Antiparticles as Particles: QED_{2+1} and Composite Fermions at $\nu = \frac{1}{2}$

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We present a continuum operator map that inverts the role of particles and antiparticles in three-dimensional QED. This is accomplished by the attachment of specific holomorphic Wilson lines to Dirac fermions. We show that this nonlocal map provides a continuum realization of Son's composite fermions at $\nu = \frac{1}{2}$. The inversion of Landau levels and the Dirac-cone–composite-fermion duality is explicitly demonstrated for the case of slowly varying magnetic fields. The role of Maxwell terms as well as the connection of this construction to a gauge-invariant formulation of 3D gauge theories is also elaborated upon.

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Introduction.—In this Letter, we construct a particle-hole symmetric description of QED in $D = 2 + 1$, while working within a Hamiltonian framework in the continuum. This construction is motivated by Son's proposed composite-fermion picture for quantum Hall states [1], which has led to a conjectured duality [2–6] between the Dirac cone action

$$S_1 = -i \int \bar{\Psi}\gamma^\mu(\partial_\mu - iA_\mu)\Psi + \cdots \tag{1}$$

and the composite fermion theory defined by the action:

$$S_2 = -i \int \bar{\chi}\gamma^\mu(\partial_\mu - ia_\mu)\chi - \frac{1}{4\pi}\int \epsilon^{\mu\nu\rho} a_\mu \partial_\nu A_\rho + \cdots. \tag{2}$$

In the second theory A is to be regarded as an external electromagnetic field while a is dynamical. (In our convention $\{\gamma^\mu = i\sigma^3, \sigma^1, \sigma^2\}$.)

It is important to note that the a_0 variation of (2) yields a constraint on the density of the composite fermions: $\rho_\chi = \langle \chi^\dagger \chi \rangle = (B/4\pi)$. B is magnetic field associated with A. On the other hand the electromagnetic charge density $\rho_e = (\delta S_2/\delta A_0) = -(b/4\pi)$, where b is the magnetic field associated with a. The associated filling fractions $\nu_e = 2\pi(\rho_2/B)$ and $\nu_\chi = 2\pi(\rho_\chi/b)$ are related as

$$2\nu_e = -\frac{1}{2\nu_\chi}. \tag{3}$$

This inversion is at the heart of the realization of the particle-hole symmetry in the composite fermion picture [5,6].

From a quantum field theory point of view, the proposed duality presents a fermionic version of the bosonic particle-vortex duality and it has been absorbed in the web of 3D dualities that have been the subject of much recent investigation [7–10]. Given the growing importance of this proposed duality in both high energy as well as condensed matter physics, it is imperative to ask if an explicit map between the two theories described by (1) and (2) may be established. In this Letter, we present a continuum realization of this duality at $\nu = \frac{1}{2}$ by finding an operator map between QED and a dual gauge theory that inverts the role of particles and antiparticles. Before presenting our construction, we briefly review some salient lessons learned from various approaches to understanding the duality which will play an important role in what is to follow.

Effective actions obtained by integrating out Ψ and χ provide important clues about how the proposed duality might work. The leading IR effective actions are as follows:

$$S_1 \to S_{1\,\text{eff}} = \pm\frac{1}{8\pi}\int \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho, \tag{4}$$

while

$$S_2 \to S_{2\text{eff}} = \mp\frac{1}{8\pi}\int \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho \mp \frac{1}{4\pi}\int \epsilon^{\mu\nu\rho} a_\mu \partial_\nu A_\rho. \tag{5}$$

As is well known, the absolute signs of half-quantized Chern-Simons in the two actions are ambiguous for the case of massless fermions. The relative signs between the various terms in the two actions are more crucial for the purposes of this Letter. The context of topological insulators [3,4] provides a microscopic physical context for understanding why the relative signs between the Chern-Simons terms are reversed [4]. For latter convenience, we note that the a_0 equation of motion from the second equation above leads to

$$b = -B. \tag{6}$$

It is easily seen that eliminating the dynamical a fields from (5) leads to (4). It is clearly desirable to understand the emergence of effective actions mentioned along with their relative signs independently of the context of TIs, and examine the duality beyond the lowest order in the IR.

In a very promising development, an explicit lattice realization of this duality was recently claimed in [11] (see also [12–14]). Reference [11] employed the formidable machinery of 2D bosonization to map operators on both sides of the duality to each other. However, no continuum analog of the lattice operator maps is presently known. While the construction presented here is defined directly in the continuum, it interestingly shares several parallels with [11].

