Negative Thermophoretic Force in the Strong Coupling Regime

Rodrigo de Miguel [1,*] and J. Miguel Rubí[2,3,†]

[1]*Department of Teacher Education, Norwegian University of Science and Technology, N-7491 Trondheim, Norway*
[2]*Department of Condensed Matter Physics, University of Barcelona, E-08028 Barcelona, Spain*
[3]*PoreLab—Center of Excellence, Norwegian University of Science and Technology, N-7491 Trondheim, Norway*

Negative thermophoresis (a particle moving up the temperature gradient) is a somewhat counterintuitive phenomenon which has thus far eluded a simple thermostatistical description. The purpose of this Letter is to show that a thermodynamic framework based on the formulation of a *Hamiltonian of mean force* has the descriptive ability to capture this interesting and elusive phenomenon in an unusually elegant and straightforward fashion. We propose a mechanism that describes the advent of a thermophoretic force acting from cold to hot on systems that are strongly coupled to a nonisothermal heat bath. When a system is strongly coupled to the heat bath, the system's eigenenergies $\mathcal{E}_j$ become effectively temperature dependent. This adjustment of the energy levels allows the system to take heat from the environment, $+d\langle\mathcal{E}_j\rangle$, and return it as work, $-d\langle T d\mathcal{E}_j/dT\rangle$. This effect can make the temperature dependence of the effective energy profile nonmonotonic. As a result, particles may experience a force in either direction depending on the temperature.

Thermal gradients may induce mass migration, even in the absence of concentration gradients [1,2]. Usually, the force induced by the thermal gradient drags the particles in the direction of the heat flow, i.e., from hot to cold. This is known as *positive* thermophoresis, and it is commonly understood as the result of more momentum transfer from solvent particles on the hot side than on the cold side. However, various particles such as colloids [3–5], polymers [4–6], charged nanoparticles [5,7–9], magnetic particles [3], fullerenes [10], proteins [5,11], and vesicles [12] have been observed to migrate from cold to hot. These observations suggest that there is more to thermophoresis than a plain momentum transfer resulting from collisions between hard particles.

Although thermophoresis has traditionally been considered only in the context of particles immersed in fluids, similar phenomena may also occur at solid interphases. In 2008, Barreiro *et al.* [13] showed experimentally that an axial temperature gradient along a single-walled carbon nanotube can cause a nanoparticle to move against the temperature gradient, thereby exhibiting behavior analog to positive thermophoresis. And, more recently, Leng *et al.* [14] have used molecular dynamics simulations to predict that negative thermophoresis can occur at solid-solid interphases when a single-walled carbon nanotube is nested inside two nanotubes at different temperatures.

As a cautionary note, the reader is warned that, in the literature, the terms *thermodiffusion* and *thermophoresis* are sometimes used interchangeably (see, e.g., [1,2,15,16]). However, while the former (also known as the Soret effect, or the Ludwig-Soret effect) refers to the formation of a concentration gradient as a result of a thermal gradient, the latter refers to the migration of a particle in a fluid due to the presence of a temperature gradient [17]. In other words, while thermodiffusion is a collective effect, thermophoresis refers to individual particles. Only in very dilute systems may both terms be safely interchanged, as, in this case, solute particles only interact with the solvent and not with one another.

In this Letter, we focus on individual particles, in particular on the thermophoretic force that allows them to migrate towards higher temperature. We present a general thermostatistical framework for the important yet elusive phenomenon of negative thermophoresis as an effect that results from the strong coupling between the migrating particle and the thermal environment. While different numerical approaches point to the possibility of negative thermophoresis when particles with high thermal conductivity are suspended in fluids at a low Knudsen number (see Ref. [18, Sec. 1], [19, Sec. 2.3] for a recent survey), to our knowledge no single satisfactory mechanism has been proposed to account for this phenomenon. However, it has recently been shown by means of saturation transfer electron spin resonance that the low temperature dynamics of proteins are strongly coupled to the dynamics of the surrounding solvent [20]. This suggests strong coupling as the reason behind anomalous low-temperature motion, as is the case of negative thermophoresis. Indeed, when the Knudsen number is very small (the near-continuum regime), the fluid may be approximated as a simple thermal field. And particles with high thermal conductivity can easily assimilate that thermal field in their nontranslational degrees of freedom, as opposed to just gaining

momentum towards the cold side as a result of collisions. The coupling between the particle and the thermal field results in an effective Hamiltonian, or a *Hamiltonian of mean force.*

