Fundamental Limitations to Local Energy Extraction in Quantum Systems

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We examine when it is possible to locally extract energy from a bipartite quantum system in the presence of strong coupling and entanglement, a task which is expected to be restricted by entanglement in the low-energy eigenstates. We fully characterize this distinct notion of "passivity" by finding necessary and sufficient conditions for such extraction to be impossible, using techniques from semidefinite programing. This is the first time in which such techniques are used in the context of energy extraction, which opens a way of exploring further kinds of passivity in quantum thermodynamics. We also significantly strengthen a previous result of Frey *et al.*, by showing a physically relevant quantitative bound on the threshold temperature at which this passivity appears. Furthermore, we show how this no-go result also holds for thermal states in the thermodynamic limit, provided that the spatial correlations decay sufficiently fast, and we give numerical examples.

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Introduction.—In the macroscopic regime, in which thermodynamic systems typically exchange energy via weak interactions, the possible flows of energy between them are easily understood in terms of the usual laws of thermodynamics. These laws, however, may become less relevant for systems where the fluctuations and the particulars of the interaction between the microconstituents are important. Moreover, in the microscopic regime, quantum effects due to, e.g., coherence or entanglement appear, and a natural question arises: how do those effects alter the flows of energy in and out of the system?

For the task of extracting energy locally from a bipartite system, one could expect the following: if the low-energy eigenstates of the system display entanglement, there are limitations when trying to get closer to them only by means of local maps (since one cannot approach entangled states with local operations). While it could be possible to decrease the energy of the system up to some mixture of those low-energy eigenstates, trying to drive the system to a lower energy state can correspond to increasing the correlations in the system beyond what is possible via local operations alone.

Inspired by this intuition, here we focus on the problem of cooling interacting multipartite systems to which only local access to a single subsystem is granted. We explore the most general type of local access to quantum systems, which is given by the completely positive trace-preserving (CPTP) maps [1], making our results relevant for any physical platform in which the subsystems are spatially separated.

This problem was first studied in Frey *et al.* [2], who gave a set of sufficient conditions for the impossibility of energy-yielding via arbitrary local operations. They called this phenomenon strong local passivity (which we refer to here as *CP*-local passivity) and showed that having a nondegenerate ground state with full Schmidt rank is a *sufficient* condition for the system to exhibit it, given a large enough population in the ground state. Here, we build on their results in two ways: (i) we find necessary and sufficient conditions for this energy extraction to be impossible and (ii) we strengthen the set of physically motivated sufficient conditions found in [2], by finding explicit bounds for the ground state population and critical temperature for which the system displays *CP*-local passivity. We also prove that these sufficient conditions hold for systems of arbitrary size provided that the spatial correlations are weak, thus extending the presence of *CP*-local passivity to strongly coupled heat baths in the thermodynamic limit. Furthermore, we highlight the relevance of the necessary and sufficient conditions we find by constructing examples where none of the sufficient conditions are met.

We also show that this effect of *CP*-local passivity, unlike the usual notion of passivity, should only be of fundamental relevance in quantum scenarios. In states without coherence or entanglement, it can only happen if the support of the states is fine-tuned and/or the

Hamiltonian is sufficiently degenerate, which constitute very strong restrictions.

Setting.—Let $\mathcal{H}_A \otimes \mathcal{H}_B$ be the Hilbert space associated with quantum systems A and B, with global Hamiltonian H_{AB}. Given a state ρ_{AB}, the maximum extractable energy under a local map on A is

$$\Delta E_{(A)B} = \min_{\mathcal{E}_A} \Delta E^{\mathcal{E}_A}_{(A)B} := \min_{\mathcal{E}_A} \mathrm{Tr}[H_{AB}(\mathcal{E}_A \otimes \mathcal{I}_B)\rho_{AB}] - \mathrm{Tr}[H_{AB}\rho_{AB}], \quad (1)$$

where $\mathcal{I}_B$ is the identity channel on B, and the optimization is over the whole set of CPTP maps on A. The above optimization can be easily written as a "semidefinite program" (see [3,4] for introductory references to the subject). Therefore, it is very practical to calculate $\Delta E_{(A)B}$ and to find the CPTP map that minimizes the energy. Moreover, we see that energy cannot be extracted when this quantity is zero, which motivates the following definition.