The analysis of the present Letter may be taken as complementary to the path integral approaches [7,8] that focus on relating currents and partition functions of (1) and (2). The present Hamiltonian approach, which uses the complex structure inherent to the spatial geometry has the advantage of allowing a precise map between microscopic degrees of freedom of QED_{2+1} and its particle-hole dual.

Gauge fields and the dual photon.—Our starting point is the realization that spatial components of gauge fields $A=\frac{1}{2}(A_1+iA_2)$ and $\bar{A}=\frac{1}{2}(A_1-iA_2)$ can be parametrized in terms of the $SL(N,\mathbb{C})$-valued complex matrices M and $M^\dagger$ as

$$A = -\partial M M^{-1}, \qquad \bar{A} = M^{\dagger -1}\bar{\partial} M^\dagger. \tag{7}$$

It is understood that $\partial = \frac{1}{2}(\partial_i + i\partial_2)$ and $z = x_i - ix_2$ This parametrization—which is at the heart of the gauge invariant formulation of gluodynamics initiated in [15]—applies in general to non-Abelian $SU(N)$ gauge fields $A = -iA^a t^a$, where t^a are the $SU(N)$ generators. An advantage of this parametrization is that local (time-independent) gauge transformations are simply realized as $M \to UM$, where U is a unitary matrix. This implies that $H = M^\dagger M$ is gauge invariant. H is the physical degree of freedom in terms of which strongly coupled gluodynamics in $D = 2+1$ was formulated in [15]. The matrix M can be regarded as a *complex* Wilson line as it satisfies (from its defining relation) $DM = 0$, where $D = \partial + A$. The real and imaginary parts of this matrix correspond to the physical and gauge degrees of freedoms encoded in A, respectively. This is seen most explicitly in the Abelian limit—which is the primary focus of this Letter—where we can express

$$M = e^\theta, \tag{8}$$

where θ is a complex scalar. In this limit the parametrization (7) reduces to the usual gradient-curl (Hodge) decomposition $A_i = \partial_i \mathrm{Im}(\theta) + \epsilon_{ij}\partial_j \mathrm{Re}(\theta)$. One can also define a set of auxiliary gauge fields which we suggestively call a, $\bar{a}$—for reasons that would soon be manifest—as

$$a = M^{\dagger -1}\partial M^\dagger, \qquad \bar{a} = -\bar{\partial} M M^{-1}, \tag{9}$$

(7) and (9) yield the relations

$$D\bar{a} = \bar{\partial} A, \qquad \bar{D} a = \partial \bar{A}, \tag{10}$$

which, in the Abelian limit, imply (6). We will now focus on the lowest Landau levels (LLLs) for fermions coupled to A and a, respectively, and show that for a given value of B, the roles of particles and holes are interchanged for fermions coupled to A and a, respectively. We first consider the case of a constant background magnetic field $B > 0$. It is also instructive—especially for comparison with [11]—to include a mass term for the fermions. The (mass-added) Hamiltonian following from (1) is

$$H[\Psi] = i\int \bar{\Psi}(\gamma^i D_i - m)\Psi = \frac{1}{2}\int \Psi^* \begin{pmatrix} m & -2iP_- \\ 2iP_+ & -m \end{pmatrix} \Psi. \tag{11}$$

We can choose the gauge where $A_i = -\frac{1}{2}\epsilon_{ij}x_j B$ and $\epsilon_{12} = +1. P_+ = -i\partial - (i/4)B\bar{z},\ P_- = -i\bar{\partial} + (i/4)Bz$. The momenta are mapped to the standard oscillator variables as $P_+ = \sqrt{(B/2)}A$, $P_- = \sqrt{(B/2)}A^\dagger$. The spectrum is given by $E^2 - 4P_-P_+ = m^2$ and it is easy to see that $\Psi_0 = \binom{\psi_1}{0}: P_+\psi_1 = 0$ is a normalizable zero mode for both $E = \pm m$. The vacuum is taken to be the state where all the negative energy levels are filled. When $E = +m$, the lowest energy state has positive energy, so the vacuum $|\Omega_\Psi\rangle$ can be identified with the Fock vacuum $|0\rangle_\Psi$. However, when $E = -|m|$, the zero mode must be regarded as part of the negative energy states. Thus the vacuum $|\Omega_\Psi\rangle$, in this case is not the Fock vacuum, but must be taken to be

