Exact Correlation Functions for Dual-Unitary Lattice Models in 1+1 Dimensions

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We consider a class of quantum lattice models in $1+1$ dimensions represented as local quantum circuits that enjoy a particular dual-unitarity property. In essence, this property ensures that both the evolution in time and that in space are given in terms of unitary transfer matrices. We show that for this class of circuits, generically nonintegrable, one can compute explicitly all dynamical correlations of local observables. Our result is exact, nonpertubative, and holds for any dimension d of the local Hilbert space. In the minimal case of qubits ($d=2$) we also present a classification of all dual-unitary circuits which allows us to single out a number of distinct classes for the behavior of the dynamical correlations. We find noninteracting classes, where all correlations are preserved, the ergodic and mixing one, where all correlations decay, and, interestingly, also classes that are both interacting and nonergodic.

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Spatiotemporal correlation functions of local observables provide the most fundamental and useful physical description of locally interacting classical or quantum many-body systems [1,2]. They characterize ergodic properties [3], as well as basic transport coefficients of many-body systems, such as conductivities, Drude weights, and kinematic viscosities [4]. Moreover, correlation functions are directly measurable quantities, e.g., via x-ray diffraction or neutron scattering in solid-state materials [4], or via quantum optical detection techniques in cold atomic gases [5].

In spite of their extraordinary relevance, an exact nonperturbative calculation of correlation functions is generically feasible only in free (Gaussian) theories, or for certain types of completely integrable models [6]. To date, no specific example of a strongly coupled, nonintegrable, local quantum many-body system with accessible correlation functions is known.

The situation is somewhat different for classical dynamical systems with few degrees of freedom, where some exactly tractable examples have been found, for instance, Arnold's cat map, Baker's map, and dispersing billiards [3,7–9]. In classical dynamical systems the decay of correlation functions leads to a rigorous criterion of ergodicity and dynamical mixing [7]. Moreover, the presence of tractable systems allows for rigorous results on quantum eigenstate ergodicity [10–12]. On the contrary, ergodic theory of quantum many-body systems is currently in its early infancy. In particular, in locally interacting systems it is exceptionally challenging to separate relaxation (mixing, or scrambling) mechanisms from the mere decaying time correlations of local operators. The former requires dynamical complexity and thus the absence of integrability [13–15], while the latter occurs even in quasifree systems [14,16,17].

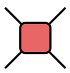
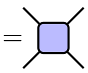
Here we take the first step toward a rigorous ergodic theory for quantum many-body systems by identifying a class of quantum lattice models in one spatial dimension with explicitly accessible spatiotemporal correlation functions for arbitrary pairs of ultralocal observables. These models are generically nonintegrable and can be formulated in terms of local quantum circuits with local gates exhibiting a particular dual-unitarity property. Specifically, they are unitary and remain unitary under a reshuffling of their indices, ensuring that the full quantum circuit defines unitary evolutions in both time and space directions. This class of models includes both the nonintegrable self-dual kicked Ising model (SDKI), where the aforementioned feature recently enabled us to find exact results on spectral correlations and entanglement entropy dynamics [18,19], and some integrable Floquet systems [20–22].

More specifically, we consider quantum systems defined on a periodic chain of $2L$ sites, where each site is equipped with a d-dimensional local Hilbert space $\mathcal{H}_1=\mathbb{C}^d$ with a basis $\{|i\rangle; i=1,2\ldots d\}$; the Hilbert space of the system is then $\mathcal{H}_{2L}=\mathcal{H}_1^{\otimes 2L}$. The time evolution is discrete and local. In particular, each time step is divided into two halves. In the first half the system is evolved by the transfer matrix $\mathbb{U}^e=U^{\otimes L}$, where $U\in \mathrm{End}(\mathcal{H}_1\otimes\mathcal{H}_1)$ is the local gate and encodes the physical properties of the system. In the second half, instead, the system is evolved by $\mathbb{U}^o=\mathbb{T}_{2L}\mathbb{U}^e\mathbb{T}_{2L}^\dagger$, where $\mathbb{T}_\ell|i_1\rangle\otimes|i_2\rangle\cdots|i_\ell\rangle\equiv|i_2\rangle\otimes|i_3\rangle\cdots|i_\ell\rangle\otimes|i_1\rangle$ is an ℓ-periodic translation by one site. This means that the transfer matrix for an entire time step is given by

