Resistive State of Superconductor-Ferromagnet-Superconductor Josephson Junctions in the Presence of Moving Domain Walls

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We describe resistive states of the system combining two types of orderings—a superconducting and a ferromagnetic one. It is shown that in the presence of magnetization dynamics such systems become inherently dissipative and in principle cannot sustain any amount of the superconducting current because of the voltage generated by the magnetization dynamics. We calculate generic current-voltage characteristics of a superconductor-ferromagnet-superconductor Josephson junction with an unpinned domain wall and find the low-current resistance associated with the domain wall motion. We suggest the finite slope of Shapiro steps as the characteristic feature of the regime with domain wall oscillations driven by the ac external current flowing through the junction.

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The ability to sustain dissipationless electric currents is assumed to be the defining property of a superconducting state. However, this fundamental concept has been challenged by the subsequent discovery of type-II superconductors, which can be driven into the mixed state characterized by the presence of Abrikosov vortices generated by the magnetic field [1]. The mixed state is a generically resistive one since, in the absence of additional constraints such as the geometrical confinement of the pinning potential, the superconductor vortices start to move under the action of any external current [2]. In such a flux-flow regime, vortex motion generates an electric field which leads to the finite resistance and Ohmic losses [3,4].

In this Letter we point out one more fundamental mechanism which can drive a superconducting system into the resistive state realized in the ideal situation for arbitrary small applied current. We find that the voltage can be generated in the superconductor-ferromagnet (SF) systems due to the interplay of two different order parameters known to produce many nontrivial effects [5–9]. The presence of superconducting condensate allows for the generation of dissipationless spin currents [10] and spin torques to manipulate the magnetic order parameter [8,9,11–25]. Here we point out that the magnetization dynamics generated in this way by the supercurrent with necessity generates an electric field and Ohmic losses in a way analogous to the Abrikosov vortex motion in the flux-flow regime. However, there is no complete analogy between these fundamental processes. In the case of a magnetic system it is the dynamics of a magnetic order parameter that generates an electric field and Ohmic losses in the superconducting state due to the Gilbert damping mechanism. The importance of this new resistive state for understanding the physics of nonequilibrium superconducting-ferromagnet systems motivates the present work.

The sketch of the system under consideration is shown in Fig. 1(a). Its magnetic part consists of a ferromagnetic strip and it is conceptually similar to the domain wall (DW) racetrack memory proposal [26,27]. The position of DWs in the strip can be controlled by the normal current j_N,

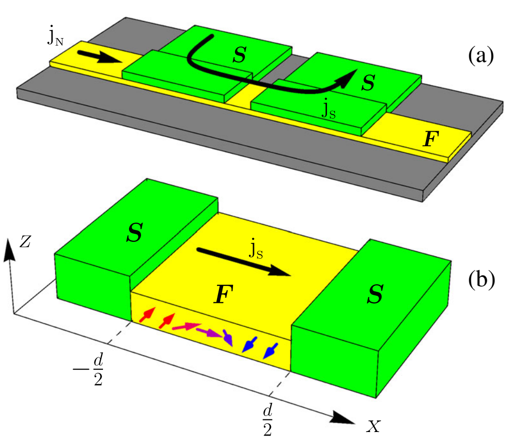

FIG. 1. (a) Sketch of the system under consideration. Superconducting electrodes forming a Josephson junction are placed on top of a ferromagnetic strip. The position of DWs in the strip can be controlled by the normal current j_N. (b) A simplified model of the Josephson junction region. A Néel-type DW is present in the interlayer. The Josephson supercurrent flows in the F region in the x direction.

which can be applied along the strip. In addition, two superconducting leads forming a Josephson junction are placed on top of a ferromagnetic strip. The Josephson current in such a geometry has been measured through CrO_2 half-metallic ferromagnet [28] for the distance between superconducting electrodes that greatly exceed the typical DW size of 20 nm. In such system the Josephson current is necessarily mediated by spin-triplet Cooper pairs [8] that can pick up the Berry phase [29] propagating through the noncoplanar spin texture or in the presence of spin-orbit coupling (SOC). This leads to the possibility of spin transfer torques (STT) generated by the supercurrent [23], so that when a DW is located inside the interlayer region of the Josephson junction, it can be moved by the Josephson current j applied between the superconducting leads.

