Depinning Transition of Charge-Density Waves: Mapping onto $O(n)$ Symmetric ϕ^4 Theory with $n \to -2$ and Loop-Erased Random Walks

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Driven periodic elastic systems such as charge-density waves (CDWs) pinned by impurities show a nontrivial, glassy dynamical critical behavior. Their proper theoretical description requires the functional renormalization group. We show that their critical behavior close to the depinning transition is related to a much simpler model, $O(n)$ symmetric ϕ^4 theory in the unusual limit of $n \to -2$. We demonstrate that both theories yield identical results to four-loop order and give both a perturbative and a nonperturbative proof of their equivalence. As we show, both theories can be used to describe loop-erased random walks (LERWs), the trace of a random walk where loops are erased as soon as they are formed. Remarkably, two famous models of non-self-intersecting random walks, self-avoiding walks and LERWs, can both be mapped onto ϕ^4 theory, taken with formally $n = 0$ and $n \to -2$ components. This mapping allows us to compute the dynamic critical exponent of CDWs at the depinning transition and the fractal dimension of LERWs in $d = 3$ with unprecedented accuracy, $z(d = 3) = 1.6243 \pm 0.001$, in excellent agreement with the estimate $z = 1.62400 \pm 0.00005$ of numerical simulations.

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The model of periodic elastic manifolds driven by an external force through a disordered medium is relevant for charge-density waves (CDWs) in disordered solids [1–3], flux-line lattices in the mixed state of disordered type-II superconductors (Bragg glass) [4–7], and disordered Wigner crystals [8–10]. It has long been known that even weak disorder destroys the long-range translational order and pins the elastic manifold [11]. Once the external driving force f exceeds a critical threshold force f_c, the manifold undergoes a depinning transition to a sliding state. The dynamics of the system in the vicinity of this transition was studied both numerically [12–16] and via field theory [17–21]. The latter requires the functional renormalization group (FRG). As scaling arguments imply that the critical behavior of a disordered elastic manifold with short-range elasticity is dominated by disorder for $d < d_{\mathrm{uc}} = 4$, any perturbative description breaks down on scales larger than the Larkin scale [22]. As a consequence, one has to follow the renormalization of the whole disorder correlator, which develops a cusp at the Larkin scale. The appearance of this nonanalyticity in the running disorder correlator accounts for metastability and a finite threshold force. As the corresponding FRG calculations are very involved, they have only recently been extended to two- [23–25] and three-loop order [26,27].

In the present Letter, we show that when the field is periodic, most properties are described by a much simpler field theory, namely, the $O(n)$ symmetric ϕ^4 model with $n \to -2$. This fact, overlooked for decades, drastically simplifies calculations of the depinning transition, since ϕ^4 theory is well known and its renormalization-group description does not require the FRG. We also prove that both models describe loop-erased random walks (LERWs) in arbitrary dimension d. In this Letter, we outline the main ideas and results, while details of the proof and calculations are published elsewhere [28].

Random walks (RWs) *without* self-intersections play an important role in mathematics, statistical physics, and quantum field theory. The two widely encountered models are self-avoiding walks (SAWs) and LERWs. The SAW describes long polymer chains with self-repulsion caused by excluded-volume effects. It can be defined as the uniform measure on all possible paths of a given length without self-intersections. While the SAW is difficult to analyze mathematically rigorously, it was discovered by de Gennes [29] that its large-scale behavior can be extracted from the $O(n)$ symmetric ϕ^4 model in the unusual limit of $n \to 0$. The LERW, which is intimately related to uniform spanning trees [30,31], is a special case of the Laplacian RW [32,33]. It is built from a RW by erasing any loop as soon as it is formed [34]. A realization of a two-dimensional LERW is shown in Fig. 1. Both models have a scaling limit in all dimensions, for instance, the end-to-end distance R scales with the RW length ℓ as $R \sim \ell^{1/z}$, where z is the fractal dimension [35].