Hamiltonian of mean force.—When a system is strongly coupled to a heat bath, it is subject to an effective Hamiltonian whose eigenvalues can be effectively perturbed by the temperature of the environment [21]. This allows the system to adjust its thermostatistics and conform to the externally dictated temperature. Taking into account that the eigenvalues $\mathcal{E}_j$ of the system's effective Hamiltonian can be affected by the local temperature T of the environment, then the usual equilibrium expression for the internal energy E

$$E = k_B T^2 \frac{d}{dT} \ln \mathcal{Z} \qquad (1)$$

(where

$$\mathcal{Z} = \sum_j e^{-\mathcal{E}_j(T)/k_B T} \qquad (2)$$

and k_B is Boltzmann's constant) of the system becomes

$$E(T) = \langle \mathcal{E}_j(T) - T\Sigma_j(T) \rangle, \qquad (3)$$

where

$$\Sigma_j(T) \equiv \frac{d\mathcal{E}_j(T)}{dT}, \qquad (4)$$

and $\langle \cdot \rangle$ represents the average over all available microstates j.

Expression (3) differs from the usual expression only by the last term in the right-hand side. This expression was first proposed by Elcock and Landsberg when they explored the statistical mechanics of what they called *temperature-dependent energy levels* in their landmark contribution [22, Eq. 2.13].

More recently, expression (3) has been proposed by Seifert [21, Eq. 10] and Talkner and Hanggi [23, Eq. 18] to describe the energy of small systems strongly coupled to a heat bath, where E is the internal energy of the system and $\mathcal{E}_j$ is the effective Hamiltonian, or Hamiltonian of mean force. The Hamiltonian of mean force is obtained by first considering the total Hamiltonian, i.e., the Hamiltonian for the system in state j, $\mathcal{H}_j^{\text{sys}}$, the Hamiltonian for the bath in state α, $\mathcal{H}_\alpha^{\text{bath}}$, and the Hamiltonian of the interaction between the system and the bath, $\mathcal{H}_{j,\alpha}^{\text{int}}$. Averaging over the bath results in a temperature-dependent effective Hamiltonian $\mathcal{E}_j$:

$$e^{-\mathcal{E}_j/k_B T} = \frac{\sum_\alpha e^{-[\mathcal{H}_j^{\text{sys}} + \mathcal{H}_{j,\alpha}^{\text{int}} + \mathcal{H}_\alpha^{\text{bath}}]/k_B T}}{\sum_\alpha e^{-\mathcal{H}_\alpha^{\text{bath}}/k_B T}}. \qquad (5)$$

In the weak coupling regime, as the interaction Hamiltonian $\mathcal{H}_{j,\alpha}^{\text{int}}$ becomes less significant, the effective Hamiltonian $\mathcal{E}_j$ becomes similar to the isolated system's $\mathcal{H}_j^{\text{sys}}$. On the other hand, at sufficiently low temperatures, the energy of the system is comparable to the interaction energy, and the interaction Hamiltonian $\mathcal{H}_{j,\alpha}^{\text{int}}$ cannot be neglected.

The temperature T of the system-bath complex is given by the usual $T = dE/dS$, which, using (3), becomes

$$T dS = d\langle \mathcal{E}_j - T\Sigma_j \rangle. \qquad (6)$$

This expression describes the energy exchange between the system and the heat bath, and it may be interpreted as follows. As the system interacts with the thermal environment, it absorbs an amount of heat $d\langle \mathcal{E}_j \rangle$. This heat increases the system's temperature by dT. Then, the system cools back down towards the heat bath's temperature by doing work $d\langle T\Sigma_j \rangle$ on the environment. That work changes the eigenstates' contribution to the system's energy, from $\langle \mathcal{E}_j \rangle$ to $\langle \mathcal{E}_j - T\Sigma_j \rangle$. The effective heat received by the system is thereby reduced from $d\langle \mathcal{E}_j \rangle$ to a smaller $dE = d\langle \mathcal{E}_j - T\Sigma_j \rangle$.

