical case, full knowledge about the state of the system allows for maximal work extraction. If Alice cannot announce correct pure states to Bob, the output will be $\bar{W} < \bar{W}^{\text{pur}}$. In fundamental contrast to the classical Szilárd scenario, a Gibbs state of a quantum system allows for infinitely many different ensembles of pure states.

Truly quantum features can be revealed when Bob wants to extract work from different decompositions $\mathcal{D}_n$. For each decomposition he has a suitable set of unitaries $U_n = \{\mathcal{U}_{k_n}^n\}$ as described above. He chooses randomly with probabilities c_n one of the sets and asks Alice which unitary out of the particular set U_n he should perform to extract the maximal amount of work. Depending on how well Alice can produce the desired decompositions, Bob will extract on average $\bar{W} \leq \sum_n c_n \bar{W}_n$.

The question now arises, under which conditions Bob can be sure that his Szilárd engine is truly quantum. He has no access to the global state $\rho_{\mathcal{SE}}$ and, therefore, cannot check whether the state is quantum correlated. The only information he gets from Alice is which unitary $\mathcal{U}_{k_n}^n$ he should use if he asks her for the decomposition $\mathcal{D}_n$. Accordingly, Bob has to certify quantumness without any assumptions about the properties (for example, the Hilbert space) of the environment $\mathcal{E}$. Such a semi-device-independent verification task is called quantum steering [54,55]. Successful steering has important implications on the objectivity [56] of the local state in the system. In a classical scenario the system state is always objective, though unknown to Bob as long as the demon does not share her knowledge with him. In the quantum case, in general, it makes no sense to assign objective system states at all, as long as no observation of the environment is made. Particularly, a thermalized quantum system is *not* in one of its energy eigenstates and does *not* fluctuate between them while time is evolving, if these fluctuations are not given relative to measured states of the bath [57]. For a closer look on how steering can rule out objective quantum dynamics see Refs. [56,58,59]. In our Szilárd scenario we can use these ideas as follows: If a local objective statistical description of Bob's system $\mathcal{S}$ holds, it can be represented by a local hidden state (LHS) model $\mathcal{F} = \{p_\xi; \rho_\xi\}$ [55]. The hidden states ρ_ξ are distributed randomly according to their probabilities p_ξ. Locally, the Gibbs state in Bob's system has to be recovered:

$$\rho_{\mathcal{S}}^{\text{Gibbs}} = \sum_\xi p_\xi \rho_\xi. \qquad (3)$$

Thus, among all the copies of his local state, a fraction p_ξ will be in state ρ_ξ. Bob does not know which state he has for a particular copy but he can assume that, if the LHS model holds, the best knowledge Alice can possibly have about his system is the particular hidden state for each of his copies. Therefore, any decomposition Alice can provide has to be either the LHS ensemble $\mathcal{F}$ itself or a coarse graining of the latter [55]. However, this does not mean that the real state $\rho_{\mathcal{S}\mathcal{E}}$ shared by Alice and Bob has to be separable. It only means that Bob could explain his statistics also by a state without quantum correlations. A truly quantum Szilárd engine can therefore be defined by the condition $\bar{W} > \bar{W}_{\mathrm{cl}}$, that is, Bob's average work output is larger than what could be obtained from a state which can be described by a LHS ensemble $\mathcal{F}$. Clearly, the work output of a single decomposition $\mathcal{D}$ can always be explained by a classically correlated state because we can always identify $\mathcal{D} = \mathcal{F}$. Bob needs at least two different sets of unitaries U_n.

We should note that the observables on Bob's side needed to perform a steering task are represented by the work extraction. In order to determine the average energy transferred to the work storage he has to measure $\mathcal{W}$ in its energy basis. According to Naimark's dilation theorem, this measurement, together with a unitary $\mathcal{U}_k$, defines a POVM on $\mathcal{S}$. The set of POVMs that can be implemented by the described work extraction scenario is strictly smaller than the set of all local POVMs on $\mathcal{S}$. For example, the only implementable projective measurement is the one diagonal in the energy eigenbasis of $H_{\mathcal{S}}$. It is an open question whether the work extraction POVMs can demonstrate steering for any steerable state $\rho_{\mathcal{S}\mathcal{E}}$ that respects the local Gibbs state.

Whether the work output on Bob's side can also be provided by a classical demon is in general not trivial to answer. As in a standard steering scenario a suitable inequality has to be derived, which depends on the properties of the work medium $\mathcal{S}$ and the work extracting unitaries $\{\mathcal{U}_k^n\}$. It is crucial for quantum steering that the inequality does not depend on the part $\mathcal{E}$ which is inaccessible for Bob.