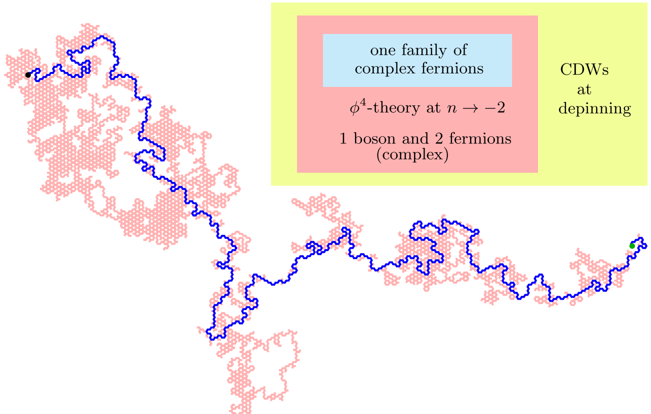


FIG. 1. Trace of a LERW in blue, with the erased loops in red, on a 2D honeycomb lattice. (Inset) Nesting of the different field theories for LERWs.

Contrary to the SAW, the LERW has no obvious field theory. Three-dimensional LERWs have been studied only numerically [36–39]. In two dimensions, LERWs can be described by the radial Schramm-Loewner evolution with parameter $\kappa = 2$, also known as SLE_2 [40,41]. It predicts a fractal dimension $z_{\mathrm{LERW}}(d = 2) = 5/4$, which is clearly different from that of SAWs $z_{\mathrm{SAW}}(d = 2) = 4/3$. Coulomb-gas techniques link this to the 2D $O(n)$ model at $n \to -2$, which is a conformal field theory with central charge $c = -2$ [42,43]. We show below that the equivalence between LERWs and $O(n)$ symmetric ϕ^4 theory at $n \to -2$ holds in any dimension d.

In [44] it was conjectured that the field theory of the depinning transition of CDWs pinned by disorder is a field theory for LERWs. This statement was based on the conjecture of Narayan and Middleton [45] that pinned CDWs can be mapped onto the Abelian sandpile model. The connection of the latter with uniform spanning trees, and thus with LERWs, is well established [31]. The two-loop predictions of [44] agree with rigorous mathematical bounds, and have been tested against numerical simulations at the upper critical dimension $d_{\mathrm{uc}} = 4$ [38], where it was found that they correctly reproduce the leading and subleading logarithmic corrections.

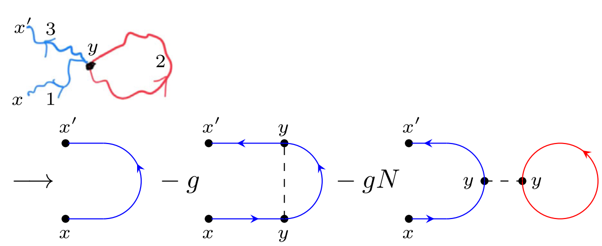
If this conjecture holds, then the ϕ^4 theory at $n \to -2$ has to reproduce the FRG picture for CDWs, at least for observables related to LERWs. Below we prove that the β function and the critical exponents z, $\nu = 1/2$, and $\eta = 0$ coincide for these theories. This is done by using a perturbative analysis of diagrams, nonperturbative supersymmetry techniques, and an explicit four-loop calculation for both models. However, this does not mean that the theories are identical, since one theory can have observables absent in the other. For instance, at depinning, CDWs exhibit avalanches [45–47], which are seemingly absent in the ϕ^4 theory. We claim that in the *sector* in which we can compare the two theories, they agree (see inset of Fig. 1).

