Radiation Reaction of a Jiggling Dipole in a Quantum Electromagnetic Field

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We show how to derive a consistent quantum theory of radiation reaction of a nonrelativistic point-dipole quantum oscillator by including the dynamical fluctuations of the position of the dipole. The proposed nonlinear theory displays neither runaway solutions nor acausal behavior without requiring additional assumptions. Furthermore, we show that quantum (zero-point) fluctuations of the electromagnetic field are necessary to satisfy the second law of thermodynamics. Our results are obtained by developing a nonperturbative technique involving a weak-coupling approximation at the level of the effective action.

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A long-standing fundamental problem in electrodynamics is the appearance of runaway and acausal solutions in the dynamics of a moving point charge interacting with its radiated electromagnetic field [1–3]. The so-called radiation reaction problem can be illustrated by the theory of a nonrelativistic point-dipole quantum oscillator interacting with the electromagnetic field, whose dynamics is described by the Hamiltonian

$$\hat{H}' = \frac{1}{2m'}[\hat{\mathbf{p}} - q\hat{\mathbf{A}}(\mathbf{R})]^2 + \frac{1}{2}\kappa\hat{\mathbf{r}}^2 + \hat{H}_{\mathrm{EM}}. \tag{1}$$

Here, m' (q) is the bare mass (charge) of the electron, $q\hat{\mathbf{r}}$ the dipole moment operator, κ the observed spring constant of the point-dipole quantum oscillator, $\hat{\mathbf{A}}(\mathbf{R})$ the potential vector operator evaluated at the position of the dipole $\mathbf{R}$ (assumed to be fixed), and $\hat{H}_{\mathrm{EM}}$ the Hamiltonian describing the dynamics of the free electromagnetic field. Note that Eq. (1) corresponds to the Lorentz-oscillator model for an electron in an atom within the long-wavelength approximation. From this Hamiltonian, one can readily show [1,3] that the dynamics of the dipole moment degrees of freedom (d.o.f.) $\mathbf{r}(t) = \langle\hat{\mathbf{r}}(t)\rangle$ is given by the Abraham-Lorentz [4,5] equation

$$m\ddot{\mathbf{r}}(t) + \kappa\mathbf{r}(t) - m\gamma\dddot{\mathbf{r}}(t) = 0, \tag{2}$$

where m is the observed mass of the electron, and $\gamma = 2q^2/(3m)$ the radiation reaction damping constant. Hereafter natural units are used. The Abraham-Lorentz equation is local in time (Markovian) and in the Fourier space reads $(\omega^2 - \omega_0^2 + i\gamma\omega^3)\mathbf{r}(\omega) = 0$, where $\omega_0^2 \equiv \kappa/m$ and $\mathbf{r}(t) = (2\pi)^{-1/2}\int d\omega e^{-i\omega t}\mathbf{r}(\omega)$. The radiation reaction problem, that is, the existence of runaways and preaccelerations, is apparent in the existence of a purely imaginary root of the characteristic polynomial $\omega^2 - \omega_0^2 + i\gamma\omega^3$ with positive imaginary part. Historically, the radiation reaction problem is circumvented by either (i) considering the charge to be extended over a sphere of radius larger than γ [1,6,7], (ii) assuming the weak-coupling regime such that one approximates the pathological term $m\dddot{\mathbf{r}}(t) \approx -\kappa\dot{\mathbf{r}}(t)$ [1,8], or (iii) imposing an ultraviolet frequency cutoff in the spectrum of the free electromagnetic field [3,9].