$$\mathbb{U}=\mathbb{U}^o\mathbb{U}^e=\mathbb{T}_{2L}U^{\otimes L}\mathbb{T}_{2L}^\dagger U^{\otimes L}. \tag{1}$$

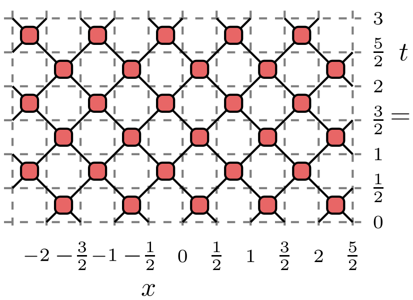
Note that from the definition (1) it immediately follows that $\mathbb{U}$ is invariant under two-site shifts $\mathbb{U}\mathbb{T}_{2L}^2=\mathbb{T}_{2L}^2\mathbb{U}$.

Before continuing, we note that the systems under examination admit a convenient diagrammatic representation. One depicts operators as boxes with a number of incoming and outgoing legs corresponding to the number of local sites they act on. Each leg (or wire) carries a Hilbert space $\mathcal{H}_1$. For instance, operators acting on a single site are represented as a line with a bullet •, while the local gate and its Hermitian conjugate are represented as

$U =$  , $\quad U^\dagger =$  . (2)

We stress that, even if we use a symmetric symbol for U, we assume no symmetry under reflection (left-to-right flip) and time reversal (up-to-down flip).

This diagrammatic representation allows us to depict the trace of the propagator for t steps as a partition function of a certain vertex model:

$\mathrm{tr}\,[\mathbb{U}^t] =$  $= \mathrm{tr}[\tilde{\mathbb{U}}^L]$. (3)

Here the transfer matrix $\mathbb{U}$ corresponds to two consecutive rows, while the dual transfer matrix $\tilde{\mathbb{U}} \in \mathrm{End}(\mathcal{H}_1^{\otimes 2t})$ corresponds to two consecutive columns, and the boundary conditions in both directions are periodic. As it is clear from the diagram, the dual transfer matrix reads as

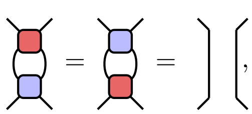
$$\tilde{\mathbb{U}} = \mathbb{T}_{2t}\tilde{U}^{\otimes t}\mathbb{T}_{2t}^{\dagger}\tilde{U}^{\otimes t}, \tag{4}$$

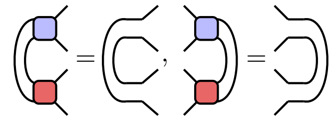
where we introduced the dual local gate $\tilde{U}$ by means of the following reshuffling:

$$\langle k| \otimes \langle \ell|\tilde{U}|i\rangle \otimes |j\rangle = \langle j| \otimes \langle \ell|U|i\rangle \otimes |k\rangle. \tag{5}$$

The dual gate defines the evolution in a circuit where the roles of time and space have been swapped.

In this Letter we consider quantum circuits with *unitary* local gates U such that $\tilde{U}$ is also unitary. Namely, we require [23]

$UU^\dagger = U^\dagger U = \mathbb{1} \;\Rightarrow$  , (6)

$\tilde{U}\tilde{U}^\dagger = \tilde{U}^\dagger\tilde{U} = \mathbb{1} \;\Rightarrow$  . (7)