$$|\Omega_\Psi\rangle = \alpha^\dagger |0\rangle_\Psi. \tag{12}$$

It is understood that $\alpha^\dagger$ is the oscillator for the zero mode in the harmonic expansion of the field given by

$$\Psi = \alpha\Psi_0 + \sum_n \alpha_n \Psi_n^+ + \sum_n \beta_n^\dagger \Psi_n^-, \tag{13}$$

where the quantization condition $\alpha|0\rangle_\Psi = 0$ is implied. Using $\frac{1}{2}[\Psi^\dagger, \Psi] = \sum_n(\alpha_n^\dagger\alpha_n - \beta_n^\dagger\beta_n) + \frac{1}{2}(\alpha^\dagger\alpha - \alpha\alpha^\dagger)$, along with the fact that the degeneracy is given by $(B/2\pi)$, we can compute the electronic charge density to be

$$\rho_e = \langle\Omega_\Psi|\frac{1}{2}[\Psi^\dagger, \Psi]|\Omega_\Psi\rangle = \frac{B}{2\pi}\langle\Omega_\Psi|\frac{1}{2}(\alpha^\dagger\alpha - \alpha\alpha^\dagger)|\Omega_\Psi\rangle, \\ = -\frac{m}{|m|}\frac{B}{4\pi}. \tag{14}$$

We can also read off the effective action (assuming slowly varying B) via

$$\rho_e = \frac{\delta}{\delta A_0} S_\Psi^{\text{eff}} = -\frac{m}{|m|}\frac{1}{8\pi}\frac{\delta}{\delta A_0}\int \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho. \quad (15)$$

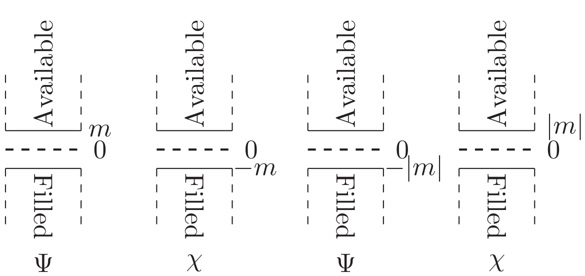
The analysis so far recapitulates the well-known picture for the emergence of an effective Chern-Simons action from LLL physics. The filling fraction here is $\nu_e = -\frac{1}{2}(m/|m|)$. However, when one couples fermions χ (with the *same* mass term) to the a fields in the same background magnetic field $B = -b$, the resultant Dirac equation is

$$\begin{pmatrix} m & -2iP(a)_- \\ 2iP(a)_+ & -m \end{pmatrix}\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = E\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}, \quad (16)$$

where $-2iP(a)_- = -(2\bar{\partial} + \frac{1}{2}zB)$ and $2iP(a)_+ = (2\partial - \frac{1}{2}\bar{z}B)$. Because of the change in the sign of the magnetic field for a relative to B, $P(a)_+$ now acts as the creation operator as opposed to the previous case. The normalizable zero modes now correspond to $E = \mp\, m$ for $(m/|m|) = \pm 1$, which is the converse of case of fermions coupled to A. Using a mode expansion for χ

$$\chi = \mathcal{A}\chi_0 + \sum_n \mathcal{A}_n \chi_n^+ + \sum_n \mathcal{B}_n^\dagger \chi_n^-, \quad (17)$$

we see that $|\Omega_\chi\rangle$ for $E = -(+)m$ is given by $\mathcal{A}^\dagger|0\rangle_\chi(|0\rangle_\chi)$. $\mathcal{A}$ is the oscillator for the zero mode of χ and $|0\rangle_\chi : \mathcal{A}|0\rangle_\chi = 0$. The complete momentum space mode expansion for Ψ and χ in a constant magnetic field is given in the Supplemental Material [16] using which it can be explicitly checked that the Landau levels of χ are obtained by inverting those for Ψ for a given fixed value of B and m. This is depicted in the figure below.



Case 1, $m > 0$ Case 2, $m < 0$

This inversion suggests that the holes of the χ field depict particles of the Ψ field and vice versa (we will substantiate this further later). Noting that the magnetic field experienced by dual χ particles is b, with the degeneracy given by $(b/2\pi)$, the induced charge density for χ is computed to be

$$\rho_\chi = \langle\Omega_\chi|\frac{1}{2}[\chi^\dagger, \chi]|\Omega_\chi\rangle = +\frac{m}{|m|}\frac{b}{4\pi},$$
$$= \frac{m}{|m|}\frac{\delta}{\delta a_0}\int\left(\frac{1}{8\pi}\epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho\right). \quad (18)$$

We note that the sign of the Chern-Simons term is reversed compared to (15), even though the sign of the mass term has *not* changed.