As a reciprocal effect to the STT, a gauge spin-dependent vector potential appears in the local spin basis due to SOC [30–38]. It produces an anomalous phase shift and, in the presence of magnetization dynamics it also produces an electromotive force [39–41]. This situation is the focus of our present study. The electromotive force should be compensated by the voltage induced at the junction. It is this voltage that maintains the DW motion, compensating the dissipation power occurring due to Gilbert damping by the work done by a power source, as it is shown below.

The model.—Figure 1(b) illustrates the simplified model of the SFS Josephson junction region, which we consider in our calculations. We assume that there is a Néel-type DW inside the F interlayer. The Rashba SOC is present in F due to a structural or internal inversion symmetry breaking. The Josephson supercurrent, which flows in F along the x direction, generates a torque on the DW [23] consisting of the adiabatic STT [42–44] and spin-orbit torques [45,46]. Under these conditions DW motion is caused even by very small currents if pinning effects are neglected. We neglect the nonadiabatic STT [47] assuming that the most part of the voltage is dropped at the interfaces and the quasiparticle nonequilibrium in the interlayer is small enough.

In the considered SFS junction, the coupled dynamics of magnetization $\boldsymbol{M}$ and Josephson phase difference φ is determined by the following closed set of equations

$$j = j_c \sin(\varphi - \varphi_0\{\boldsymbol{M}\}) + \frac{\dot{\varphi} - \dot{\varphi}_0\{\boldsymbol{M}\}}{2eRS}. \tag{1}$$

$$\frac{\partial \boldsymbol{M}}{\partial t} = -\gamma \boldsymbol{M} \times \boldsymbol{H}_{\mathrm{eff}} + \frac{\alpha}{M} \boldsymbol{M} \times \frac{\partial \boldsymbol{M}}{\partial t} + \boldsymbol{T}, \tag{2}$$

Eq. (1) represents the nonequilibrium current-phase relation (CPR) generalizing a resistively shunted Josephson junction (RSJ) model. This relation is written in a gauge-invariant form amended to include the anomalous phase shift [12,24,29,48–74] $\varphi_0\{\boldsymbol{M}\}$ defined by SOC and magnetic texture. For strong ferromagnets only spin-triplet pairs can penetrate into F. Then the transport can be calculated in the local spin basis for spin-up and spin-down Fermi surfaces separately with an effective $U(1)$ spin-dependent gauge field $\boldsymbol{Z}$ that yields [29]

$$\varphi_0\{\boldsymbol{M}\} = -2 \int_{-d/2}^{d/2} Z_x(x,t)dx, \tag{3}$$

where $\boldsymbol{Z} = \boldsymbol{Z}^m + \boldsymbol{Z}^{so}$. Here $Z_i^m = -i\mathrm{Tr}(\hat{\sigma}_z \hat{U}^\dagger \partial_i \hat{U})/2$ is the texture-induced part, where $\hat{U}(\boldsymbol{r},t)$ is the time- and space-dependent unitary 2×2 matrix that rotates the spin quantization axis z to the local frame determined by the exchange field.

The term $Z_j^{so} = (M_i B_{ij})/M$ appears due to SOC, where B_{ij} is the constant tensor coefficient describing the linear spin-orbit coupling of the general form $\hat{H}_{so} = \sigma_i B_{ij} p_j/m$. Here we assume that $\hat{H}_{so}$ is of a Rashba type: $\hat{H}_{so} = (B_R/m)(\sigma_x p_y - \sigma_y p_x)$. $\boldsymbol{Z}^m$ is nonzero only for noncoplanar magnetic structures, and in our case $\boldsymbol{Z}^m = 0$ [75]. The electromotive force can also occur due to the noncoplanarity of the moving DW or the presence of the nonadiabatic (antidamping) torque [76–81]. However, for the case of Rashba SOC and the Néel DW, presented in Fig. 1(b), the moving DW remains coplanar.