The strong interactions between the system and the heat bath induce a change in the Hamiltonian, which is responsible for the temperature dependence of the effective eigenenergies $\mathcal{E}_j$. When examining the temperature-dependence of energy levels in polarons, Whitfield and Engineer [24] showed that two distinct types of temperature-dependence emerge from the coupling between the system and the heat bath. While the $\mathcal{E}_j(T)$ appearing in (2) determine the probability that a state is occupied, the

$$\Phi_j(T) \equiv \mathcal{E}_j(T) - T\Sigma_j(T) \qquad (7)$$

appearing in (3) determine the contribution made to the total energy by that state when it is occupied. They showed that, while the $\mathcal{E}_j$ are monotonically increasing functions of temperature, the Φ_j given by (7) exhibit the opposite behavior at low temperature, as illustrated in Fig. 1. Indeed, detailed calculations based in T-matrix theory have recently confirmed that, at low temperatures, polaron energy does decrease with increasing temperature [25]. Proteins have also been shown to exhibit dynamics that are strongly coupled to the surrounding solvent [20], and they have been observed to migrate from cold to hot at low enough temperatures [5,11], suggesting that their energy is a nonmonotonic function of the temperature. More recently, lipid vesicles have been shown to also exhibit thermophilic motion over a wide range of ambient temperatures [12]. And notably, the thermophilic accumulation of lipids and nucleic acids has been proposed as a key step in the origin of life as it leads to the self-assembly of nucleic acids [26] and protocells [27].
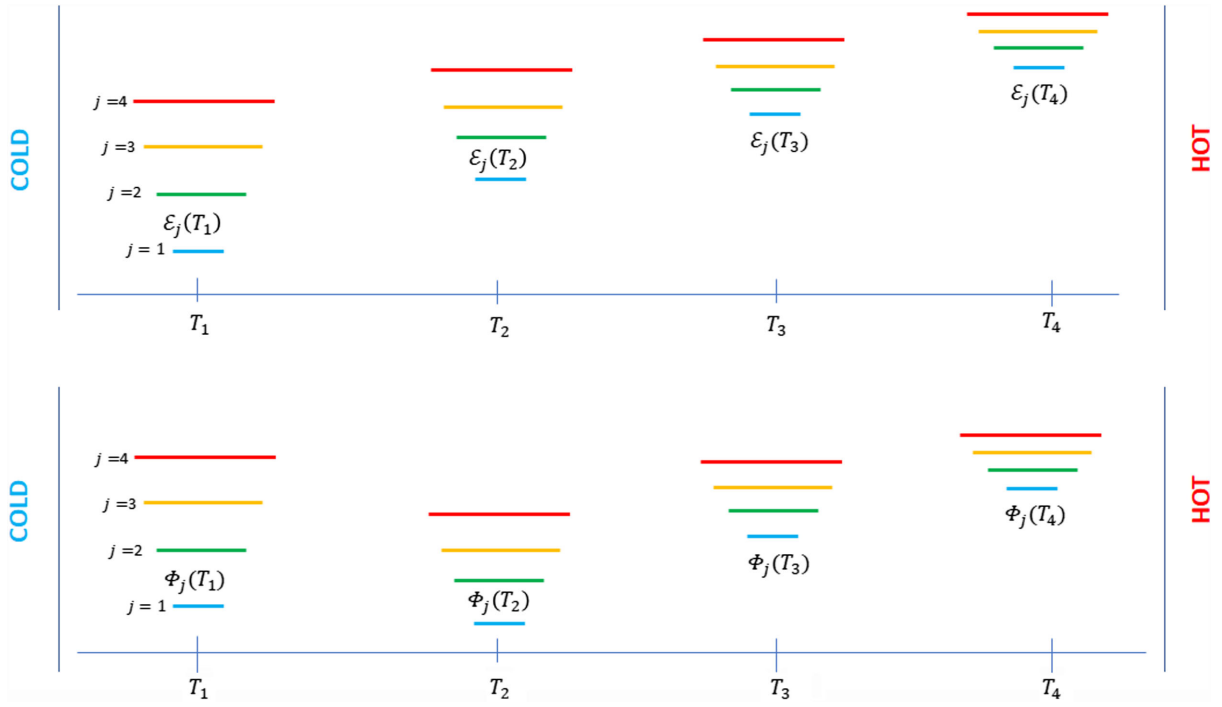
FIG. 1. The energy levels $\mathcal{E}_j(T)$ determine the probability that state j is occupied. Once level j is occupied, its contribution to the system's energy is given by $\Phi_j(T)$. While the $\mathcal{E}_j(T)$ are monotonically increasing functions of temperature, at low enough temperature the $\Phi_j(T)$ exhibit the opposite behavior.