In this Letter, we show that by simply promoting the position of the dipole $\mathbf{R}$ to a dynamical quantum d.o.f. $\hat{\mathbf{R}}$, one obtains, without further requirements, a consistent quantum theory that does not display the radiation reaction problem. That is, we describe the radiation reaction of a dipole using the extended Hamiltonian

$$\hat{H} = \hat{H}' + \frac{\hat{\mathbf{P}}^2}{2M} + V(\hat{\mathbf{R}}), \tag{3}$$

where $\hat{\mathbf{P}}$ is the conjugate momentum operator of the dipole position operator $\hat{\mathbf{R}}$, M the total observed mass of the dipole, and V an external potential for the dipole position d.o.f. By replacing $\mathbf{R} \to \hat{\mathbf{R}}$ in Eq. (1), $\hat{H}$ contains a threefold coupling term between the electromagnetic field, the dipole moment associated to the motion of the bounded electron, and the center-of-mass position of the dipole. In the electric dipole representation, this coupling term reads $q\hat{\mathbf{r}}\cdot\hat{\mathbf{E}}(\hat{\mathbf{R}})$, where $\hat{\mathbf{E}}$ is the electric field operator. We show how to trace out the electromagnetic field and the center-of-mass d.o.f. in order to get an effective dynamical equation for the dipole moment d.o.f. $\hat{\mathbf{r}}(t)$. We assume that the center of mass of the dipole does not move on average, $\langle\hat{\mathbf{R}}(t)\rangle = 0$, but does fluctuate, $\langle\hat{\mathbf{R}}^2(t)\rangle \neq 0$. Our analytical procedure leads to a dynamical equation that amends the Abraham-Lorentz Eq. (2) and does not suffer from the

radiation reaction problem. The amended dynamical equation leads to Eq. (2) in the limit $M \to \infty$, that is, if the center of mass does no longer fluctuate. Furthermore, we show within the developed theory that the zero-point quantum fluctuations of the electromagnetic field are necessary to satisfy the second law of thermodynamics [10].

In the following, we sketch the derivation of our result using the closed-time-path (CTP) formalism and the influence functional method with path integrals [11]. The detailed derivation can be found in the Supplemental Material [12]. The starting point of the theory is the action of the closed total system, which in the electric dipole representation is given by

$$S[\mathbf{R},\mathbf{r},A^\mu] = S_{\mathrm{CM}}[\mathbf{R}] + S'_{\mathrm{Dip}}[\mathbf{r}] + S_{\mathrm{EM}}[A^\mu] + S_{\mathrm{Int}}[\mathbf{R},\mathbf{r},A^\mu]. \tag{4}$$

We use the electromagnetic four-potential $A^\mu = (\phi, \mathbf{A})$. The first three terms are the actions describing the free dynamics of the subsystems. The action of the dipole center of mass is

$$S_{\mathrm{CM}}[\mathbf{R}] = \int_{t_{\mathrm{in}}}^{t_f} d\lambda \left(\frac{M}{2}\dot{\mathbf{R}}^2(\lambda) - V[\mathbf{R}(\lambda)] \right). \tag{5}$$

The action of the dipole internal d.o.f. is

$$S'_{\mathrm{Dip}}[\mathbf{r}] = \int_{t_{\mathrm{in}}}^{t_f} d\lambda \left[\frac{m}{2}\dot{\mathbf{r}}^2(\lambda) - \frac{\kappa'}{2}\mathbf{r}^2(\lambda) \right], \tag{6}$$

where κ' is the bare spring constant of the dipole [13]. The action of the free electromagnetic field is

$$S_{\mathrm{EM}}[A^\mu] = \frac{1}{2}\int d^4x [\mathbf{E}^2(x^\mu) - \mathbf{B}^2(x^\mu)], \tag{7}$$

where $\mathbf{B}$ is the magnetic field. We have used the four-vector coordinate $x^\mu = (x^0, \mathbf{x})$ and the notation $\int d^4x = \int_{t_{\mathrm{in}}}^{t_f} dx^0 \int d\mathbf{x}$. The fourth term in Eq. (4) accounts for the threefold interaction between the jiggling point dipole and the electromagnetic field and is given by

$$S_{\mathrm{Int}}[\mathbf{R},\mathbf{r},A^\mu] = q\int_{t_{\mathrm{in}}}^{t_f} d\lambda \mathbf{r}(\lambda)\cdot\mathbf{E}[R^\mu(\lambda)], \tag{8}$$

where $R^\mu(\lambda) = (\lambda, \mathbf{R}(\lambda))$. The initial state of the total system is assumed to be the product state $\hat{\rho}(t_{\mathrm{in}}) = \hat{\rho}_{\mathrm{CM}} \otimes \hat{\rho}_{\mathrm{Dip}} \otimes \hat{\rho}_{\mathrm{EM}}$.