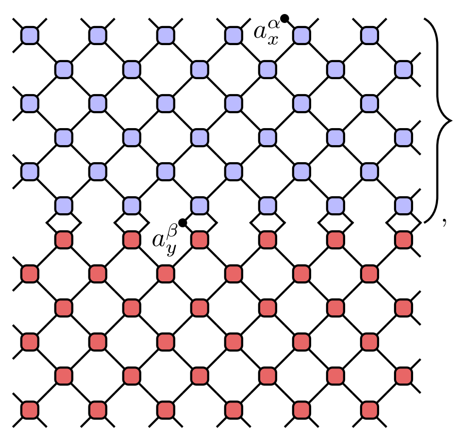
We call "dual unitary" local gates fulfilling both Eqs. (6) and (7) (these conditions immediately imply that $\mathbb{U}$ and $\tilde{\mathbb{U}}$ are also unitary). In the following we show that dual-unitary gates provide a remarkable testing ground for studying dynamical correlations in many-body quantum systems. They allow us to classify a number of qualitatively different physical behaviors [29].

Here we consider dynamical correlation functions of local operators in the general time-translation invariant, tracial, or infinite temperature state. These quantities are defined as follows:

$$D^{\alpha\beta}(x,y,t) \equiv \frac{1}{d^{2L}}\mathrm{tr}[a_x^\alpha \mathbb{U}^{-t} a_y^\beta \mathbb{U}^t], \tag{8}$$

where $x, y \in \frac{1}{2}\mathbb{Z}_{2L}$, $t \in \mathbb{N}$ [the space-time lattice of the circuit is drawn in Eq. (3)] and $\{a_x^\alpha\}_{\alpha=0}^{d^2-1}$ denotes a basis of the space of local operators at site x, i.e., a basis of $\mathrm{End}(\mathcal{H}_1)$. We assume that a^α are Hilbert-Schmidt orthonormal $\mathrm{tr}[(a^\alpha)^\dagger a^\beta] = d\delta_{\alpha,\beta}$ and choose $a^0 = \mathbb{1}$, so all other a^α are traceless, i.e., $\mathrm{tr}[a^\alpha] = 0$ for $\alpha \neq 0$.

The expression (8) is represented diagrammatically as

$\frac{1}{d^{2L}}$  , (9)

where, again, boundary conditions in both directions are periodic. Since $\mathbb{U}^{-t}a_x^0\mathbb{U}^t = a_x^0$, we have for all $\alpha \neq 0$,

$$D^{00}(x,y,t) = 1,$$
$$D^{0\alpha}(x,y,t) = D^{\alpha 0}(x,y,t) = 0. \tag{10}$$

Moreover, using the two-site shift invariance of $\mathbb{U}$, we find

$$D^{\alpha\beta}(x,y,t) = \begin{cases} C_{-}^{\alpha\beta}(x-y,t) & 2y\,\text{even} \\ C_{+}^{\alpha\beta}(x-y,t) & 2y\,\text{odd}, \end{cases} \tag{11}$$

where we set $C_{\pm}^{\alpha\beta}(x,t) \equiv D^{\alpha\beta}(x+(1\mp 1)/4,(1\mp 1)/4,t)$.

We are now in a position to derive the main result of this Letter: an exact closed-form expression for Eq. (8). The calculation can be subdivided into two main steps, summarized in the following two properties.

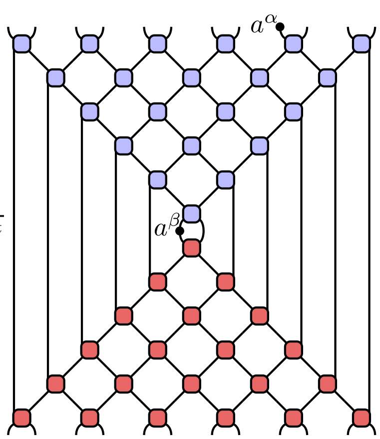
Property 1.—If U is dual unitary, the dynamical correlations for $t \le L/2$ are nonzero only on the edges of a light cone spreading at speed 1:

$$C_{\nu}^{\alpha\beta}(x,t) = \delta_{x,\nu t} C_{\nu}^{\alpha\beta}(\nu t,t), \quad \nu = \pm, \quad \alpha,\beta \neq 0. \tag{12}$$

Before proceeding with the rigorous proof we note that Property 1 has a clear physical interpretation. Because of the dual unitarity of the dynamics, correlations have a causal cone in space, together with that in time. Since they can only propagate along the intersection of the two light cones, we must have $x = \pm t$.