In fact, Eq. (1) is quite general and is applicable to a wide class of Josephson systems exhibiting an anomalous phase shift. We derive it [82] microscopically for the case of a strong ferromagnetic interlayer [29]. Besides that, in contrast to the previously used gauge noninvariant formulations [13,17], Eq. (1) describes the normal spin-galvanic effects when $j_c = 0$ such as the electromotive force and charge current generated in the ferromagnet due to the time derivative of the Berry phase [76–80,83,84]. The analogous equation is also valid for a more general nonsinusoidal CPR.

The magnetization dynamics driven by the spin-polarized supercurrent is described by LLG Eq. (2) where α is the Gilbert damping constant, γ is the gyromagnetic ratio. The last term in Eq. (2) is the current-induced spin torque $\boldsymbol{T} = (\gamma/M)(\boldsymbol{J}_s \boldsymbol{\nabla})\boldsymbol{M} + (2\gamma/M)(\boldsymbol{M} \times \boldsymbol{B}_j) J_{s,j}$. The first term here is the adiabatic spin-transfer torque generated by the spin current $\boldsymbol{J}_s$ in the local spin basis. The second term is the spin-orbit induced torque determined by the spin vector $\boldsymbol{B}_j = (B_{xj}, B_{yj}, B_{zj})$ corresponding to the jth spatial component of the SOC tensor B_{ij}. Below we assume $R_\uparrow \ll R_\downarrow$ and neglect for simplicity the spin-down contribution to the current. In this case $\boldsymbol{J}_s \approx \boldsymbol{j}/2e$.

DW motion.—It is convenient to parametrize the magnetization as $\boldsymbol{M} = M(\sin\theta \sin\delta, \cos\theta, \sin\theta\cos\delta)$, where the both angles depend on (x,t). At zero applied supercurrent the equilibrium shape of the DW is given by $\delta = \pi/2$ and

$$\cos\theta = -\tanh[(x - x_0)\pi/d_W], \tag{4}$$

where $d_W = \pi\sqrt{A_{ex}/K}$ is the DW width. Here it is assumed that $K > 0$ and $K_\perp > 0$ are the anisotropy constants for the easy and hard axes, respectively, and A_{ex} is the constant describing the inhomogeneous part of the exchange energy. The effective magnetic field $\boldsymbol{H}_{\text{eff}} = (1/M^2)(KM_y\boldsymbol{y} - K_\perp M_z\boldsymbol{z} + A_{ex}\partial_x^2\boldsymbol{M})$.

For dealing with the SOC-induced torque it is convenient to define the dimensionless SOC constant $\beta = -2B_R d_W/\pi$. For small applied supercurrents $j \ll d_W Me/(\pi t_d \mu_B|\alpha - \beta|)$ the DW moves as a coplanar object correspondingly to $\theta(x,t)$ defined by Eq. (4) with $x_0(t) = \int_{t_0}^t v(t')dt'$ and $\delta \equiv \delta(t)$. The exact solution for $v(t)$ following from Eq. (2) for the situation when the electric current is switched on at $t = 0$ is presented in the Supplemental Material [82]. On a characteristic timescale $t_d = (1+\alpha^2)M/(\alpha\gamma K_\perp)$, the DW velocity reaches its stationary value

$$v_{st} = -u\beta/\alpha, \tag{5}$$

where $u = \gamma J_s/M$.

In the considered case of a Néel DW and a Rashba SOC, $Z_x^{so} = (\pi\beta M_y)/(2d_W M)$, which according to (3) yields the anomalous phase shift:

$$\varphi_0(t) \approx -2\pi\beta x_0(t)/d_W. \tag{6}$$

Our consideration is strictly applicable if $|d/2 \pm x_0| \gg d_W$, that is if the DW is not close to the SF interface.