The goal is to obtain the effective equation of motion for the dipole internal d.o.f. $\hat{\mathbf{r}}$ under the influence of the electromagnetic field and the fluctuations of its center-of-mass position. For this purpose, we trace out first the electromagnetic field and subsequently the center-of-mass d.o.f. The first step benefits from the fact that the dependence on the electric field in Eq. (8) is linear. The functional integrals that are required to trace out the electromagnetic field within the CTP formalism can be analytically calculated by assuming the initial state of the electromagnetic field to be Gaussian (e.g., thermal state at temperature $T = \beta^{-1}$), see Refs. [12,14,15].

The second step, namely, tracing out the center-of-mass d.o.f., is more involved due to its nonlinear dependence in Eq. (8). We develop a nonperturbative approximation technique based on expanding the influence functional in powers of q. We perform the functional integrations over the center of mass in the first term of the series. Then, having a formal expression for the influence action, we show that the first order expansion in the coupling corresponds to retain on the dynamics the contribution coming from the nth power of this first term. A perturbative expansion at the level of the action implies a nonperturbative approximation on the dynamics. Within the quantum theory of many-particle system with Green's functions, this technique resembles a first-order approximation in the self-energy of an interacting system, which implements a nonperturbative approximation for the interacting system [16]. This procedure allows us to obtain a quadratic influence action for the dipole internal d.o.f., which gives a CTP action of the form $S_{\mathrm{CTP}}[\mathbf{r},\mathbf{r}'] = S_{\mathrm{Dip}}[\mathbf{r}] - S_{\mathrm{Dip}}[\mathbf{r}'] + S^{(1)}_{\mathrm{RR}}[\mathbf{r},\mathbf{r}']$. Here, S_{Dip} has the same form as Eq. (6), but with the observed (renormalized) spring constant $\kappa' \to \kappa$. The influence action $S^{(1)}_{\mathrm{RR}}$ accounts for the dynamics of both the electromagnetic field and the dipole center-of-mass motion. Although $S^{(1)}_{\mathrm{RR}}$ is quadratic, its form is different from what one obtains for standard dissipative environments (e.g., Brownian motion [8,17]). Our problem contains a threefold nonlinear coupling that leads to a complicated influence action that becomes quadratic after the nonperturbative method. We remark that this scenario and approach is different from perturbative techniques applied to twofold nonlinear interactions [18].

The final step is to obtain the equation of motion for the dipole internal d.o.f., which is done by minimizing the CTP action, namely, $(\delta S_{\mathrm{CTP}}/\delta\mathbf{r})|_{\mathbf{r}'=\mathbf{r}} = 0$ [11]. This leads to the main result of this Letter, namely, the amended Abraham-Lorentz equation

$$m\ddot{\mathbf{r}}(t) + \kappa\mathbf{r}(t) - 2m\gamma\int_{t_{\mathrm{in}}}^{t} dt' D(t-t')\mathbf{r}(t') = 0, \tag{9}$$

where the memory function is given by

$$D(\tau) = \int_0^\infty \frac{d\omega}{(2\pi)^2}\omega^3 \exp\left[-\frac{\omega^2}{2}\Delta^2(\tau)\right]\left(\cos\left[\frac{\omega^2}{2}G(\tau)\right] \times \mathrm{Re}[\Gamma(\omega,\tau)] + 2\sin\left[\frac{\omega^2}{2}G(\tau)\right]\mathrm{Im}[\Gamma(\omega,\tau)]\right). \tag{10}$$