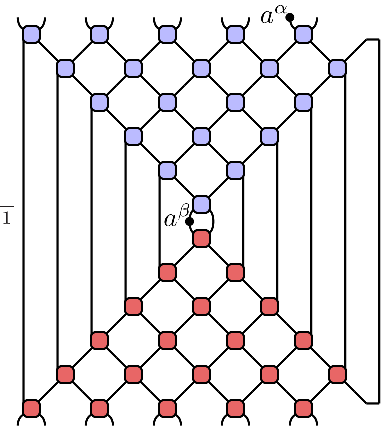
Proof.—The most intuitive way to prove this property is by using the diagrammatic representation (2) and (9). Let us consider the case $\nu = +$, while the procedure for $\nu = -$ is analogous.

By repeated use of the unitarity property (6) we can simplify the circuit (9) out of the light cone spreading at speed 1 from a_0^{β}. This is a simple consequence of the causal structure of the time evolution. Pictorially, we have

$$C_{+}^{\alpha\beta}(x,t) = \frac{1}{d^{4t}}$$


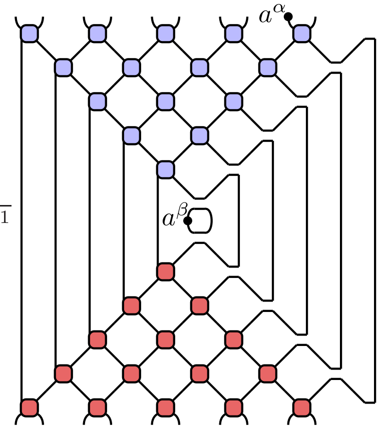
. (13)

At this point, it is convenient to distinguish three cases: (i) $x = t$, (ii) $x = t - \frac{1}{2}$, and (iii) $x \neq t - \frac{1}{2}, t$. Let us start by considering case (iii): using the unitarity of $\tilde{U}$, i.e., Eq. (7), we have

$$C_{+}^{\alpha\beta}(x,t) = \frac{1}{d^{4t-1}}$$


. (14)

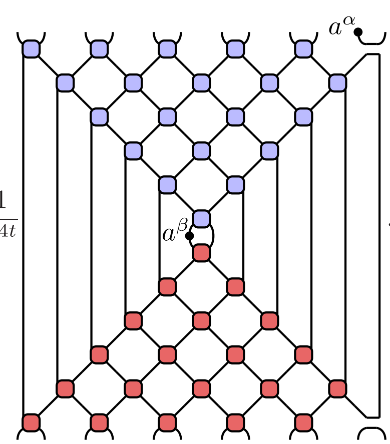
From this picture it is clear that (7) can be telescoped until the operator a^{β} is encountered. Namely,

$$C_{+}^{\alpha\beta}(x,t) = \frac{1}{d^{4t-1}}$$


, (15)

where the central loop represents the trace of a^{β} factoring out. Using that for $\beta \neq 0$ the operators a^{β} are traceless, we then conclude that the correlation vanishes.