Resistive state.—Suppose that we apply a constant electric current $I = jS$ (here S is the junction area) to the Josephson junction and consider a steady motion of the DW across the junction with a constant velocity defined by Eq. (5). In this case Eq. (1) can be easily solved and the time-averaged voltage induced at the junction is

$$\overline{V(t)} = RS\sqrt{j^2 - j_c^2} + \frac{\pi\beta^2 u}{e\alpha d_W}, \tag{7}$$

where the first term represents the well-known Josephson voltage, which is generated at $j > j_c$. The second term V_M is nonzero even at $j < j_c$ and reflects the fact that the Josephson junction is in the resistive state if the DW is moved by the current. The corresponding *IV* characteristics of the junction are shown in Fig. 2. In principle, in small-area or point Josephson junctions with a large resistance, the capacitance or inductance shunting can also lead to the finite slope of the supercurrent branch [85–87]. At the same time, the experimentally realized Josephson junctions via metallic ferromagnets [28,88] practically do not demonstrate noticeable slopes of the supercurrent branches. However, even if a finite slope due to the interaction with the environment is present, it can be distinguished experimentally from the effect discussed in our Letter by comparing *IV* characteristics of the same junction in the presence and in the absence of the domain wall. It is also

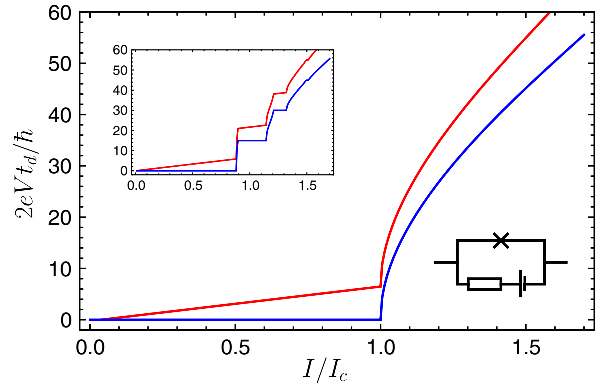

FIG. 2. *IV* characteristics of the SFS junction with a DW at rest (blue) and a moving DW (red). $\beta = 1$, $\alpha = 0.1$, $eKd_W/(\pi j_c) = 5$, $t_d = 40t_J$, where $t_J = 1/2eRI_c$. Upper-left insert: Shapiro steps for $I(t) = I + 0.3I_c\cos\omega t$, $\omega = 15t_d^{-1}$. Axis labeling is the same as in the main figure. Bottom-right insert: the equivalent circuit scheme of the junction.

worth noting that Eq. (1) does not yield a ratchet potential for the Josephson phase because the anomalous phase shift can be compensated by the change of origin. Therefore, our model yields no rectification effects typical for asymmetric Josephson systems [89–100] and the *IV* characteristics remain purely antisymmetric with respect to the current reversal.

For numerical estimates of V_M we take $\alpha = 0.01$, $d_W = 60$ nm, $u \approx 1$ m/s, what corresponds to the maximal Josephson current density [28] through the CrO_2 nanowire $j_c \sim 10^9$ A/m^2. The dimensionless SOC constant β can vary in wide limits. Having in mind that experimentally our predictions can be realized, for example, for hybrid interlayers consisting of a ferromagnet–heavy-metal bilayers, $\beta = 1$–10 considering that the SOC $\alpha_R = B_R/m$ ranges from 3×10^{-11} to 3×10^{-10} eV m at interfaces of heavy-metal systems [101]. Then we can obtain $V_M|_{j=j_c}$ up to $10^{-5} - 10^{-3}$ V.

The resistance of the junction at $j < j_c$ caused by the DW motion is given by

$$R_{\text{DW}} = \left(\frac{\partial V}{\partial I}\right)_{I<I_c} = \frac{\pi\gamma\beta^2\hbar}{2e^2 S\alpha d_W M}, \tag{8}$$

It is interesting that, according to Eq. (8), R_{DW} per unit area does not depend on the Josephson junction parameters, such as j_c and R, and is determined only by the characteristics of the magnetic subsystem. It can be naturally understood if we take into account that in this case the work done by a power source is exactly equal to the energy losses due to the Gilbert damping. Indeed, the dissipation power due to Gilbert damping can be calculated as [102]

$$P_G = \frac{\alpha}{\gamma M}\int dx\left(\frac{d\boldsymbol{M}}{dt}\right)^2 \tag{9}$$

For the stationary DW motion described by Eq. (4) with $\dot{x}_0 = v_{st}$ we get $P_G = j(\pi u\beta^2/e\alpha d_W)$, which exactly coincides with the power of jV provided by the source.