But what action does (16) follow from? It is relatively easy to see that the action whose quantization in a constant magnetic field background generates (16) is given precisely by

$$S(\chi) = -i\int \bar{\chi}(\gamma^\mu(\partial_\mu - i a_\mu) - m)\chi + \frac{m}{|m|}\frac{1}{4\pi}\int \epsilon^{\mu\nu\rho} a_\mu \partial_\nu A_\rho. \quad (19)$$

The gauge fields a_μ are not constrained at the level of the action. Quantization of this action in the Hamiltonian picture in the $a_0 = 0$ gauge requires that the Gauss law constraint $\rho_\chi = \langle\chi^\dagger\chi\rangle = (B/4\pi)$ be solved. This is achieved via the parametrization (9) of the spatial components of a_μ (which imply $b = -B$), resulting in a consistent Hamiltonian quantization of $S(\chi)$. Using (18), we also see that $\nu_\chi = +\frac{1}{2}(m/|m|)$ and satisfies (3). The complete effective action for χ is given by the Chern-Simons term computed from ρ_χ as well as the mixed term already present in $S(\chi)$.

$$S_\chi^{Eff} = \frac{m}{|m|}\frac{1}{8\pi}\int \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho + \frac{m}{|m|}\frac{1}{4\pi}\int \epsilon^{\mu\nu\rho} a_\mu \partial_\nu A_\rho, \quad (20)$$

which agrees with (5). The two signs in (5) correspond to the two possible values of $(m/|m|)$. It is also understood that S_χ^{eff} and S_Ψ^{eff} are the leading order effective actions. In general, there might be further terms depending on details of the UV completion employed; however they are not immediately relevant for the present discussion.

Fermions and dual fermions.—We have so far seen that fermions coupled to the auxiliary gauge fields a satisfy the properties of Son's composite fermions at half-filling. However to really establish a duality we need a map between the Ψ and χ fermions. This is what we do next.

Let us define composite fermions χ in terms of the original spinor Ψ by the following operator map:

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} M(M^{\dagger -1})^t \chi_2^* \\ M^{\dagger -1} M^t \chi_1^* \end{pmatrix} = \exp\left[2i\text{Im}(\theta)\right]\begin{pmatrix} \chi_2^* \\ \chi_1^* \end{pmatrix} \quad (21)$$

and

$$\begin{pmatrix} \psi_1^* \\ \psi_2^* \end{pmatrix} = \begin{pmatrix} (M^\dagger)^t M^{-1}\chi_2 \\ (M^{-1})^t M^\dagger \chi_1 \end{pmatrix} = \exp\left[-2i\text{Im}(\theta)\right]\begin{pmatrix} \chi_2 \\ \chi_1 \end{pmatrix}. \quad (22)$$

A straightforward computation now shows that, under this map,

$$H[\Psi] = i \int \bar{\Psi}(\gamma^i D_i - m)\Psi = i \int \bar{\chi}(\gamma^i d_i - m)\chi = H[\chi], \tag{23}$$

where d_i is the standard covariant derivative in which A is replaced by a. This gives us an explicit operator map from the original electronic theory to the Hamiltonian for the dual (χ) fermions directly in the continuum. This is the central result of the Letter, which we explore further below.

A few words about the nature of the dual fermions are in order at this point. First, we note that (21) and (22) preserve the symplectic structure in the Abelian theory. Further, the mass terms for the two fermions are mapped to each other without a relative sign being introduced, just as was reported in the lattice construction [11]. The mapping of the mass terms to each other is also crucial for consistency with the LLL analysis performed earlier. In the Abelian case, we can explicitly express M as a holomorphic Wilson line. $M = \exp(\int_{-\infty}^{z} iA)$. The factor $\exp[2i\mathrm{Im}(\theta)]$ in the definition of χ can thus be viewed as a field dependent gauge transformation generated by attaching two complex Wilson lines to the original fermion Ψ. The factors of M in (21) and (22) thus play a role similar to the monopole operators used in [7,8]. Further, we note that both $(\chi\Psi) \to e^{i\mathrm{Im}(\theta)}(\chi\Psi)$ under $U(1)$ gauge transformations. The common $U(1)$ charge allows us to interpret χ as describing bonafide particles (as opposed to simply antiparticles of Ψ) coupled to the gauge field a_μ. Finally, we note that (21) and (22) imply that the mode creation or annihilation operators of Ψ and χ in constant magnetic field backgrounds are related as

$$\alpha = \mathcal{A}^\dagger, \qquad \mathcal{B}_n = \alpha_n \quad \text{and} \quad \mathcal{A}_n = \beta_n. \tag{24}$$

