Erratum: Finite-Temperature Conformal Field Theory Results for All Couplings: O(N) Model in 2+1 Dimensions [Phys. Rev. Lett. 122, 231603 (2019)]

Paul Romatschke

(Received 23 October 2019; published 13 November 2019)

DOI: 10.1103/PhysRevLett.123.209901

In Ref. [1], it is stated that the O(N) model with quartic interactions in $2+1$ dimensions at large N is a conformal field theory (CFT) for all coupling values. This statement is incorrect. Also, the explicit expression of the pressure in Eq. (12) of Ref. [1] contains a typographical error in the second term. The correct expression is

$$P = T\frac{\ln Z}{\partial V} = -N\left[J(\sqrt{z^*}) - \frac{z^{*2}}{16\lambda}\right], \tag{1}$$

from which the trace of the energy-momentum tensor is found to be

$$T^\mu_\mu = sT - 3P = \frac{\xi^4 T^4 N}{16\lambda}, \tag{2}$$

where ξ is the solution to the gap equation

$$\frac{T}{\lambda}\xi^2 = -\frac{\xi}{\pi} - \frac{2}{\pi}\ln(1 - e^{-\xi}). \tag{3}$$

The solution to the gap equation for weak and strong coupling implies that $T^\mu_\mu = 0$ for $(\lambda/T) \to 0, \infty$ but not in between. Therefore, the main results from Ref. [1], such as the weak-strong ratio of the entropy density $s_{\text{strong}}/s_{\text{free}} = \frac{4}{5}$ remain unchanged.

The correct statement is that the O(N) model with sextic interactions in $2+1$ dimensions at a large N, with the Euclidean Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\vec{\phi})(\partial_\mu\vec{\phi}) + \frac{1}{2}m^2\vec{\phi}^2 + \frac{\lambda_2}{N^2}(\vec{\phi}^2)^3, \tag{4}$$

is a CFT for all coupling values. This is a special case of potential $U(\sigma) = \lambda_2\sigma^3$ discussed in Ref. [1] where the pressure of the theory becomes

$$P = -N\left[J(\sqrt{z^*}) - \frac{z^*\sigma^*}{2} + U(\sigma^*)\right], \tag{5}$$

and z^*, σ^* are the locations of the saddle points of the action

$$\sigma^* = I(\sqrt{z^*}), \qquad z^* = 6\lambda_2\sigma^{*2}. \tag{6}$$

Writing $z^* = \xi^2 T^2$, the relevant gap equation for ξ for the O(N) model with sextic interaction becomes

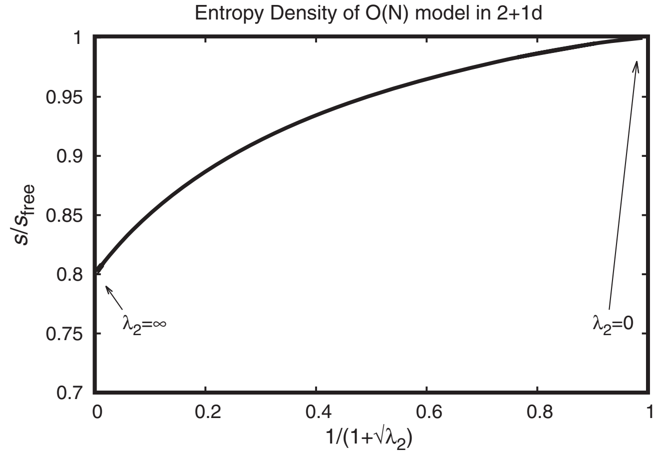

FIG. 1. The ratio s/s_{free} for the $\mathrm{O}(N)$ model with sextic interactions in $2+1$ dimensions in the large N limit for all temperatures and couplings. Results are shown using a compactified interval $(1/1+\sqrt{\lambda_2}) \in [0,1]$ in order to show all coupling values. Arrows indicate free theory result and strong coupling result.

$$\frac{4\xi}{\sqrt{6\lambda_2}} = -\frac{\xi}{\pi} - \frac{2}{\pi}\ln(1-e^{-\xi}), \tag{7}$$

while the pressure and entropy density are given by

$$P = \frac{NT^3}{2\pi}\left[\mathrm{Li}_3(e^{\xi}) - \xi \mathrm{Li}_2(e^{\xi}) - \frac{\xi^3}{3}\left(\frac{1}{2} - \frac{2\pi}{\sqrt{6\lambda_2}}\right) - \frac{i\pi\xi^2}{2}\right],$$
$$s = \frac{NT^2}{4\pi}\left[\xi^3 + \xi^2 \ln\frac{1-e^{-\xi}}{(1-e^{\xi})^3} - 6\xi \mathrm{Li}_2(e^{\xi}) + 6\mathrm{Li}_3(e^{\xi})\right].$$

The trace of the energy momentum tensor becomes

$$T^{\mu}_{\mu} = sT - 3P = \frac{\xi^2 T^3 N}{4}\left(\frac{\xi}{\pi} + \frac{2}{\pi}\ln(1-e^{-\xi}) + \frac{4\xi}{\sqrt{6\lambda_2}}\right),$$
$$= 0 \quad \forall\ \lambda_2, \tag{8}$$

where Eq. (7) was used. Therefore, the massless $\mathrm{O}(N)$ model with sextic interaction is a CFT for all values of the coupling λ_2. The ratio s/s_{free} can be evaluated numerically for all values of the coupling λ_2. The corresponding result is shown in Fig. 1.

[1] Paul Romatschke, Finite-Temperature Conformal Field Theory Results for All Couplings: $\mathrm{O}(N)$ Model in $2+1$ Dimensions, Phys. Rev. Lett. **122**, 231603 (2019).