In the regime $j < j_c$ the normal current through the Josephson junction is zero in spite of the nonzero voltage generated at the junction. This follows directly from Eq. (1) because for $j < j_c$ it has the solution $\dot{\varphi}(t) = \dot{\varphi}_0(t)$. The equivalent circuit scheme of the junction is presented in the insert to Fig. 2. The voltage is compensated by the electromotive force induced in the junction by the emergent electric field $(\hbar/e)\dot{\mathbf{Z}}_{so}$.

If there are n DWs inside the junction, then under the assumptions above R_{DW} expressed by Eq. (8) is multiplied by n. If the Josephson junction is driven by an ac component of the voltage or current having the frequency ω, then the dependence $V(I)$ manifest horizontal steps at $V_k = k\omega/2e$, which are known as the Shapiro steps [103,104]. If a moving DW is present in the junction, then the Shapiro steps acquire a nonzero slope, which is determined by Eq. (8). The reason is that in this case the oscillation frequency of the Josephson current is determined by $\overline{\dot{\varphi} - \dot{\varphi}_0}$ and does not coincide with 2 eV anymore. The Shapiro steps occur just when the oscillation frequency of the Josephson current equals to a multiple integer of the external frequency. The IV characteristic demonstrating the inclined Shapiro steps is shown in the insert of Fig. 2.

In real setups the time of the DW movement through the junction is limited by the finite junction length: $t_{\mathrm{DW}} \approx d/v_{st} = (\alpha/\beta)(d/u)$. Therefore, the voltage should be averaged over $t < t_{\mathrm{DW}}$. Although experimental data on the DW motion in Josephson junctions are not yet available, for the estimates we take $d = 0.5 \times 10^{-6}$ m and $u \approx 1$ m/s. Then $t_{\mathrm{DW}} \geq 0.5(\alpha/\beta) \times 10^{-6}$ s. For other experiments, where the Josephson current carried by equal-spin triplet correlations was reported [105,106], this time can be several orders of magnitude higher due to much less values of the critical current density.

The IV characteristic presented in Fig. 2 was obtained under the assumption of a steady DW motion. In fact, $V(t)$ is driven by $\dot{\varphi}_0 \sim v(t)$. For a steplike applied electric current, $V(t)$ saturates exponentially at the characteristic time t_d except for the short Josephson pulses (see below). Therefore, in order to be able to measure the resistance expressed by Eq. (8) it is important to have $t_{\mathrm{DW}} > t_d$. For estimations of t_d we use the material parameters of the $\mathrm{CrO_2}$ nanostructures [28,107]. Taking the saturation magnetization $M = 4.75 \times 10^5$ A/m, $K = 1.43 \times 10^5$ erg/cm^3 and $K_\perp = 4\pi M^2$, and $\alpha = 0.01$ we obtain $t_d \approx 10^{-9}$ s. Consequently, the ratio $t_{\mathrm{DW}}/t_d > 1/\beta$ and, for not very large values of the SOC constant $\beta \lesssim 1$, the condition $t_{\mathrm{DW}} > t_d$ is realistic.

In practice DW motion can be induced by large current pulses. For short pulses $j(t) = j\theta(t)\theta(T - t)$ with $T < t_{\mathrm{DW}}$, the DW does not go out of the junction during the impulse time. The exact expression for the DW velocity $v(t)$ is to be found from the LLG equation and is calculated in the Supplemental Material [82]. The resulting voltage signal consists of two parts of different physical origin. The first part is the purely Josephson response with the characteristic time $t_J = 1/2eRI_c$ and the other part is of purely magnetic origin, has a timescale t_d and vanishes if there is no moving DW in the junction. Taking for estimates of t_J the material parameters of the $\mathrm{CrO_2}$ nanostructures $j_c \sim 10^9$ A/m^2, $R \sim 0.3$–$1.5\ \Omega$, $S = 7.5 \times 10^{-14}$ m^2 we obtain $t_J = 0.3 \times 10^{-11} - 1.5 \times 10^{-11}$ s. According to this estimate $t_J \ll t_d$. Then the Josephson voltage signal should decay much faster than the DW signal.