We use $\tau = t - t'$. The memory function includes the effect of the coupling to the electromagnetic field via the function

$$\Gamma(\omega,\tau) \equiv \theta(\tau)\sin(\omega\tau) + \frac{i}{2}\coth\left(\frac{\beta\omega}{2}\right)\cos(\omega\tau), \quad (11)$$

where $\theta(\tau)$ is the Heaviside step function. The imaginary part of Eq. (11) accounts for the fluctuations of the electromagnetic field, where $\coth(\beta\omega/2) = 1 + 2\bar{n}(\beta\omega)$ separates the quantum (zero-point) fluctuations from the classical (thermal) fluctuations. The effect of the center-of-mass fluctuations is included in the memory function via the functions $\Delta^2(t-t') \equiv \mathrm{tr}(\hat{\rho}_{\mathrm{CM}}[\hat{R}_j(t) - \hat{R}_j(t')]^2)$ and $G(t-t') = i\theta(t-t')\mathrm{tr}([\hat{R}_j(t), \hat{R}_j(t')])$, which are independent of the axis of motion j and only depend on the time difference for dynamics described by isotropic and quadratic Hamiltonians. Moreover, G is state independent in this case.

The amended Abraham-Lorentz Eq. (9) contains several features originating from the nonlinear threefold coupling in Eq. (8). On the one hand, the equation is non-Markovian with a memory function given by Eq. (10). This is due to the participation of the center of mass in the dipole moment dynamics via the threefold nonlinear coupling that generates delays through the energy exchange between the subsystems. Indeed, in the limit $M \to \infty$, the functions Δ, G vanish and $D(t-t') \to -\partial^3\delta(t-t')/\partial t'^3$. Therefore, in the fixed-dipole limit $M \to \infty$, Eq. (9) leads to the Abraham-Lorentz equation Eq. (2). On the other hand, the damping perceived by the dipole moment depends on the state of the electromagnetic field, namely, its temperature, which is a new feature in the radiation reaction scenario. Furthermore, the super-Ohmic nature of the electromagnetic field is not altered, as shown by the presence of the factor ω^3 in the memory function Eq. (10). However, the function $\exp[-\omega^2\Delta^2(t-t')/2]$ in Eq. (10) acts as a cutoff to the frequency integral since $\Delta^2(t-t') \geq 0$ for any $t - t' \geq 0$. This is a natural cutoff provided by the center-of-mass fluctuations that prevents the localization of the integral in time, similarly to what is achieved by assuming an ultraviolet frequency cutoff in the electromagnetic field [3,9].

In order to show that the amended Abraham-Lorentz equation (9) is free from the radiation reaction problem, it is convenient to write Eq. (9) in Fourier space as

$$[\omega^2 - \omega_0^2 + i\omega\mu(\omega + i0^+)]\mathbf{r}(\omega) = 0. \quad (12)$$

Here, the spectral distribution function $\mu(\omega + i0^+)$ is defined as the boundary value on the real axis of the function $\mu(z) \equiv 2\gamma(iz)^{-1}\int_0^\infty d(t-t')D(t-t')e^{iz(t-t')}$, where we have chosen $t_{\mathrm{in}} \to -\infty$. In this form, one can use the results of Ford, Lewis, and O'Connell (FLO) [10] to show that if $\mu(z)$ is a positive real function, then the following three general physical principles are fulfilled: (i) causality, which requires $\mu(z)$ to be analytical in the upper half plane $\mathrm{Im}[z] > 0$, (ii) the second law of thermodynamics, which enforces the real part of the spectral distribution to be positive $\mathrm{Re}[\mu(\omega + i0^+)] \geq 0$ in all the real axis, and (iii) that $\hat{\mathbf{r}}$ is Hermitian, which requires $\mu(\omega + i0^+) = [\mu(-\omega + i0^+)]^*$. Note that the Abraham-Lorentz equation Eq. (2) does not fulfill the FLO criteria [3,9], which is another manifestation of the radiation reaction problem. In contrast, we show in the following that the amended Abraham-Lorentz Eq. (9) does fulfil the FLO criteria when the initial states of the center of mass and the electromagnetic field have the same temperature.