Before we demonstrate the relation between CDWs and the n-component ϕ^4 theory at $n \to -2$, we outline how the latter can be used to study LERWs in arbitrary dimension d. First of all, it is convenient to rewrite the ϕ^4 theory in terms of $N = n/2$ complex bosons Φ, with action

$$\mathcal{S}[\vec{\Phi}] \coloneqq \int_x \nabla\vec{\Phi}^*(x)\nabla\vec{\Phi}(x) + m^2\vec{\Phi}^*(x)\vec{\Phi}(x) + \frac{g}{2}[\vec{\Phi}^*(x)\vec{\Phi}(x)]^2. \tag{1}$$

It is known perturbatively that for $N = -1$ the full two-point correlation function $\langle \Phi_i^*(x)\Phi_j(x')\rangle$ reduces to the free-theory value independent of g [48–51]. It can be proven nonperturbatively by mapping onto complex fermions. Indeed, in Feynman diagrams for a bosonic Φ^4 theory, each loop carries a factor of N. In a fermionic Φ^4 theory with M fermions, a closed fermion loop carries a factor of $-M$, so that a theory with N bosons is equivalent to a theory with $N + M$ bosons and M fermions, where N and M can be continued to arbitrary real numbers. In particular, $N = -1$ corresponds to $M = 1$, where the term quartic in fermionic fields vanishes, proving nonrenormalization of the propagator.

We now sketch the equivalence, referring to the Supplemental Material [52] for details and an alternative proof based on Ref. [54]. In Fourier space, the two-point correlator $\langle \Phi_i^*(k)\Phi_j(-k)\rangle$ can be viewed as the Laplace transform of the k-dependent Green's function for a RW. It is convenient to draw the trajectory of the RW in blue and, when it hits itself, color the emerging loop in red instead of erasing it. Going to the lattice and studying configurations with exactly one self-crossing, the contributions from perturbation theory are



(2)

The first line is a graphical representation of the RW used to construct a SAW or LERW. It starts at x and ends in x', passing through the segments numbered 1–3. By assumption, it crosses once at point y, but nowhere else. The second line contains all one-loop diagrams of Φ^4 theory. de Gennes [29] showed that setting $N \to 0$ yields the perturbative expansion of SAWs, a fact that can also be proven algebraically [48]. In our formulation, the idea of the proof is as follows: As we consider configurations with exactly one self-intersection, and since we are working on a lattice, the choice $g = 1$ cancels the first two terms, while the last one is absent at $N = 0$. Thus, there is no configuration with a self-intersection for SAWs. Now consider $g = 1$ and $N \to -1$, for which the first two and last two terms cancel. This implies that the free propagator can be rewritten as the last diagram, which has the advantage to distinguish between red and blue parts of the trace, as long as the limit of $N \to -1$ is not yet taken. The final step is to pass to the field theory. The latter has a β function with an attractive fixed point g^* governing the large-distance behavior, implying that the choice $g = 1$ taken above can be relaxed to an arbitrary $g > 0$.

What we need now is an operator that measures the length of the blue backbone in (2). This is achieved by the crossover operator [55–57],

$$\mathcal{O}(y) := \Phi_1^*(y)\Phi_1(y) - \Phi_2^*(y)\Phi_2(y). \tag{3}$$

It checks whether point y is part of the *blue* trace, as it vanishes in a *red* loop. The fractal dimension z of a LERW is extracted from the length of the blue part via

$$\left\langle \int_y \mathcal{O}(y) \right\rangle \sim m^{-z}. \tag{4}$$

We now turn back to CDWs, which in the presence of disorder can be described by the Hamiltonian [5,23]

$$\mathcal{H} = \int_x \left(\frac{1}{2}[\nabla u(x)]^2 + \frac{m^2}{2}(u(x) - w)^2 + V(x, u(x)) \right), \tag{5}$$