The details leading to the above are presented in the Supplemental Material [16]. As an added consistency check we note that

$$\langle\Omega_\chi|\frac{1}{2}[\chi^\dagger, \chi]|\Omega_\chi\rangle = \langle\Omega_\Psi\frac{1}{2}[\chi^\dagger, \chi]|\Omega_\Psi\rangle. \tag{25}$$

The left-hand side refers to the computation already presented in section II, where $|\Omega_\chi\rangle$ was computed after explicitly imposing the constraint $B = -b$ on the a field (while no relationship was assumed among Ψ and χ) and the quantization condition for $|0\rangle_\chi$ was the usual one: $\mathcal{A}|0\rangle_\chi = 0$. The right-hand side uses the explicit map between the fermions [(21) and (24)] and utilizes the fact that the quantization condition for χ changes on $|\Omega_\Psi\rangle$, since (24) implies that $\mathcal{A}^\dagger|0\rangle_\psi = 0$. The BF term in (20) imposes precisely this altered quantization condition. This is further evident when we note that integrating out A_μ in (20) amounts to averaging over the constraints and it yields $S_\chi^{\mathrm{eff}} = -(m/|m|)\frac{1}{8\pi}\int \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$: the expected Chern-Simons term for χ coupled to gauge fields a_μ without any constraints *vis-á-vis* A_μ.

The analysis presented above extends to the massless case. The LLL computation presented in the previous section included a fermionic mass term, which served as a regulator, making it manifest that the spectra of $H[\Psi]$ and $H[\chi]$ approach the zero energy state (in the massless limit) from opposite directions (this is clear from the earlier figure). However, the operator map [(21) and (22)] allows us to work directly in the massless limit. For example, the relations (24) implied by the map ensure the correct relative signs between the induced Chern-Simons terms of χ and Ψ once a Fock vacuum is defined for the operator algebra ($\alpha|0\rangle = 0$). It is straightforward to check that all other statements made in the previous section also apply directly to the massless limit once the quantization condition for χ is fixed via the operator maps (21), (22), and (24).

Maxwell terms.—Finally we focus on making sense of the ellipses in (1) and (2), which stand for potential Maxwell terms. Having analyzed the mapping of the fermionic Hamiltonians, we now show how the Maxwell terms for the A and a fields transform into each other under (21) and (22). For this purpose we employ the gauge invariant Hamiltonian framework for 3D gauge fields coupled to fermions developed in [17], where it was shown how the photonic Hamiltonian is sensitive to the choice of gauge invariant fermionic variables. In particular, the sign of the induced Chern-Simons term uniquely dictates how gauge invariant fermionic variables may be constructed by attaching holomorphic Wilson lines to Dirac fermions. For the purposes of specificity, we fix $(m/|m|) = -1$. Following [17], the gauge invariant fermionic variable compatible with the induced CS term for a is

$$\tilde{\Lambda} = \begin{pmatrix} \tilde{\lambda}_1 \\ \tilde{\lambda}_2 \end{pmatrix} = \begin{pmatrix} \mathcal{M}^\dagger \chi_1 \\ \mathcal{M}^{-1}\chi_2 \end{pmatrix}, \tag{26}$$

where $\mathcal{M} = M^{\dagger -1}$. This change of variables can be regarded as a 2D chiral transformation, whose Jacobian is a Wess-Zumino-Witten (WZW) functional [17]. The measure on Hilbert space of the composite fermion transforms under the above as [17] $d\chi^* d\chi \to e^{-2\mathcal{S}(H^{-1})} d\check{\tilde{\Lambda}} d\tilde{\Lambda}$, where $\check{\tilde{\Lambda}}$ is the canonical conjugate to $\tilde{\Lambda}$. $\mathcal{S}$ denotes the WZW functional [17,18] and it is the boundary piece of the Chern-Simons term induced by the fermions in the effective action picture. Using the currents,