In particular, we analytically show that the FLO criteria is met for the paradigmatic case of a free dipole [$V(\mathbf{R}) = 0$] with an initial motional state $\hat{\rho}_{\mathrm{CM}}$ given by a thermal state in a harmonic potential of frequency ω_I. This choice is crucial in order to have the dipole interacting with an environment (the electromagnetic field and the center-of-mass motion) at thermal equilibrium. Under this assumption FLO criteria (ii) is applicable. Such an initial Gaussian state is determined by $\langle\hat{R}_i(t_{\mathrm{in}})\rangle = \langle\hat{P}_i(t_{\mathrm{in}})\rangle = \langle\{\hat{R}_i(t_{\mathrm{in}}), \hat{P}_i(t_{\mathrm{in}})\}\rangle = 0$, and the initial thermal fluctuations $\langle\hat{R}_i^2(t_{\mathrm{in}})\rangle = [2\bar{n}(\beta\omega_I) + 1]/(2\omega_M\omega_I)$ and $\langle\hat{P}_i^2(t_{\mathrm{in}})\rangle = [2\bar{n}(\beta\omega_I) + 1]\omega_M\omega_I/2$, where $\omega_M \equiv M$ is the Compton frequency. We denote the ratio between the two relevant frequencies describing the center-of-mass dynamics as $\chi \equiv \omega_I/\omega_M$. With the center of mass of the dipole being in this initial thermal state, its mean position does not evolve within the free dynamics described by the action Eq. (5) with $V(\mathbf{R}) = 0$, but it does fluctuate. The thermal wave packet spreads, and this dynamics leads to the following particular expressions for $\Delta^2(t-t') = [2\bar{n}(\beta\omega_I) + 1](t-t')^2\omega_I/(2\omega_M)$ and $G(t-t') = \theta(t-t')(t-t')/\omega_M$. By plugging these functions into Eq. (10), which represent the influence that the center-of-mass d.o.f. exerts on the dynamics of the dipole moment d.o.f., one can obtain an expression for $\mu(z)$. It can be written as

$$\mu(z) = \frac{i\mu_0}{\pi z}\int_0^\infty dx x^2 \int_{-\infty}^\infty dy \frac{u(x,y)}{y + z/\omega_I}. \quad (13)$$

Here, we have defined the positive constant $\mu_0^{-1} \equiv 4\sqrt{\pi^3\chi\coth(\beta\omega_I/2)}/(\gamma\omega_I^2)$ and the function $u(x,y) = K_+(x,y) + \bar{n}(\beta\omega_I x)[K_+(x,y) + K_-(x,y)]$ with

$$K_\pm(x,y) = \exp\left[-\frac{(y \pm x + \chi x^2/2)^2}{\chi x^2\coth(\beta\omega_I/2)}\right] - \exp\left[-\frac{(y \mp x - \chi x^2/2)^2}{\chi x^2\coth(\beta\omega_I/2)}\right]. \quad (14)$$

The spectral distribution $\mu(\omega + i0^+)$ can be calculated from Eq. (13) using the distribution identity $i/(x + i0^+) = i\mathcal{P}(1/x) + \pi\delta(x)$. One can then readily prove that Eq. (13) meets the FLO criteria by showing that (i) the Cauchy-Riemann equations in the upper half plane are fulfilled for any nonzero value of M, (ii) the integrand of the real part of the spectral distribution is positive for any

value of χ and β (see Supplemental Material [12] for further details), and (iii) the real (imaginary) part of the spectral distribution is symmetric (antisymmetric) with respect to ω. In accordance with the FLO criteria [3,9], the amended Abraham-Lorentz equation (9) is causal, does not contain runaway solutions, and is consistent with the second law of thermodynamics. While this result has been explicitly (and analytically) shown for the paradigmatic case of a free dipole, it is expected to hold for other center-of-mass dynamics [e.g., assuming $V(\mathbf{R})$ is a harmonic potential].