Definition 1.—*CP*-local passivity: The pair $\{\rho_{AB}, H_{AB}\}$ is *CP*-local passive with respect to subsystem A if and only if

$$\Delta E_{(A)B} = \Delta E^{\mathcal{I}_A}_{(A)B} = 0. \quad (2)$$

That is, a system is *CP*-local passive if the best local strategy for extracting energy (as measured by the global Hamiltonian H_{AB}) is to act trivially on it. The word "passive" is used here in analogy to the commonly known passive states [5], from which energy cannot be extracted under unitary maps. Throughout, we assume that the time evolution given by the Hamiltonian H_{AB} does not play a role. This means that this setting applies to situations in which the local actions happen quickly, in the same spirit as that of fast local quenches or pulses in other quantum-thermodynamic settings [6,7].

Let us now outline how this might be possible. First, let us rewrite the term corresponding to the average energy of the system after applying a local map, as follows:

$$\mathrm{Tr}[H_{AB}(\mathcal{E}_A \otimes \mathcal{I}_B)\rho_{AB}] = \mathrm{Tr}[C_{AA'}E_{AA'}], \quad (3)$$

where $E_{AA'}$ is the Choi-Jamiołkowski operator for an arbitrary channel $\mathcal{E}_A : A \to A'$, and $C_{AA'} \in \mathcal{H}_A \otimes \mathcal{H}_{A'}$ the Hermitian operator $C_{AA'} \equiv \mathrm{Tr}_B[\rho^{\Gamma_A}_{AB} H_{A'B}]$, with $\rho^{\Gamma_A}_{AB}$ the partial transpose on A [8].

Let us now assume that *CP*-local passivity holds, such that for all $E_{AA'}$ the energy of the system does not decrease after the local action

$$\mathrm{Tr}[C_{AA'}E_{AA'}] \geq \mathrm{Tr}[H_{AB}\rho_{AB}], \quad (4)$$

We can rewrite the right-hand side, using the fact that $E_{AA'}$ satisfies $\mathrm{Tr}_{A'}[E_{AA'}] = \mathbb{I}_A$, and defining $d_A|\Phi\rangle\langle\Phi|$ as the Choi-Jamiołkowski operator for the identity channel as

$$\begin{aligned}\mathrm{Tr}[H_{AB}\rho_{AB}] &= \mathrm{Tr}[d_A|\Phi\rangle\langle\Phi|C_{AA'}] \\ &= \mathrm{Tr}_A[\mathrm{Tr}_{A'}[d_A|\Phi\rangle\langle\Phi|C_{AA'}]\mathrm{Tr}_{A'}[E_{AA'}]] \\ &= \mathrm{Tr}[(\mathrm{Tr}_{A'}[d_A|\Phi\rangle\langle\Phi|C_{AA'}] \otimes \mathbb{I}_{A'})E_{AA'}]. \end{aligned} \quad (5)$$

Since this holds for all $E_{AA'}$, this suggests that *CP*-local passivity will hold whenever the following operator inequality is true:

$$C_{AA'} \geq \mathrm{Tr}_{A'}[d_A|\Phi\rangle\langle\Phi|C_{AA'}] \otimes \mathbb{I}_{A'}. \quad (6)$$

Complete conditions.—The previous inequality in fact gives the necessary and sufficient condition. This constitutes our first main result.

Theorem 1.—The pair $\{\rho_{AB}, H_{AB}\}$ is *CP*-local passive with respect to subsystem A if and only if $\mathrm{Tr}_{A'}[d_A|\Phi\rangle\langle\Phi|C_{AA'}]$ is Hermitian and

$$C_{AA'} - \mathrm{Tr}_{A'}[d_A|\Phi\rangle\langle\Phi|C_{AA'}] \otimes \mathbb{I}_{A'} \geq 0, \quad (7)$$

where $\mathcal{H}_{A'}$ is a copy of the Hilbert space $\mathcal{H}_A$, $C_{AA'} \in \mathcal{H}_A \otimes \mathcal{H}_{A'}$ is a Hermitian operator defined as $C_{AA'} \equiv \mathrm{Tr}_B[\rho^{\Gamma_A}_{AB} H_{A'B}]$, with $\rho^{\Gamma_A}_{AB}$ as the partial transpose on A, and $d_A|\Phi\rangle\langle\Phi|$ is the (maximally entangled) Choi-Jamiołkowski operator of the identity channel.