where $F(x, u) = -\partial_u V(x, u)$ is a random Gaussian force with zero mean and variance $\overline{F(x, u)F(x', u')} = \Delta(u - u')\delta^d(x - x')$. The function $\Delta(u)$ is even with period 1. The overdamped dynamics of CDWs is given by the equation of motion $\partial_t u(x, t) = -\delta\mathcal{H}[u]/\delta u(x, t)$ [19]. Considering the system driven by increasing w [58], which means that the driving force f fluctuates around its *self-organized* critical value f_c, we arrive at the dynamic field theory [24,25]

$$\mathcal{S}^{\mathrm{CDW}} = \int_{x,t} \tilde{u}(x, t)(\partial_t - \nabla^2 + m^2)[u(x, t) - w] - \frac{1}{2}\int_{x,t,t'} \tilde{u}(x, t)\tilde{u}(x, t')\Delta(u(x, t) - u(x, t')). \tag{6}$$

The statistical tilt symmetry implies nonrenormalization of the gradient and mass terms, equivalent to exponents $\nu = 1/2$ and $\eta = 0$ in the ϕ^4 model at $n \to -2$.

One checks that in the theory (6) all Taylor coefficients in the expansion of $\Delta(u)$ at $u = 0$ are relevant coupling constants for $d < 4$ so that one has to follow renormalization of the whole function. This can be achieved by using the FRG [17–21,23–25]. The flow equation to one-loop order is

$$-m\partial_m\Delta(u) = \varepsilon\Delta(u) - \frac{1}{2}\frac{d^2}{du^2}[\Delta(u) - \Delta(0)]^2, \tag{7}$$

where $\varepsilon = 4 - d$. The analysis of the FRG flow shows that the fixed point (FP) with period 1 has the form $\Delta(u) = \Delta(0) - (g/2)u(1 - u)$ for $u \in [0, 1]$ with a cusp at the origin. In the absence of higher-order terms in u, the renormalization group flow closes in the space of polynomials of degree 2, and for the quadratic term one is left with the renormalization of a single coupling constant g. This form of the FP has been confirmed explicitly to three-loop order and presumably holds to all orders [26,27].

In order to connect to the Φ^4 theory introduced above, let us use supersymmetry to average over disorder [59–61]. The validity of this method at depinning is justified by the fact that the periodic FPs describing depinning and equilibrium have the same value of g and differ only by $\Delta(0)$. At equilibrium, the FP is potential, i.e., $\int_{-\infty}^{\infty} du\Delta(u) = 0$, and thus g also determines $\Delta(0)$. At depinning, g is not enough to get the whole two-point function, and some information is absent. The disorder average of any observable $\mathcal{O}[u_i]$ is [59–61]

$$\overline{\mathcal{O}[u_i]} = \int \prod_{a=1}^{2} \mathcal{D}[\tilde{u}_a]\mathcal{D}[u_a]\mathcal{D}[\bar{\psi}_a]\mathcal{D}[\psi_a]\mathcal{O}[u_i] \times \exp\left[-\int_x \tilde{u}_a(x)\frac{\delta\mathcal{H}[u_a]}{\delta u_a(x)} + \bar{\psi}_a(x)\frac{\delta^2\mathcal{H}[u_a]}{\delta u_a(x)\delta u_a(y)}\psi_a(y) \right]. \tag{8}$$

Here the integral over the auxiliary bosonic fields $\tilde{u}_a$ implies that u_a is at a minimum of $\mathcal{H}$, while the integrals over fermionic fields $\bar{\psi}_a$ and ψ_a cancel the functional determinant appearing in the integration over u_a.

It is known that direct application of this method with one copy fails beyond the Larkin length, leading to the so-called dimensional reduction [59,60]. The key point is that we introduced two copies $a = 1, 2$ of the system in (8) to get access to the second cumulant of the disorder distribution that we want to renormalize. As was shown in Ref. [61], one recovers the FRG flow equation (7) of the statics, which in turn leads to the appearance of a cusp in the running disorder correlator at the Larkin scale, thus avoiding dimensional reduction. It can also be viewed as a breaking of supersymmetry.