The resulting voltage signals for $j < j_c$ are shown in Fig. 3(a). In this regime the typical $V(t)$ curve consists of an initial, sharp Josephson voltage impulse decaying at $t \sim t_J$, a final sharp impulse of the same nature, and a gradual voltage increase and decrease of purely magnetic origin, which takes the form $V(t) = -\pi\beta v(t)/ed_W$. The signal saturates at time t_d to the voltage defined by the steady DW motion velocity. In the absence of DW there is no generated voltage between the initial and final Josephson pulses as shown by dashed curves in Fig. 3(a).

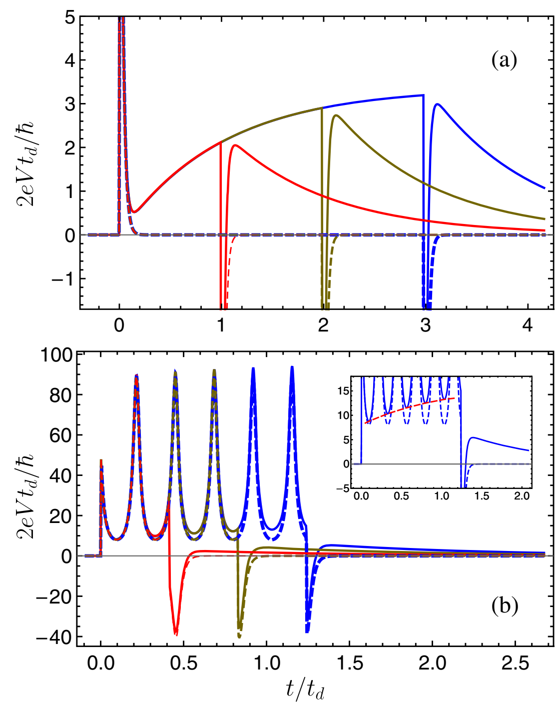

FIG. 3. $V(t)$ for rectangular current impulses. Different curves correspond to different impulse periods T. For all the panels the solid lines correspond to $\beta = 1$ (the anomalous phase due to the DW motion is nonzero) and the dashed lines are for $\beta = 0$ (the anomalous phase shift is zero). (a) $j = 0.5j_c$, $T = 3t_d$ (blue), $T = 2t_d$ (yellow), $T = t_d$ (red); (b) $j = 1.2j_c$, $T = 1.25t_d$ (blue), $T = 0.83t_d$ (yellow), $T = 0.42t_d$ (red). Insert: $j = 1.2j_c$, $T = 1.25t_d$ (the part of the main panel on a large scale). For all the curves $\alpha = 0.1$, $eKd_W/(\pi j_c) = 5$, $t_d = 40t_J$.

The regime $j > j_c$ is characterized by the voltage signal Josephson oscillations during the impulse time as shown in Fig. 3(b). Nevertheless, the gradual increase of the voltage due to the DW motion is also present. It results in the increasing difference between the solid and the dashed curves minima as marked by a red dashed line in the insert.

To conclude we have generalized the RSJ equation to describe the new resistive state generated by magnetization dynamics in the interlayer of a SFS junction. Taking into account the emergent vector potential originated from the SOC and/or magnetization texture, we obtained the gauge-invariant system of coupled equations which governs the dynamics of the magnetization and superconducting phase. Using this model we have shown that in the presence of magnetization dynamics the Josephson junction is in the resistive state even at $j < j_c$. In this regime the junction can be used for electrical detection of the dynamics. Experimentally DW motion inside the Josephson junction can also be observed through the nonzero slope of Shapiro steps.

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