To conclude the discussion of the results, let us turn to a subtle but intriguing observation. At finite temperatures, one could be tempted to ignore the quantum (zero-point) fluctuations of the electromagnetic field, in particular since the radiation reaction problem is known to appear also in a classical description of a radiating particle [2]. The effect of the fluctuations of the electromagnetic field is encoded in the term of the memory function Eq. (10) which contains the function $\mathrm{Im}[\Gamma(\omega,\tau)] = \coth(\beta\omega/2)\cos(\omega\tau)/2$. This function can be separated into two terms via $\coth(\beta\omega/2) = 1 + 2\bar{n}(\beta\omega)$. The first term accounts for the quantum (zero-point) fluctuations and the second term for the classical (thermal) fluctuations. Ignoring the quantum (zero-point) fluctuations, namely, considering a stochastic classical theory for the electromagnetic field, would result in an amended Abraham-Lorentz equation with the same form as Eq. (9), but with a memory function Eq. (10) that contains a modified Γ function given by $\Gamma_c(\omega,\tau) = \theta(\tau)\sin(\omega\tau) + i\cos(\omega\tau)/(\beta\omega)$, as opposed to Eq. (11). The Fourier transform of this modified memory function reads as Eq. (13) but with a modified $u(x,y)$ function given by

$$u_c(x,y) = \frac{1}{2}[K_+(x,y) - K_-(x,y)] + \frac{1}{\beta\omega_I x}[K_+(x,y) + K_-(x,y)]. \quad (15)$$

One can then readily show that while the FLO criteria (i) and (iii) are still fulfilled, and the theory thus maintains causality, there is a broad range of parameters χ and β for which the FLO criterion (ii) is not fulfilled (see Supplemental Material [12] for further details). Therefore, the quantum (zero-point) fluctuations of the electromagnetic field are necessary to have a theory that is consistent with the second law of thermodynamics. In other words, a radiation reaction theory for a dipole that includes its center-of-mass dynamics would be causal but to be physically consistent, the quantum (zero-point) fluctuations of the electromagnetic field have to be accounted for in order to respect the second law of thermodynamics. We remark that this observation is only pertinent for a moving dipole. In the fixed-dipole limit $M \to \infty$, the term in the memory function Eq. (10), which contains the relevant function $\mathrm{Im}[\Gamma(\omega,\tau)]$ distinguishing classical versus quantum fluctuations of the electromagnetic field, vanishes.

In summary, in this Letter we have provided an amended Abraham-Lorentz theory for a point-dipole quantum oscillator that is physically consistent and it can be derived from nonrelativistic quantum electrodynamics without additional assumptions. The crucial point to circumvent the long-standing radiation reaction problem for the case of a charged harmonic oscillator is to account for the center-of-mass d.o.f. of the point dipole. In this context, we have shown that the quantum (zero-point) fluctuations of the electromagnetic field are crucial to obtain a physically consistent theory, even at finite temperatures, since otherwise the theory would violate the second law of thermodynamics. From a technical point of view, accounting for the center-of-mass dynamics renders the electrodynamical problem of a point-dipole quantum oscillator interacting with the electromagnetic field nonlinear. We were able to obtain analytical results by using path integral techniques [11], which are proven to be very well suited to this problem. In particular, we developed a nonperturbative approximation assuming weak coupling at the level of the action.

Furthermore, our results open new research directions. While we have here focused on the effective dynamics of the dipole moment d.o.f., it should be possible to use similar techniques to describe the effective dynamics of the center-of-mass d.o.f. In particular, it might be feasible to obtain a consistent theory that describes the equilibration of the center of mass of a dipole interacting with a thermal electromagnetic field, a long-standing open question originally discussed by Einstein and Hopf [19,20]. Furthermore, while we have shown how to circumvent the radiation reaction problem of a point dipole by including additional d.o.f., one could investigate whether the same can be achieved for a moving free point charge (e.g., an electron) by including spin d.o.f. Last but not least, it would be very exciting to explore whether such fundamental questions can be addressed experimentally with the new generation of experiments trapping atoms and dielectric nanoparticles in high vacuum near photonic nanostructures, where large light-matter couplings can be engineered, as a complement to the recent radiation reaction experiments with high energy electrons [21–23].

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