Notice that Eq. (7) only depends on ρ_{AB} and H_{AB} through the operator $C_{AA'}$. In fact, Eq. (3) guarantees that this operator contains all the information about how much energy can be extracted through local operations. Once it is constructed, the operator inequality can be easily checked to find whether the pair $\{\rho_{AB}, H_{AB}\}$ is *CP*-local passive or not. If it is not, the semidefinite program can be solved to find the amount of energy that can be extracted, as well as the minimizing CPTP map. The proof can be found in the Supplemental Material [9], together with details on semidefinite programing duality theory, which we use in a similar manner as in the proof of the Holevo-Yuen-Kennedy-Lax conditions for quantum state discrimination [10–13].

On top of this characterization, we show that the condition of Theorem 1 is robust to errors, by using a recent result concerning convex channel optimization problems [14]. Roughly, if the operator on the lhs of Eq. (7) has the smallest eigenvalue $-\varepsilon \leq 0$, then the amount of energy that can be extracted is bounded as $\Delta E_{(A)B} \geq -\varepsilon d_A$. We give the precise statement and the proof in the Supplemental Material [9].

Sufficient conditions.—The condition of Theorem 1, even though it is simple to verify, makes no direct reference to physical properties of the pair $\{\rho_{AB}, H_{AB}\}$. It is important, however, to find physically relevant situations in which *CP*-local passivity holds. To that end, we derive sufficient conditions for steady states $\rho_{AB} = \sum_{i=0}^{d_A \times d_B - 1} p_i |E_i\rangle\langle E_i|$ of Hamiltonians $H_{AB} = \sum_{i=0}^{d_A \times d_B - 1} E_i |E_i\rangle\langle E_i|$ of full Schmidt

rank with a nondegenerate ground state. Steady states are always trivially CP-local passive for $p_0 = 1$, and Frey *et al.* [2] found qualitative conditions under which there exists a threshold ground state population p_* such that the pair $\{\rho_{AB}, H_{AB}\}$ remains CP-local passive for all $p_0 \geq p_*$. Here, we provide explicit upper bounds on p_* in terms of ground state entanglement and the energy gap with the first excited state.

Theorem 2.—Threshold ground state population: Let the ground state $|E_0\rangle$ of the Hamiltonian H_{AB} be nondegenerate and with full Schmidt rank. All pairs $\{\rho_{AB}, H_{AB}\}$ with $\rho_{AB} = \sum_i p_i |E_i\rangle\langle E_i|$ and $p_0 \geq p_*$ are CP-local passive with respect to A, with the threshold ground state population bounded from above by

$$p_* \leq \left(1 + \frac{E_1 (q^{AB}_{0,\min})^2}{\max_{i\geq 1}[E_i (q^{AB}_{i,\max})^2]}\right)^{-1}, \tag{8}$$

where $\{q^{AB}_{i,\alpha}\}_{\alpha=0}^{d_A-1}$ denotes the Schmidt coefficients of $|E_i\rangle$ and $q^{AB}_{i,\min} \equiv \min_\alpha[q^{AB}_{i,\alpha}]$, $q^{AB}_{i,\max} \equiv \max_\alpha[q^{AB}_{i,\alpha}]$.

See the Supplemental Material [9] for the proof and an example illustrating the tightness of the bound. The idea behind it is that, if the ground state population is high enough, the energetic changes caused by any CPTP map will be dominated by the energy gained by exciting the ground state into higher energy levels, making the total change non-negative.

For thermal states, this result implies that, if the ground state has full Schmidt rank, there exists a threshold temperature $T_* > 0$ below which CP-local passivity holds (note that, if $T = 0$, CP-local passivity holds trivially). Moreover, this threshold temperature is such that

$$\langle H\rangle_{\beta_*} \geq E_1 p_0 (q^{AB}_{0,\min})^2, \tag{9}$$

where $\langle H\rangle_{\beta_*}$ is the average energy in the thermal state of inverse temperature β_*.

We now describe when we expect this bound to be of importance. An entangled state of full Schmidt rank is typical in first-neighbor interactions where the local Hamiltonians do not commute with the interaction ones. However, given that $q^{AB}_{i,\min} \leq 1/d_A$, the bound weakens as the size of A grows (and it trivializes once $d_A > d_B$). Also, a unique ground state and a finite energy gap is needed. On top of that, frustration is required, as we show in the following. Let us rewrite the Hamiltonian as $H_{AB} = H_A + H_B + V_{AB}$. The "frustration energy" of H_{AB} is defined as

$$E_f \equiv E_0^{H_{AB}} - E_0^{H_A+H_B} - E_0^{V_{AB}}, \tag{10}$$

where E_0^H is the ground state energy of Hamiltonian H. This quantity measures the degree of frustration of H_{AB} with respect to a particular decomposition into local and interaction terms of H_{AB}. The main result of [15] then states that

$$\frac{E_f}{\max_{i\in\{A,B\}} E_1^i} \geq 1 - q^{AB}_{0,\max} \geq (d_A - 1) q^{AB}_{0,\min}, \tag{11}$$

where E_1^i is the gap of the local Hamiltonian H_i. This shows precisely that a certain level of frustration is necessary to have entanglement (in particular, with full Schmidt rank) in a unique ground state.