Introducing center-of-mass coordinates

$$u_{1,2}(x) = u(x) \pm \frac{1}{2}\phi(x), \qquad \tilde{u}_{1,2}(x) = \frac{1}{2}\tilde{u}(x) \pm \tilde{\phi}(x), \tag{9}$$

the effective action becomes, after some cumbersome but straightforward calculation shown in the Supplemental Material [52],

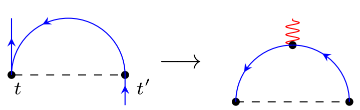
$$\begin{aligned}\mathcal{S} = \int_x & \tilde{\phi}(x)(-\nabla^2 + m^2)\phi(x) + \tilde{u}(x)(-\nabla^2 + m^2)u(x) \\ & + \sum_{a=1}^{2} \bar{\psi}_a(x)(-\nabla^2 + m^2)\psi_a(x) \\ & + \frac{g}{2}\tilde{u}(x)\phi(x)\left[\bar{\psi}_2(x)\psi_2(x) - \bar{\psi}_1(x)\psi_1(x) - \frac{1}{4}\tilde{u}(x)\phi(x)\right] \\ & + \frac{g}{2}[\tilde{\phi}(x)\phi(x) + \bar{\psi}_1(x)\psi_1(x) + \bar{\psi}_2(x)\psi_2(x)]^2. \end{aligned} \tag{10}$$

It is easy to check that, while $u(x)$ and $\tilde{u}(x)$ have nontrivial expectations, the terms depending on them (the second term in the first line, and the third line) do not contribute to the renormalization of g and thus can be dropped. What remains in action (10) is a Φ^4-type theory with one ($N = 1$) complex boson and two ($M = 2$) complex fermions. As we showed above, this can equivalently be viewed as complex ϕ^4 theory with $N \to -1$, or real ϕ^4 theory with $n \to -2$. We thus proved that both models have the same effective coupling g, and thus the same β function for g. This allows us to reconstruct $\Delta(u)$ in the statics and up to the constant $\Delta(0)$ also at depinning.

We show now that this relation between the two models allows one to determine the dynamic exponent z at depinning. The dynamic theory has an additional renormalization of friction or time, which shows up in corrections to the term $\int_{x,t} \tilde{u}(x,t)\dot{u}(x,t)$ in action (6). Using this action to construct all diagrams in which one field $\tilde{u}$ and one field u remain, the latter has the form $u(x,t) - u(x,t')$ and can be expanded as $\dot{u}(x,t)(t-t')$. The time difference, when appearing in the expression for a diagram, together with a response function given in Fourier by $R(k,t) = \Theta(t)\mathrm{e}^{-t(k^2+m^2)}$, can be treated as an insertion of an additional point into the line, for the latter using the relation

$$tR(k,t) = \int_0^t dt' R(k,t')R(k,t-t'). \tag{11}$$

One can check perturbatively that the diagrams renormalizing the term $\tilde{u}(x,t)\partial_t u(x,t)$ in the CDW action (6) reduce to the two-point function of model (1) with an insertion of the crossover operator (3). This identifies the dynamic exponent of CDWs at depinning with the crossover exponent of the Φ^4 theory. Let us demonstrate this on the example of the one-loop dynamic diagram



(12)

The wavy line is the crossover operator defined in Eq. (3). Using a short-time expansion, the lhs of Eq. (12) is evaluated to [24]

$$\int_{x,t,t'} \tilde{u}(x,t)[\Delta'(0^+) + \Delta''(0)(t-t')\dot{u}(x,t)]R_{0,t-t'}, \tag{13}$$

where $R_{x,t}$ is the response function in real space. The first term $\sim\Delta'(0^+)$ renormalizes the critical force, while the second one corrects the friction. Using relation (11) and integrating over times, the resulting expression is the one-loop diagram of Φ^4 theory for the observable (3), i.e., the rhs of Eq. (12). Following this strategy, we checked that this property persists up to four-loop order. This can be proven graphically to all orders [28].