Thermophoresis.—We consider a particle strongly coupled to a nonisothermal heat bath $T(x)$. The particle's position is x, and its energy is

$$E(x) = \langle \Phi_j(T(x)) \rangle. \tag{8}$$

The energy contributions of (7) present in (8) have the nature of a free energy. They account for the energy of each eigenstate, and also for an entropic interaction with the environment. Even if the environment is uniform and it presents no geometrical constraints for the particle, the strong coupling induces the entropic term $T\Sigma_j$ to account for the strength of the environmental perturbations. Hence, in the strong coupling regime, we may think of the energy (1) resulting from (5) and (2) as a free energy of solvation for the particle in the particle-bath complex. If the particle is free to move along the temperature gradient, it will find a position that minimizes this energy. The force F driving that motion is given by

$$F = -\nabla E(x) = -\frac{d\langle \Phi_j \rangle}{dT} \nabla T. \tag{9}$$

Not surprisingly, the thermophoretic force is proportional to the temperature gradient. Local temperature gradients may indeed be very large over length scales relevant to nanoparticles [28–30].

It should be noted that the derivative multiplying ∇T in (9) is the specific heat as it would be obtained from the partition function (2) based on the Hamiltonian of mean force (5). For systems strongly coupled to a low-temperature heat bath, this specific heat may become negative [31]. This negative specific heat has previously been explained as the difference between the specific heat of the system-bath complex and that of the heat bath alone, asserting that, while this difference may become negative, each of the two involved specific heats is strictly positive [31].

However, in the case of strong coupling, the function $E = \langle \phi_j \rangle$ should not be interpreted as the system's internal energy, but instead as a free energy of solvation. Hence, the derivative multiplying ∇T in (9) is not a heat capacity whose negativity endangers the stability of the system. Instead, it shows the system's tendency to evolve along the free energy landscape provided by the temperature gradient.

Indeed, as opposed to the eigenvalues $\mathcal{E}_j(T)$, at low temperature, the energy contributions $\Phi_j(T)$ in strongly coupled systems decrease with increasing temperature. This implies that, at low temperature, the temperature derivative present in (9) is negative, resulting in thermophilic behavior (i.e., negative thermophoresis). Indeed, noting that

$$\langle \Phi_j(T) \rangle = \frac{\sum_j \Phi_j(T) e^{-\mathcal{E}_j(T)/k_B T}}{\sum_j e^{-\mathcal{E}_j(T)/k_B T}}, \tag{10}$$

the thermophoretic force (9) may be written as the sum of two distinct contributions:

$$F = \left\langle T \frac{d^2 \mathcal{E}_j}{dT^2} \right\rangle \nabla T + \frac{\sigma^2}{k_B} \nabla \frac{1}{T}, \tag{11}$$

where

$$\sigma^2 \equiv \langle \Phi_j^2 \rangle - \langle \Phi_j \rangle^2. \tag{12}$$

While the first term in the right-hand side of (11) points from cold to hot, the second term points from hot to cold. It is the temperature that determines which of these two contributions dominates.