Note, however, that while these conditions are sufficient, they are by no means necessary. In fact, we provide simple examples of pairs that are CP-local passive but in which (i) the ground state is not entangled, (ii) the ground state is degenerate, and (iii) the state is not diagonal in the energy eigenbasis. These can be found in the Supplemental Material [9].

Thermodynamic limit.—The bound in Eq. (8) trivializes when the system B becomes very large, as the energy E_i grows with it. However, we show that for thermal states with weak spatial correlations, one can increase the size of system B indefinitely without breaking CP-local passivity. Hence, this phenomenon can hold even in the thermodynamic limit. First, we need the following definition.

Definition 2.—Clustering of correlations: A state ρ on a finite square lattice $\mathbb{Z}^D$ has $\epsilon(l)$ clustering of correlations if

$$\max_{M,N} |\mathrm{Tr}[M \otimes N\rho] - \mathrm{Tr}[M\rho]\mathrm{Tr}[N\rho]| \leq ||M||||N||\epsilon(l), \tag{12}$$

where the operator M has support on region A and N on region B, and $l \leq \mathrm{dist}(A, B)$, with $\mathrm{dist}(A, B)$ as the Euclidean distance on the lattice.

For a state ρ with $\epsilon(l)$ clustering of correlations, it is reasonable to expect that CP-local passivity is only determined by the vicinity of the region in which we act. We make this intuition precise in the following result. Let H_{AB} be a Hamiltonian on regions A, B in a d-dimensional finite square lattice. Let B_1, B_2 be any splitting of B (see Fig. 1), with $l \equiv \mathrm{dist}(A, B_2)$ as the distance over which B_1 shields A from B_2, with a boundary between B_1, B_2 of size $|\partial B_2|$. More precisely, H_{AB} takes the form

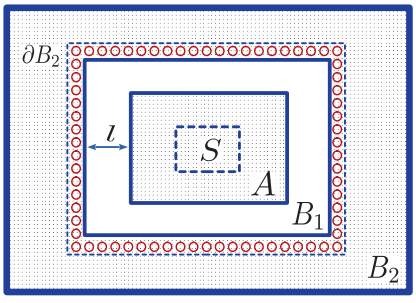

FIG. 1. Regions on the lattice for Theorem 3. The map acts on a region $S \subset A$, which is shielded from the region B_2 by B_1, by a distance of l. The boundary of the lattice between B_1 and B_2 is defined as ∂B_2 and has a number of sites $|\partial B_2|$.

$$H_{AB} = H_A + V_{AB_1} + H_{B_1} + V_{B_1B_2} + H_{B_2}. \quad (13)$$

We shall denote $H_{AB_1} \equiv H_A + V_{AB_1} + H_{B_1}$, and define $E_i^{AB_1}, q_{i,\alpha}^{AB_1}$ as the eigenvalues and Schmidt coefficients of H_{AB_1}. Let region $S \subseteq A$ be such that no site in S interacts with any site outside of the region A under H_{AB} (see Fig. 1). The result is as follows.

Theorem 3.—Consider a Hamiltonian H_{AB} as in Eq. (13) and let $\tau_{AB}^{\beta} = e^{-\beta H_{AB}}/Z_{AB}$ be its thermal state with $\epsilon(l)$ clustering of correlations. There exists a finite temperature β_* such that all pairs $\{\tau_{AB}^{\beta}, H_{AB}\}$ with $\beta \geq \beta_*$ are *CP*-local passive with respect to local operations on S if the regions B_1, B_2 can be chosen such that

$$E_1^{AB_1}(q_{0,\min}^{AB_1})^2 > \lambda(l), \quad (14)$$

where

$$\lambda(l) = K d_A^2 ||H_A|| |\partial B_2| (\epsilon(l/2) + c_1 e^{-c_2 l}). \quad (15)$$

Moreover, β_* is such that

$$\mathrm{Tr}[e^{-\beta_* H_{AB_1}}]^{-1} \leq \left(1 + \frac{\lambda(l)}{\max_{i\geq 1}[E_i^{AB_1}(q_{i,\max}^{AB_1})^2]}\right) \times \left(1 + \frac{E_1^{AB_1}(q_{0,\min}^{AB_1})^2}{\max_{i\geq 1}[E_i^{AB_1}(q_{i,\max}^{AB_1})^2]}\right)^{-1}, \quad (16)$$

where $K, c_1, c_2 > 0$ are constants.