Qubit work medium.—To illustrate the concept we consider a qubit work medium $\mathcal{S}$ with local Hamiltonian $H_{\mathcal{S}} = |1\rangle\langle 1|$. Its thermalized Gibbs state is given by

$$\rho_{\mathcal{S}}^{\mathrm{Gibbs}} = \frac{1+\eta}{2}|1\rangle\langle 1| + \frac{1-\eta}{2}|0\rangle\langle 0|, \qquad (4)$$

with $\eta = (e^{-\beta} - 1)/(e^{-\beta} + 1)$ and $\beta = 1/k_B T$. As stated above, Bob needs at least two different sets of work extracting unitaries to verify a quantum Szilárd engine. Let us assume that he would like to extract work from two dichotomic pure state decompositions $\mathcal{D}_1$ and $\mathcal{D}_2$. The first one is a decomposition into energy eigenstates $\{|0\rangle, |1\rangle\}$, the second one is given by the two Bloch vectors $\vec{r}_\pm = (\pm\sqrt{1-\eta^2}, 0, \eta)$. The local unitaries for $\mathcal{D}_1$ are $\mathcal{U}_z^1 = \sigma_x$ for the state $|1\rangle$ and $\mathcal{U}_z^0 = \mathbb{1}$ for the state $|0\rangle$. For $\mathcal{D}_2$ the suitable unitaries $\mathcal{U}_x^\pm$ are rotations around

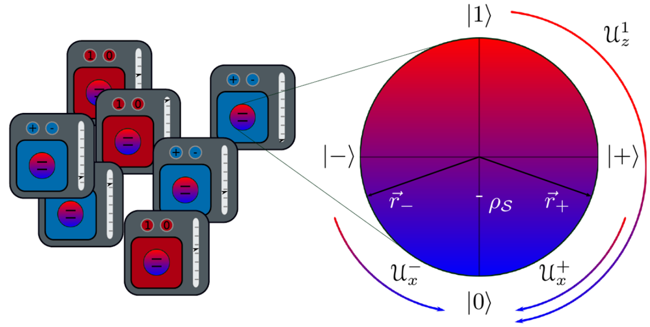

FIG. 1. Work extraction. Bob has blue and red work extraction cells with locally thermal qubits. The reduced state $\rho_{\mathcal{S}}$ can be decomposed into statistical mixture of $|1\rangle$ and $|0\rangle$ or the two states given by the Bloch vectors $\vec{r}_\pm$. Different unitaries can be used to bring the states to the ground state extracting some work. The red cells can apply the $\mathcal{U}_z^1$ and the identity operation. Qubits in the blue cells can be manipulated by two unitaries $\mathcal{U}_x^\pm$. Bob gets the information which unitary he should use from Alice. After the coupling process the work storage is measured in its energy basis.

the y axis about an angle $\alpha = \pm\arctan(\eta/\sqrt{1-\eta^2})$ (see Fig. 1). Accordingly, Bob needs two different kinds of work extraction devices. We represent them by red and blue cells which both have two buttons to trigger the different work extraction unitaries and measure the work storage $\mathcal{W}$ in the energy basis (Fig. 1). In each cell Bob can place one qubit. The red cells can perform $\mathcal{U}_z^1$ and $\mathcal{U}_z^0$, the blue cells apply either $\mathcal{U}_x^+$ or $\mathcal{U}_x^-$. In the Supplemental Material [45] we construct an explicit model, how the energy conserving unitaries can be realized by using only two qubit interactions.

Let us first assume that Alice prepares the global state $\rho_{\mathcal{S}\mathcal{E}}^{\mathcal{D}_1} = \frac{1}{2}(1+\eta)|1\rangle\langle 1| \otimes |1\rangle\langle 1| + \frac{1}{2}(1-\eta)|0\rangle\langle 0| \otimes |0\rangle\langle 0|$, compatible with the local Gibbs state. Bob places his qubit into a red cell and asks Alice which button he should press. Alice measures $\mathcal{E}$ in the σ_z basis and tells Bob to press the button 1 if the outcome is 1 and button 0 if the outcome is 0. The cell will apply either $\mathcal{U}_z^1$ or $\mathcal{U}_z^0$. On average—Bob has many red cells which he wants to charge—he will extract $\bar{W}_z = \frac{1}{2}(1+\eta)$ because Alice tells him to press button 1 with probability $p_z^1 = \frac{1}{2}(1+\eta)$.