Consider now case (ii). Using (7) we find

$$C_{+}^{\alpha\beta}(t-\tfrac{1}{2},t) = \frac{1}{d^{4t}}$$


. (16)

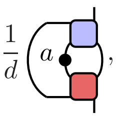
Here the loop giving $\mathrm{tr}[a^\alpha]$ factors out, so we again conclude that the whole expression vanishes. We then showed that the only remaining possibility is case (i). This concludes the proof. ∎

Property 2.—The light cone correlations $C_+^{\alpha\beta}(t,t)$ and $C_-^{\alpha\beta}(-t,t)$ are given by

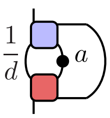
$$C_\nu^{\alpha\beta}(\nu t,t)=\frac{1}{d}\mathrm{tr}[\mathcal{M}_\nu^{2t}(a^\beta)a^\alpha], \tag{17}$$

where we introduced the linear maps over $\mathrm{End}(\mathbb{C}^d)$:

$$\mathcal{M}_+(a)=\frac{1}{d}\mathrm{tr}_1\left[U^\dagger(a\otimes\mathbb{1})U\right]=\frac{1}{d} \tag{18}$$



$$\mathcal{M}_-(a)=\frac{1}{d}\mathrm{tr}_2\left[U^\dagger(\mathbb{1}\otimes a)U\right]=\frac{1}{d} \tag{19}$$

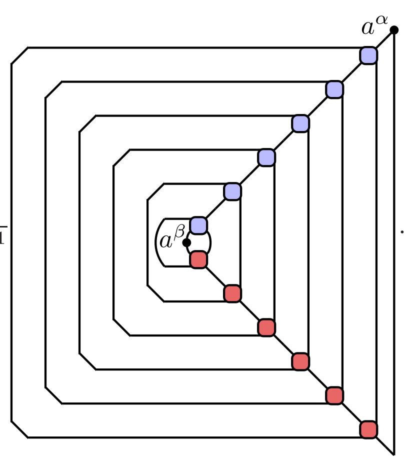


$\mathrm{tr}_i[A]$ denote partial traces over the ith site ($i=1,2$).

Proof.—We again prove the property for $C_+^{\alpha\beta}(t,t)$, using the diagrammatic representation. A completely analogous reasoning applies for $C_-^{\alpha\beta}(-t,t)$.

By repeated use of the unitarity property (6), we can reduce $C_+^{\alpha\beta}(t,t)$ to the following form:

$$C_+^{\alpha\beta}(t,t)=\frac{1}{d^{2t+1}} \tag{20}$$



Using the definition (18) we see that (20) is precisely the diagrammatic representation of (17). ∎

Properties 1 and 2 have a very powerful consequence: all dynamical correlations of local operators in dual-unitary quantum circuits are determined by the two linear single-qudit channels $\mathcal{M}_\pm$. These maps are trace preserving, completely positive, and unital (meaning that they map the identity operator to itself). Moreover, as it is apparent from their definition, they are completely determined by a single $d^2\times d^2$ unitary matrix (U in our case). Maps with these properties are known in the literature as unistochastic maps [31–33]. These maps are generically nondiagonalizable; however, they are contractive. Namely, the eigenvalues $\{\lambda_{\nu,\gamma}\}_{\gamma=0}^{d^2-1}$ of $\mathcal{M}_\nu$ lie on the unit disk, and those that are on the unit circle have coinciding algebraic and geometric multiplicity [23]. This means that the dynamical correlations take the following general form:

$$C_\nu^{\alpha\beta}(x,t)=\delta_{\nu x,t}\sum_{\gamma=1}^{d^2-1}c_{\nu,\gamma}^{\alpha\beta}(\lambda_{\nu,\gamma})^{2t},\qquad \alpha,\beta\neq 0, \tag{21}$$

where $|\lambda_{\nu,\gamma}|\leq 1$, and for eigenvalues corresponding to nontrivial Jordan blocks, the "constant" $c_{\nu,\gamma}^{\alpha\beta}$ is a polynomial in t. Note that since a^β are orthogonal, we excluded the trivial eigenvalues $\lambda_{\pm,0}=1$ corresponding to the identity operator.

This gives a systematic way to classify dual-unitary circuits based on the increasing level of ergodicity of ultralocal observables.

(i) Noninteracting behavior: All $2(d^2-1)$ nontrivial eigenvalues $\lambda_{\nu,\gamma}$ are equal to 1, meaning that all dynamical correlations remain constant.