At high temperature, energy becomes a locally linear function of the temperature as it is equipartitioned amongst the system's degrees of freedom [32], and the average second derivative of the energy levels $\mathcal{E}_j$ quickly approaches zero. On the other hand, the thermal fluctuations σ^2 scale as $\sim T^2$ [32]. As a result, the second term on the right-hand side of (11) dominates, making the thermophoretic force point from hot to cold, i.e., positive thermophoresis.

As temperature becomes lower, the populated energy levels $\mathcal{E}_j(T)$ come closer to their minimum value, where their second derivative is most positive. Then, at mild temperatures, the first term in the right-hand side of (11), albeit small, may become significant. At the same time, thermal fluctuations σ^2 tend to vanish, making the second term smaller. Therefore, at a sufficiently low temperature, the sign of the thermophoretic force may be inverted, making it point from cold to hot, i.e., negative thermophoresis.

It should be stressed that negative thermophoretic behavior is a consequence of the strong coupling between the particle and the heat bath. At low temperature, the system's energy becomes comparable to the interaction energy; in this regime, the system-bath separability assumption of classical thermodynamics does not apply, and their interaction may manifest itself with directly observable anomalous motion, as is the case of negative thermophoresis. As the particle's temperature increases, interactions become less significant until the second term in (11) dominates and the particle ceases to be thermophilic.

Indeed, thermophoresis has been shown to exhibit this type of temperature dependence. Experiments show that, at high temperatures, particles exhibit thermophobic behavior (a tendency to move from hot to cold), and, below a certain temperature, their behavior becomes thermophilic (from cold to hot) [3,3–12].

Example.—Proteins may be modeled as a set of harmonic oscillators [33]. And models have also been proposed that treat the heat bath as a large collection of oscillators coupled to the particle in solution [34,35]. Coupling between oscillators occurs when they are allowed to interact and exchange energy. The spectrum of the

system is then modified into vibrational modes whose frequencies differ from the original oscillator modes. If the coupling is very small compared to the system's energy, then the modification of the original energies is negligible. On the other hand, when coupling is strong, the spectrum of the system can change significantly [36].

As an illustrative toy example we consider a harmonic system with j energy units of size $\hbar\omega$. If the oscillator is strongly coupled to the oscillators in the heat bath, then its spectrum will be modified as a result of the energy exchange, which, for a given system, is regulated by the heat bath's temperature T. As a result, the spectrum frequency ω and the vibrational energy levels $\mathcal{E}_j$ are temperature dependent [21]:

$$\mathcal{E}_j(T) = (j + 1/2)\hbar\omega(T). \tag{13}$$

The partition function (2) is then given by

$$\mathcal{Z} = \sum_{j=0}^{\infty} e^{-(j+1/2)\hbar\omega(T)/k_B T}, \tag{14}$$

and the energy (3) of the system becomes

$$E(T) = \frac{1}{2} \coth\left(\frac{\hbar\omega(T)}{2k_B T}\right) \hbar[\omega(T) - T\omega'(T)]. \tag{15}$$

In the limit of vanishing coupling, the oscillator's frequency ω is not affected by the heat bath's temperature, the rightmost term in (15) vanishes, and the usual energy expression for a harmonic oscillator is recovered.

However, in the strong coupling regime, the spectrum frequency ω is affected by the energy of the oscillators around it, i.e., by the heat bath's temperature T. In the low temperature limit, the oscillator's frequency has a value ω_0 corresponding to the oscillator's vacuum energy $\hbar\omega_0/2$. As the temperature increases, the system becomes softer and the force constant becomes smaller. And, in the high temperature limit, the force constant (i.e., the square of the frequency) vanishes. This physical behavior may be modeled as $\omega(T) = \omega_0 e^{-\alpha T}$ (with a positive coupling constant α). The resulting energy function (15) displays one (and only one) minimum at a positive temperature $T = T^*$. This temperature is known as the *reversal temperature* [5], i.e., the temperature at which the particle's motion switches from thermophobic to thermophilic. Below the reversal temperature, the first term in the right-hand side of (11) is largest, and above this temperature, the second term dominates. As in [5], we find that the reversal temperature is a single-particle property [note that, for N noninteracting particles, only a multiplicative factor N would appear in Eq. (15)]. For instance, an oscillator with $\omega_0 = 10^{15}$ Hz and $\alpha = 1/120$ K^{-1} will have a reversal temperature $T^* = 24\,°$C.