If Bob wants to charge his blue cells, Alice could help him by preparing the state $\rho_{\mathcal{S}\mathcal{E}}^{\mathcal{D}_2} = \rho_+ \otimes |1\rangle\langle 1| + \rho_- \otimes |0\rangle\langle 0|$, where $\rho_\pm$ are the density matrices corresponding to the Bloch vectors $\vec{r}_\pm$. Locally, the Gibbs state is again recovered. Depending on her outcome, Alice tells Bob to press either the button which applies $\mathcal{U}_x^+$ or $\mathcal{U}_x^-$ (see Fig. 1). On average Bob can again extract $\bar{W}_x = \frac{1}{2}(1+\eta) = \bar{W}_z$. Thus, the decomposition $\mathcal{D}_1$ into energy eigenstates is by no means better for the work extraction than decomposition $\mathcal{D}_2$. Both $\rho_{\mathcal{S}\mathcal{E}}^{\mathcal{D}_1}$ and $\rho_{\mathcal{S}\mathcal{E}}^{\mathcal{D}_2}$ are separable and describe situations where Alice exploits only classical correlations. We call such a demon a classical one because the same result can be obtained from a local statistical model for ρ_S without any reference to a global quantum state $\rho_{S\mathcal{E}}$.

When is the demon really quantum?—In the remainder we will consider the case where Bob would like to charge both the red and the blue cells. He has N blue cells, M red cells, and $N+M$ thermal qubits. The ratio between red and blue cells is given by $c = N/M$. First, he distributes the qubits over the cells, which fixes the decomposition he needs to extract maximal work with any given cell. Subsequently, he announces the color of each single cell to Alice and asks her which button he should press. In the end, he can read off the extracted work from each work meter and average over them to see how efficient the procedure has been. If Alice's knowledge for each single qubit is described by $\rho^{\mathcal{D}_1}_{S\mathcal{E}}$ or $\rho^{\mathcal{D}_2}_{S\mathcal{E}}$, his average work output in the limit $N \to \infty$ can never reach the optimal $\bar{W}_{\rm opt} = \frac{1}{2}(1+\eta)$ (we assume that $c > 0$ is kept constant). The blue and the red cells are not compatible with the same statistical mixture of states. On the other hand, the entangled state of the form

$$|\Psi\rangle_{S\mathcal{E}} = \sqrt{\frac{1+\eta}{2}}|1\rangle_S \otimes |1\rangle_\mathcal{E} + \sqrt{\frac{1-\eta}{2}}|0\rangle_S \otimes |0\rangle_\mathcal{E} \qquad (5)$$

would do the job. Alice could measure either σ_z or σ_x depending on the color of the cell for which Bob would like to know which button he should press. If Alice is indeed a demon who can prepare Bob's thermal state to be the partial trace of a pure entangled state, he can extract the optimal average $\bar{W}_{\rm opt}$.

We will now calculate which average work Bob can maximally obtain if a LHS model holds. The best Alice can do if she knows the state ρ_ξ for each cell is to tell Bob which button he should press in order to obtain the maximal work output. Accordingly, for the red cells Alice would tell him to press the button triggering $\mathcal{U}^1_z$ whenever $z_\xi = \mathrm{Tr}\{\sigma_z\rho_\xi\} > 0$ for the state in this cell. The average work output for the red cells will then be $\bar{W}_z = \frac{1}{2}\sum_\xi p_\xi(|z_\xi| + \eta)$ [45]. For the blue cells Alice announces the button for $\mathcal{U}^+_x$ if $x_\xi = \mathrm{Tr}\{\sigma_x\rho_\xi\} > 0$ and the button for $\mathcal{U}^-_x$ if $x_\xi \le 0$. On average, the blue cells will then reach $\bar{W}_x = \frac{1}{2}(\eta + \eta^2 + \sum_\xi p_\xi|x_\xi|\sqrt{1-\eta^2})$ [45]. The work average over all cells that can be expected for a LHS model is, thus, given by $\bar{W}_{\rm cl} = (\bar{W}_z + c\bar{W}_x)/(1+c)$. For the choice $c = 1/\sqrt{1-\eta^2}$ we can bound the average work for any LHS model by [45]

$$\bar{W}_{\rm cl} \le \frac{\eta\left(\sqrt{1-\eta^2}+\eta+1\right)+\sqrt{2-2\eta^2}}{2\left(\sqrt{1-\eta^2}+1\right)}. \qquad (6)$$

If Bob extracts an average work beyond the classical limit $\bar{W}_{\rm cl}$ he can be satisfied that Alice is indeed a quantum demon who exploits nonclassical correlations. Any local statistical model has to be discarded in this case.