(ii) Nonergodic (and generically interacting and nonintegrable) behavior: There are n, $1\leq n<2(d^2-1)$, nontrivial eigenvalues $\lambda_{\nu,\gamma}$ equal to 1, meaning that some dynamical correlations remain constant.

(iii) Ergodic but nonmixing behavior: All nontrivial eigenvalues $\lambda_{\nu,\gamma}$ are different from 1, but there exists at least one eigenvalue with unit modulus. In this case, all time averaged dynamical correlations vanish at large times, reproducing the infinite-temperature state value.

(iv) Ergodic and mixing behavior: All nontrivial eigenvalues are within unit disk, $|\lambda_{\nu,\gamma}|<1$. In this case, all time dynamical correlations vanish at large times, reproducing the infinite-temperature state value even without time averaging.

An example of (i) is the SWAP gate $U|i\rangle\otimes|j\rangle=|j\rangle\otimes|i\rangle$, which is clearly self-dual, i.e., $U=\tilde{U}$. Note that, since dual-unitary gates are generically not parity invariant, we can have chiral cases where the number of nondecaying modes (i.e., with $\lambda_{\nu,\gamma}=e^{i\theta}$) propagating to the left and to the right is different.

We point out that Eq. (21) gives direct access to time correlations among extensive operators of the form $A_\nu^\alpha\equiv\sum_{x\in\mathbb{Z}_L}a_{x+(\nu-1)/4}^\alpha$. Specifically, one finds

$$\frac{1}{Ld^{2L}}\mathrm{tr}[A_\nu^\alpha\mathbb{U}^{-t}A_\mu^\beta\mathbb{U}^t]=\delta_{\nu,\mu}\sum_{\gamma=1}^{d^2-1}c_{\nu,\gamma}^{\alpha\beta}(\lambda_{\nu,\gamma})^{2t}. \tag{22}$$

These correlations are able to distinguish dynamical mixing from the mere decaying local correlators. Indeed, even if all dynamical correlations (21) (at fixed distance x) vanish in the infinite time limit, the correlation (22) vanishes only if the system is ergodic and mixing (all nontrivial eigenvalues are within unit disk). In particular, if the mode a_x^α is conserved [i.e., $\mathcal{M}_\nu(a^\alpha) = a^\alpha$], A_ν^α is a proper conserved charge of the system. Note that, since we considered modes localized on a single site, these conservation laws are of single-body form; i.e., they do not couple different sites. Finally, we remark that our classification here only concerns ergodicity of ultralocal observables and their extensive sums. For instance, circuits in (iv) may in principle still exhibit nonergodic or nonmixing behavior for nonlocal operators or local operators with a larger support.

The proposed classification can be explicitly carried out for $d = 2$. Indeed, in this minimal case it is possible to parametrize all dual-unitary local gates [23]. The result reads as

$$U = e^{i\phi}(u_+ \otimes u_-)V[J](v_- \otimes v_+), \tag{23}$$

where $\phi, J \in \mathbb{R}$ [34], $u_\pm$, $v_\pm \in \mathrm{SU}(2)$, and

$$V[J] = \exp\left[-i\left(\frac{\pi}{4}\sigma^x \otimes \sigma^x + \frac{\pi}{4}\sigma^y \otimes \sigma^y + J\sigma^z \otimes \sigma^z\right)\right]. \tag{24}$$

Quantum circuits with local gates of the form (23) include both integrable [20] and nonintegrable cases. For instance, $U_{XXZ} = V[J]$ is a full parameter line of the integrable trotterized XXZ chain [21,22] and

$$U_{\mathrm{SDKI}} = (e^{-ih\sigma^z} e^{i(\pi/4)\sigma^x} \otimes e^{i(\pi/4)\sigma^x})\tilde{V}[0](e^{-ih\sigma^z} \otimes \mathbb{1}), \tag{25}$$

with $\tilde{V}[0] = (e^{-i(\pi/4)\sigma^y} \otimes e^{-i(\pi/4)\sigma^y})V[0](e^{i(\pi/4)\sigma^y} \otimes e^{i(\pi/4)\sigma^y})$, is a quantum circuit representation [23] (see also Ref. [30]) of the nonintegrable self-dual kicked Ising chain studied in Refs. [18,19]. In other words, for $d = 2$, integrable [20] and dual-unitary quantum circuits form two distinct but overlapping sets.