$$J_l = \partial H H^{-1}, \qquad J_r = -H^{-1}\partial H, \tag{27}$$

the Maxwell Hamiltonians for the A and a fields take on the following forms:

$$H[A] = \frac{e^2}{2\pi}\int\left(-J_l\frac{\delta}{\delta J_l} + \frac{1}{(x-y)^2}\frac{\delta}{\delta J_l(x)}\frac{\delta}{\delta J_l(y)}\right) + \frac{1}{2e^2}\int B^2 \tag{28}$$

and

$$H[a] = \frac{e^2}{2\pi}\int\left(-J_r\frac{\delta}{\delta J_r} + \frac{1}{(x-y)^2}\frac{\delta}{\delta J_r(x)}\frac{\delta}{\delta J_r(y)}\right) + \frac{1}{2e^2}\int b^2. \tag{29}$$

We will refer to [17,18] for a derivation of the gauge invariant photonic Hamiltonians (we present a brief outline in the Supplemental Material [16]). Here we draw attention to the fact that the first terms in $H[A]$ and $H[a]$ act as mass terms for the photon, and they are a direct consequence of a the nontrivial measure ($\mathcal{S}$) on the configuration space. The second term in the Hamiltonians is nothing but the two-point function in the OPE of the WZW model [15]. Noting that $J_l = \partial[\mathrm{Re}(\theta)]$, $J_r = -\partial[\mathrm{Re}(\theta)]$, and $b = -B$, it is trivial to see that the two Hamiltonians map to each other term by term. As with the fermionic Hamiltonian analyzed earlier, the photonic analysis continues to hold when $m = 0$ so long as care is taken to let m approach zero from a fixed side of the real axis. Finally, we point out that the WZW functional is well known in the context of 2D bosonization, which, in turn, was central to the lattice approach to this duality [11]. The interchange of the left and right currents $J_l \leftrightarrow J_r$ between the A and a descriptions is very reminiscent of a similar phenomenon observed on the lattice [11]. It would be extremely interesting to explore if a closer connection exists between the gauge invariant continuum approach presented here and lattice bosonization methods.

Conclusions and future directions.—In the present Letter we have shown that $d = 2+1$ QED with a single species of Dirac fermion can be mapped to a dual description via an operator map (21) and (22) where (i) the dual fermions possess properties expected of Son's composite fermions including the inversion of filling fractions (3) and the generation of the expected effective actions [(15) and (20)]. The operator map explicitly inverts the roles of particles and holes while endowing both Ψ and χ with the same $U(1)$ transformation properties. (ii) The gauge field A is mapped in a Hamiltonian framework to a dual photon a defined by (10). The relationship between the gauge fields solves the Gauss's law constraint for the action of the composite fermion (19). And (iii) parity violating mass terms for the fermions—which map to each other—act as regulators in this analysis. The entire analysis continues to hold in the $m = 0$ limit so long as care is taken to let m approach zero keeping its sign fixed. The upper and lower signs in (4) and (5) correspond to m approaching zero from the positive and negative halves of the real axis, respectively.

Finally, a gauge invariant formulation allows us to map the corresponding Maxwell terms of the two theories to each other in the $A_0 = a_0 = 0$ gauges.

While not a hindrance to the analysis and operator maps presented in this Letter, the theories studied here are known to be anomalous (as seen from the half-quantized induced Chern-Simons term in the effective actions). This can be amended in the special case of flat spacetimes by adding a second fermionic species to both sides of the duality with a large parity violating mass term. As discussed in [8] the more general solution valid in curved spaces involves the addition of gravitational Chern-Simons terms to both sides of the duality.

Several additional issues also require further study. The constraint $B = -b$ restricts the construction to $\nu = \frac{1}{2}$. Can this analysis be generalized to other filling fractions? Following the tell-tale signs noted earlier, is there a possible connection between the gauge-invariant framework described here and bosonization techniques employed in the coupled-wire constructions? Finally, the present results—reliant as they are on the complex structure provided by the flat geometry of the plane—are valid in flat spacetimes. The starting point of the present Letter (7) needs to augmented by auxiliary fields to account for zero modes of A in spaces of nontrivial topologies. The duality studied here was formulated in spaces with nontrivial geometries admitting magnetic fluxes in [8]. Can the present operator map be generalized to such geometries? We hope future work will clarify these issues.

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