If Alice can, for example, indeed prepare the entangled state (5), the work value that can be reached is $\bar{W}_{\rm qu} = \bar{W}_{\rm opt} = \frac{1+\eta}{2}$. Thus, Alice can violate the inequality for $\eta > -\sqrt{2(\sqrt{2}-1)} \approx -0.91$, that is, for temperatures $k_B T > 0.33$. For the case of infinite temperature ($\eta \to 0$) the steering inequality simplifies to $\bar{W}_{\rm cl} \le 1/(2\sqrt{2})$, while $\bar{W}_{\rm opt} = \frac{1}{2}$.

For an intermediate regime we can consider the following mixture,

$$\rho_{S\mathcal{E}} = q\rho_{\rm qu} + (1-q)\rho_{\rm cl}, \qquad (7)$$

where $\rho_{\rm qu} = |\Psi\rangle\langle\Psi|_{S\mathcal{E}}$ as in Eq. (5) and $\rho_{\rm cl}$ is the classically correlated state $\rho_{\rm cl} = \frac{1}{2}(1+\eta)|1\rangle\langle 1|_S \otimes |1\rangle\langle 1|_\mathcal{E} + \frac{1}{2}(1-\eta)|0\rangle\langle 0|_S \otimes |0\rangle\langle 0|_\mathcal{E}$. The parameter q tunes between the fully quantum case ($q=1$) and the scenario that represents a classical Szilárd demon ($q=0$). The extractable work in the blue cells is now $\bar{W}_x = \frac{1}{2}(q + \eta + \eta^2 - q\eta^2)$ [45]. Figure 2 shows the relation between the nonclassicality of the demon and the two parameters η and q. For parameters above the red line Alice can demonstrate that she is a quantum Szilárd demon.

We should note that the steering inequality (6) is not ideal for the detection of nonclassical correlations in the state $|\Psi\rangle_{S\mathcal{E}}$. It is well known that any pure entangled state is steerable [54]; thus, $|\Psi\rangle_{S\mathcal{E}}$ is steerable for any $-1 < \eta < 1$. More generally, any nonzero temperature Gibbs state is mixed and therefore has an entangled and steerable purification. However, we have restricted Bob's observables to a special class of operations, namely, the work extraction. It is therefore not surprising that the given inequality cannot detect every steerable state. We could

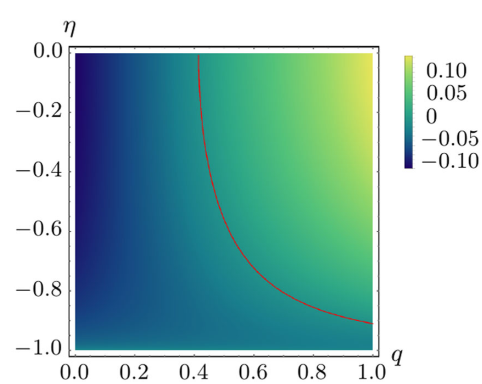

FIG. 2. The plot shows by how much Alice can violate the classical work bound. For parameters in the region below the red line Bob can explain the extracted work with a local hidden state model.

improve the bound by adding additional work extraction options on Bob's side, but this does not add anything conceptually new to the framework. Furthermore, motivated by the concept of a Szilárd engine, the inequality is based on the assumption that Bob's reduced state is indeed a Gibbs state. This property can of course be locally verified by Bob. It has to be emphasized that the construction of the steering inequality only depends on the device-dependent part of the steering task, such as the Hilbert space of Bob's system $\mathcal{S}$ and the work extracting operations he uses. There are no assumptions made about the structure of the environment or the operations Alice performs.

Conclusions.—In this Letter we have shown how the concept of quantum steering can be applied to quantum thermodynamics in order to verify quantumness. The violation of a steering inequality is connected to the macroscopic average work. The use of a quantum steering task for the verification of quantumness is motivated by the asymmetric setting in quantum heat engines. The work system under control is taken to be the device-dependent part in the scenario, whereas the environment is treated device independently.

Our concept is of particular interest for the investigation of bath-induced fluctuations in quantum thermodynamics. A violation of the steering inequality rules out any possible objective (though statistical) description of fluctuations in the system. Notably, the assumption that a system fluctuates between its energy eigenstates is not valid if genuine quantum correlations are taken into account. Statements about the fluctuations in the system can only be made with respect to the observed fluctuations of the environment which will depend on how the environment is measured.

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