The reversal temperature is specific to each system. Experiments show that charged polystyrene particles in aqueous solution turn thermophilic below about 20 °C [7], while ionic micelles do so only below 4 °C [5]. In similar conditions, lysozome proteins become thermophillic below about 20 °C [5,7,11], while dextran does so below 45 °C [37]. Poly(N-isopropylacrylamide) in ethanol becomes thermophilic below 34 °C [38]. And, while some types of vesicles only exhibit negative thermophoretic mobility below 10 °C, others have been observed to do so at temperatures higher than 50 °C [12]. These experiments were all carried out with polar solvents. Polar solvents are more likely to strongly couple to the internal degrees of freedom of a particle in solution, which may in turn result in observable anomalous thermophilic motion. Nonpolar solvents, on the other hand, resemble classical particles with whom energy may mostly be exchanged via momentum transfer, and, in such a case, only thermophobic motion is expected.

Conclusion.—We propose a mechanism that describes the advent of a thermophoretic force acting from cold to hot on systems strongly coupled to a nonisothermal heat bath. When a system is strongly coupled to a heat bath, the eigenenergies become temperature dependent. This adjustment of the energy levels allows the system to take heat from the environment $+d\langle \mathcal{E}_j \rangle$ and return it as work $-d\langle T\Sigma_j \rangle$. This effect can make the temperature dependence of the effective energy profile nonmonotonic. As a result, the sign of the thermophoretic force can become temperature dependent, as experiments have shown.

A better understanding of the mechanisms behind phoretic effects at small scales is important for the design and operation of advanced nanosystem properties. In applications such as drug delivery, where local heating is easier than local cooling, it is desirable to guide self-propelled particles towards locally heated targets [39]. The thermophilic motion of nanoparticles may also be used to detect and capture DNA and other biological indicators in serum, allowing for the detection of quantities otherwise unachievable in a thermally homogeneous sample [40]. Thermophilic drift is also believed to be a key mechanism in the self-assembly of the nucleic acids [26] and protocells [27] which lead to the origin of life. Further progress in any of these directions calls for a better understanding of negative thermophoresis, a phenomenon which had thus far eluded a thermostatistical description.

*rodrigo.demiguel@ntnu.no
†mrubi@ub.es

[1] S. Duhr and D. Braun, Proc. Natl. Acad. Sci. U.S.A. **103**, 19678 (2006).

[2] W. Kohler and K. I. Morozov, J. Non-Equilib. Thermodyn. **41**, 151 (2016).

[3] I. Turek, J. Štelina, C. Musil, M. Timko, P. Kopčansky, M. Koneracká, and L. Tomčo, J. Magn. Magn. Mater. **201**, 167 (1999).

[4] B.-J. de Gans, R. Kita, B. Müller, and S. Wiegand, J. Chem. Phys. **118**, 8073 (2003).

[5] S. Iacopini, R. Rusconi, and R. Piazza, Eur. Phys. J. E **19**, 59 (2006).

[6] M. Giglio and A. Vendramini, Phys. Rev. Lett. **38**, 26 (1977).

[7] S. A. Putnam, D. G. Cahill, and G. C. L. Wong, Langmuir **23**, 9221 (2007).

[8] R. Cabreira Gomes, A. Ferreira da Silva, M. Kouyat, G. Demouchy, G. Mriguet, R. Aquino, E. Dubois, S. Nakamae, M. Roger, J. Depeyrot, and R. Perzynski, Phys. Chem. Chem. Phys. **20**, 16402 (2018).

[9] M. Kouyaté, C. L. Filomeno, G. Demouchy, G. Mériguet, S. Nakamae, V. Peyre, M. Roger, A. Cēbers, J. Depeyrot, E. Dubois, and R. Perzynski, Phys. Chem. Chem. Phys. **21**, 1895 (2019).

[10] J. Rodrigo Vélez-Cordero and J. Hernández-Cordero, Langmuir **31**, 10066 (2015).