The proof can be found in the Supplemental Material [9]. It relies on a result from [16] (which builds on [17]), which shows how clustering of correlations implies that the marginals of many-body thermal states can be efficiently estimated by looking only at subregions of the lattice. Crucially, the bound on β_* in Eq. (16) only depends on parameters of the Hamiltonian H_{AB_1} and on $\lambda(l)$ and is independent of B_2 (in particular, on its size) except for the boundary factor $|\partial B_2| \sim l^{D-1}$, with D as the dimension of the lattice. Hence, the best possible bound on β_* for an arbitrary system size is achieved by choosing a partition AB_1B_2 such that the marginals on A of τ_{AB} and τ_{AB_1} are close enough, and the size of AB_1 is not too large to render the bound useless.

A choice of regions (or rather, the choice of l) giving a nontrivial bound is possible provided that the correlations of the thermal state decay fast enough. More concretely, as long as we can find an l such that Eq. (14) holds, the upper bound on p^* of Eq. (16) is nontrivial. We expect this to be possible in a large class of models, as the gap rarely closes faster than polynomially with system size (if at all), and having an exponentially decaying $\epsilon(l)$ clustering of correlations at finite temperature is a property of many lattice models [18–20]. In Fig. 2, we provide a numerical example of a model in which we calculate how the threshold

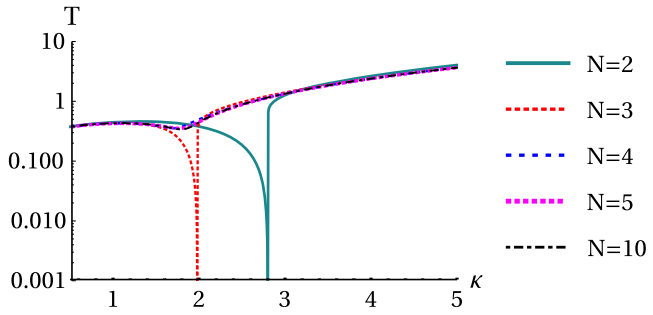

FIG. 2. Threshold temperature for the 1D Hamiltonian $H_{AB} = \sum_{l=1}^{N} \sigma_Z^{(l)} + \kappa\{[(1+\gamma)/2]\sigma_X^{(l)}\sigma_X^{(l+1)} + [(1-\gamma)/2]\sigma_Y^{(l)}\sigma_Y^{(l+1)}\}$ as a function of the coupling strength κ, fixing the anisotropy parameter $\gamma = 0.7$. The system A on which the maps act is the leftmost qubit $l = 1$. For $N > 3$ the curves overlap, showing that increasing system B beyond a certain size does not affect the threshold temperature appreciably. The threshold temperature was determined using the condition of Theorem 1.

temperature changes as we increase the system size. Note that the curves converge as N becomes large, showing that larger system sizes do not affect the threshold temperature appreciably.

Classical CP-local passivity.—This phenomenon can appear in certain classical situations (for instance, when the Hamiltonian is noninteracting and the initial state is $\rho_{AB} = |0\rangle\langle 0| \otimes \rho_B$), but we argue that either coherence or entanglement are necessary for it to be nontrivial. We do this by showing that *CP*-local passivity, in a classical setting, only happens in very restricted situations. Let us consider a fully classical model, with an incoherent state ρ_{AB} and a Hamiltonian with product eigenstates, such that

$$H_{AB} = \sum_{i,j} E_{i,j} |i\rangle\langle i| \otimes |j\rangle\langle j|, \quad (17)$$

$$\rho_{AB} = \sum_{i,j} p_{i,j} |i\rangle\langle i| \otimes |j\rangle\langle j|. \quad (18)$$