We generated all diagrams contributing to $\mathcal{O}(y)$ at five-loop order, and to the renormalization of the coupling constant at four-loop order, using the diagrams computed in a massless minimal subtraction scheme in [50,51]. This yields for the dynamical exponent z of CDWs at depinning in dimension $d = 4 - \varepsilon$, equivalent to the fractal dimension d_f of LERWs in the same dimension,

$$\begin{aligned} z = 2 - \frac{\varepsilon}{3} - \frac{\varepsilon^2}{9} + \left[\frac{2\zeta(3)}{9} - \frac{1}{18}\right]\varepsilon^3 \\ - \left[\frac{70\zeta(5)}{81} - \frac{\zeta(4)}{6} - \frac{17\zeta(3)}{162} + \frac{7}{324}\right]\varepsilon^4 \\ + \left[\frac{121\zeta(3)}{972} - \frac{8\zeta(3)^2}{81} + \frac{17\zeta(4)}{216} - \frac{103\zeta(5)}{243} - \frac{175\zeta(6)}{162} \right. \\ \left. + \frac{833\zeta(7)}{216} - \frac{17}{1944}\right]\varepsilon^5 + \mathcal{O}(\varepsilon^6), \end{aligned} \tag{14}$$

where $\zeta(s)$ is the Riemann zeta function. This result agrees with the dynamic critical exponent of CDWs at depinning computed using the FRG to two- [24] and four-loop order [28], the four-loop result for the crossover exponent of the $O(n)$ symmetric ϕ^4 theory computed in Ref. [56], setting $n \to -2$ and its extension to six-loop order [62]. Using Borel resummation of the latter yields $z = 1.244 \pm 0.01$ in $d = 2$, where the exact value is $z = 5/4$ [40,41], and $z(d = 3) = 1.6243 \pm 0.001$. This can be compared to the most precise numerical simulations to date by Wilson [39], $z(d = 3) = 1.62400 \pm 0.00005$.

To summarize, we showed that CDWs at depinning are equivalent to the $O(n)$ symmetric ϕ^4 theory with $n \to -2$, and that both field theories describe LERWs. We gave both a perturbative proof of this equivalence and a proof based on supersymmetry. This was checked by an explicit four-loop calculation. Using the $O(n)$ symmetric ϕ^4 theory, we calculated the dynamic critical exponent for CDWs at depinning and the fractal dimension of LERWs to fifth order in $\varepsilon = 4 - d$, in excellent agreement with known numerical results. Our findings are surprising, since a simple ϕ^4 theory allows one to obtain the FRG fixed point of CDWs, which is a glassy disordered system. However, it does not provide all information about pinned CDWs, for instance, the two-point dynamic correlation function.

Our understanding is that both field theories are not isomorphic, but when restricted to the same *physical sector* make the same predictions. This opens a path to eventually tackle other systems, which currently necessitate the FRG, such as random-field magnets [63–65], using a simpler effective field theory.

Our results provide a strong support for the Narayan-Middleton conjecture [45] that CDWs pinned by disorder can be mapped onto the Abelian sandpile model and on LERWs [44]. As a consequence, the dynamic critical exponent of a 2D CDW at depinning is exactly $z(d=2)=5/4$. Remarkably, while CDWs at depinning map onto Abelian sandpiles, disordered elastic interfaces at depinning map onto Manna sandpiles [66,67]. Thus, each main universality class at depinning corresponds to a specific sandpile model.

Finally, the mapping of ϕ^4 theory at $n\to-2$ onto LERWs provides not only the fractal dimension of the latter, but also the correction-to-scaling exponent ω. We propose to measure it in simulations by erasing loops with probability $p<1$. Its ε expansion at six-loop order [51] is only slowly converging, and we estimate $\omega=0.83\pm0.01$.

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