[11] S. Iacopini and R. Piazza, Europhys. Lett. **63**, 247 (2003).

[12] E. L. Talbot, J. Kotar, L. Parolini, L. Di Michele, and P. Cicuta, Nat. Commun. **8**, 15351 (2017).

[13] A. Barreiro, R. Rurali, E. R. Hernandez, J. Moser, T. Pichler, L. Forro, and A. Bachtold, Science **320**, 775 (2008).

[14] J. Leng, Z. Guo, H. Zhang, T. Chang, X. Guo, and H. Gao, Nano Lett. **16**, 6396 (2016).

[15] T. Tsuji, K. Kozai, H. Ishino, and S. Kawano, Micro Nano Lett. **12**, 520 (2017).

[16] D. Lüsebrink, M. Yang, and M. Ripoll, J. Phys. Condens. Matter **24**, 284132 (2012).

[17] E. Bringuier, Philos. Mag. **91**, 1653 (2011).

[18] R. W. Bosworth, A. L. Ventura, A. D. Ketsdever, and S. F. Gimelshein, J. Fluid Mech. **805**, 207 (2016).

[19] R. W. Bosworth, *An Experimental Study of Positive and Negative Thermophoresis* (University of Colorado, Colorado Springs, CO, 2016).

[20] Y.-H. Kuo and Y.-W. Chiang, ACS Cent. Sci. **4**, 645 (2018).

[21] U. Seifert, Phys. Rev. Lett. **116**, 020601 (2016).

[22] E. W. Elcock and P. T. Landsberg, Proc. Phys. Soc. London Sect. B **70**, 161 (1957).

[23] P. Talkner and P. Hanggi, Phys. Rev. E **94**, 022143 (2016).

[24] G. Whitfield and M. Engineer, Phys. Rev. B **12**, 5472 (1975).

[25] H. Hu, B. C. Mulkerin, J. Wang, and X.-J. Liu, Phys. Rev. A **98**, 013626 (2018).

[26] D. Niether, A. Afanasenkau, J. K. G. Dhont, and S. Wiegand, Proc. Natl. Acad. Sci. U.S.A. **113**, 4272 (2016).

[27] I. Budin, R. J. Bruckner, and J. W. Szostak, J. Am. Chem. Soc. **131**, 9628 (2009).

[28] A. Lervik, F. Bresme, S. Kjelstrup, D. Bedeaux, and M. Rubí, Phys. Chem. Chem. Phys. **12**, 1610 (2010).

[29] F. Romer, F. Bresme, J. Muscatello, D. Bedeaux, and J. M. Rubí, Phys. Rev. Lett. **108**, 105901 (2012).

[30] K. Okabe, N. Inada, C. Gota, Y. Harada, T. Funatsu, and S. Uchiyama, Nat. Commun. **3**, 705 (2012).

[31] G.-L. Ingold, P. Hanggi, and P. Talkner, Phys. Rev. E **79**, 061105 (2009).

[32] V. P. Carey, *Statistical Thermodynamics and Microscale Thermophysics* (Cambridge University Press, Cambridge, England, 1999).

[33] V. Alexandrov, U. Lehnert, N. Echols, D. Milburn, D. Engelman, and Mark Gerstein, Protein Sci. **14**, 633 (2005).

[34] H. Grabert, P. Schramm, and G.-L. Ingold, Phys. Rep. **168**, 115 (1988).

[35] R. Kupferman, A. M. Stuart, and J. R. Terry, Stoch. Dyn. **02**, 533 (2002).

[36] P. Törmä and W. L. Barnes, Rep. Prog. Phys. **78**, 013901 (2015).

[37] R. Sugaya, B. A. Wolf, and R. Kita, Biomacromolecules **7**, 435 (2006).

[38] R. Kita, G. Kircher, and S. Wiegand, J. Chem. Phys. **121**, 9140 (2004).

[39] J. R. Baylis, K. Y. T. Chan, and C. J. Kastrup, Thrombosis research **141**, S36 (2016).

[40] L.-H. Yu and Y.-F. Chen, Ann. Chem. **87**, 2845 (2015).