Plugging the form (23) into the definitions (18) and (19), and writing the corresponding matrices in the Pauli basis $\{a^0, a^1, a^2, a^3\} = \{\mathbb{1}, \sigma^x, \sigma^y, \sigma^z\}$, we find

$$\mathcal{M}_\pm = \begin{bmatrix} 1 & 0 \\ 0 & R[u_\pm] \end{bmatrix} \mathcal{M}[J] \begin{bmatrix} 1 & 0 \\ 0 & R[v_\pm] \end{bmatrix}. \tag{26}$$

Here, to lighten the notation, we denoted the matrices associated with $\mathcal{M}_\pm$ with the same symbols. Moreover, we denoted by $R[w]$ the 3×3 adjoint representation of $w \in \mathrm{SU}(2)$, and, finally, we introduced $\mathcal{M}[J] \equiv \mathrm{diag}(1, \sin(2J), \sin(2J), 1)$, the matrix of the map associated with the gate $V[J]$.

Since the spectrum is invariant under similarity transformations, the eigenvalues of $\mathcal{M}_\pm$ depend only on J and on the products $v_\pm u_\pm \in \mathrm{SU}(2)$, thus, in principle, on four real parameters. The matrix $\mathcal{M}[J]$, however, is invariant under rotations around the z axis, so the eigenvalues of $\mathcal{M}_\pm$ depend on three real parameters only (J is a common parameter). Analyzing the spectra of $\mathcal{M}_\pm$ as functions of the parameters, we identify all four types of ergodic behavior [23]. For instance, for the SDKI chain the spectra of $\mathcal{M}_+$ and $\mathcal{M}_-$ coincide and are given by $\{1, \cos(2h), 0\}$. This means that, for generic h, the model is in the ergodic and mixing class, while at the integrable point $h = 0$, it is in the class (ii). At this special point, the y magnetizations on the integer and half-odd integer sublattices are conserved. Instead, in the case of the trotterized XXZ chain the matrices $\mathcal{M}_\pm$ coincide with $\mathcal{M}[J]$. Therefore, they are always in the class (ii) except for $J = \pi/4$, when they correspond to the SWAP gate and are in the noninteracting class (i). For $J \neq \pi/4$, the charges associated to the conserved modes are z magnetizations on the integer and half-odd integer sublattices.

The results presented in this Letter admit several generalizations and extensions. First of all, we note that the proofs of Properties 1 and 2 do not rely on translational invariance either in time or in space. This means that (17) directly generalizes to cases where some inhomogeneity or randomness is introduced either in space or in time [35–37]: one simply needs to replace $\mathcal{M}_\pm^{2t}(a)$ in (17) with a product of $2t$ different operator maps, each one determined by a different local gate depending on the space-time point. Moreover, our treatment can be straightforwardly repeated to find correlation functions of local observables with larger support. This will, for instance, allow one to find exactly solvable circuits with more complex, "many-body," local conservation laws. Such circuits are currently attracting considerable attention, see, e.g., Refs. [38,39], because they are regarded as toy models for generic closed systems. Another very interesting direction is to approach generic quantum circuits by a perturbative expansion around ergodic and mixing dual-unitary ones. Indeed, the fact that in the dual-unitary point the dynamics have a sort of exponential space-time clustering hints that an expansion might have a finite radius of convergence. Finally, a construction very similar to the one presented here can be carried out for higher dimensional quantum circuits, where, instead of on chains, one considers local sites disposed on hypercubes of any dimension. Requiring dual unitarity in all directions again constrains the correlations to the edges of a light cone, and allows one to express them in terms of unistochastic maps.

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