Without loss of generality, we can order the energies such that $E_{i,j} \leq E_{i+1,j}$ and $E_{i,j} \leq E_{i,j+1}$. The optimal local cooling strategy is straightforward: map the initial eigenstates to the eigenstates of lower energy that can be accessed with local maps. Let us write

$$\begin{aligned} \Delta E_{(A)B}^{\mathcal{E}_A} &= \mathrm{Tr}[H_{AB}(\mathcal{E}_A \otimes \mathcal{I}_B)\rho_{AB}] - \mathrm{Tr}[H_{AB}\rho_{AB}] \\ &= \sum_{i,k}\sum_{j} E_{ij} \sum_{l} p_{kl}\delta_{j,l}\langle i|\mathcal{E}_A(|k\rangle\langle k|)|i\rangle - \delta_{i,k} \\ &\equiv \sum_{i,k} \tilde{E}_{i,k}(\langle i|\mathcal{E}_A(|k\rangle\langle k|)|i\rangle - \delta_{i,k}), \end{aligned} \quad (19)$$

where $\tilde{E}_{i,k} = \sum_j E_{i,j} p_{k,j}$. The optimal CPTP map is such that $\mathcal{E}_A^{\mathrm{opt}}(|k\rangle\langle k|) = |i_k^*\rangle\langle i_k^*| \; \forall \, k$, where $i_k^* = \mathrm{argmin}_i \tilde{E}_{i,k}$, and thus

$$\Delta E_{(A)B} = \Delta E_{(A)B}^{\mathcal{E}_A^{\mathrm{opt}}} = \sum_k \tilde{E}_{i_k^*,k} - \tilde{E}_{k,k}, \tag{20}$$

which is non-negative if and only if $i_k^* = k \ \forall \ k$, in which case $\{\rho_{AB}, H_{AB}\}$ is CP-local passive. This happens only if the matrix $\tilde{E}_{i,k}$ is such that the smallest number in each row (indexed by k) is in the diagonal. This condition, however, can only be met by states with a particular support or by highly degenerate Hamiltonians. To be more precise, let us look at the individual terms of Eq. (20) for every $k > 1$,

$$\tilde{E}_{k-1,k} - \tilde{E}_{k,k} = \sum_j (E_{k-1,j} - E_{k,j}) p_{k,j}. \tag{21}$$

Since $E_{k-1,j} - E_{k,j} \leq 0$ by definition, the only way Eq. (21) can be non-negative is if either $p_{k,j} = 0$ or $E_{k-1,j} = E_{k,j} \ \forall \ j$, which constitutes a strong restriction on the support of the initial state and H_{AB}. For instance, no thermal state (with full support) of a Hamiltonian with any nondegeneracy on index k will obey this condition.

Discussion.—We have found necessary and sufficient conditions for CP-local passivity, which take the form of a simple inequality of operators of size $d_A \times d_A$. We also derived simpler sufficient conditions that show definite physical situations in which this phenomenon appears, and we provide numerical examples illustrating the general picture.

Our proof of the necessary and sufficient conditions, of Theorem 1, uses tools from the theory of convex optimization, widely used in quantum information, but which, apart from a few exceptions [21,22], have not yet been exploited in quantum-thermodynamic contexts. In fact, this is, to our knowledge, the first time that the theory of semidefinite programing has been used in the context of energy extraction and passivity. We expect these tools to be of further use in similar situations in which the actions allowed on the state are limited in different physically motivated ways. The fact that we optimize over a linear function of the channels (the energy of the output) made the derivations particularly simple, but, in fact, recent results [14] easily allow for extensions to arbitrary nonlinear functions.

A further set of previous results (e.g., [23–25]) identify entanglement in the initial state as a useful resource in energy extraction when one has access to global operations and the Hamiltonians are noninteracting. Here we explore a different side of the general picture, by showing that entanglement in the eigenstates can forbid the possibility of energy extraction via local operations when the interactions are strong.

The underlying principle here is that entanglement in the low-energy eigenstates causes a fundamental lack of local control in systems at low temperature, provided that the CPTP maps are fast compared to the dynamics of the system. This effect could potentially also include quenches and/or pulses that are commonly taken as the steps of quantum thermal cycles in which "work" is exchanged [6,7,26,27], in which case our results should put constraints on their regime in which those machines can perform.

A further study on CP-local passivity could be the characterization of scenarios in which this passivity can be circumvented by allowing classical communication. This type of setting goes under the name of quantum energy teleportation (QET) [28–30]. Our necessary and sufficient conditions could help design better QET-based protocols, which have been applied both in quantum field theory [31] and algorithmic cooling in quantum information processing [32].

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