Integrable Many-Body Quantum Floquet-Thouless Pumps

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We construct an interacting integrable Floquet model featuring quasiparticle excitations with topologically nontrivial chiral dispersion. This model is a fully quantum generalization of an integrable classical cellular automaton. We write down and solve the Bethe equations for the generalized quantum model and show that these take on a particularly simple form that allows for an exact solution: essentially, the quasiparticles behave like interacting hard rods. The generalized thermodynamics and hydrodynamics of this model follow directly, providing an exact description of interacting chiral particles in the thermodynamic limit. Although the model is interacting, its unusually simple structure allows us to construct operators that spread with no butterfly effect; this construction does not seem possible in other interacting integrable systems. This model exemplifies a new class of exactly solvable, interacting quantum systems specific to the Floquet setting.

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Introduction.—Periodically driven (or "Floquet") quantum systems have become a major theme in many-body physics [1–15]: driving enables one to engineer exotic states of matter experimentally [16–19] to realize phases that are absent in equilibrium [7–9,20–29]. Floquet dynamics are captured by a unitary $\hat{F}$, which evolves the system by a single period; when $\hat{F}$ is not smoothly connected to the identity, the resulting dynamics are distinct from those of any "static" Hamiltonian ($e^{i\hat{H}t}$ is always deformable to the identity). This is particularly transparent in noninteracting Floquet systems: their band structure is compactified in both quasimomentum *and* quasienergy, which allows for band structures that wind nontrivially in quasienergy [Fig. 1(a)], which cannot be realized in local lattice Hamiltonians [6–10,20,21]. Thus, Floquet systems can host unpaired chiral modes, while Hamiltonian dynamics only admit chiral modes on the boundaries of higher-dimensional systems [6,8,30,31]. In systems with nontrivial quasienergy winding, each single-particle state has a quantized time-averaged current; protocols that accomplish such adiabatic particle transfer are called "Thouless pumps" [6,32,33].

These topological features remain long-lived in certain interacting models [33–35]; however, interactions generally heat a system up to infinite temperature, unless it is integrable or many-body localized (MBL) [36–43]. Although a MBL system can protect Floquet topological phases [22,43–57], such *localized* phases do not host chiral modes. *Interacting* integrable systems are another broad class of systems that do not thermalize [58–61]; whether or not distinctively Floquet versions exist has been less discussed [62–66].

This Letter presents an interacting integrable Floquet model that hosts quasiparticles with a nontrivial winding number, i.e., an integrable Thouless pump. Unlike previously proposed interacting integrable Floquet systems, this model is not smoothly connected to any Hamiltonian and is thus inherently Floquet, rather than an "integrable Trotterization" [66]. This model is a fully quantum extension of an integrable cellular automaton known as rule 54, or the Floquet Fredrickson-Andersen model (FFA) [67–69]. The FFA model's simplicity has elucidated various puzzles concerning hydrodynamics and operator growth in generic interacting integrable systems [68–75]. The FFA model can be written as a Floquet unitary comprising local gates, but it is classical in that it maps computational-basis product states to one another. Although the FFA model has chiral quasiparticles, they do not disperse, but instead all have one of two group velocities, ± 1. The dispersing FFA (DFFA) generalization we introduce here alternates the FFA dynamics with that of a particular strictly local Hamiltonian, making the model fully quantum by restoring dispersion while preserving integrability. This generalization remains simple enough that the Bethe equations can be solved analytically—a remarkable feature for an interacting model. This model is simple because the quantization of either quasiparticle species depends only on the total number of quasiparticles of each species, and *not* on their rapidities. This simplicity also manifests in the existence of special local operators that remain lightly entangled at all times, as in the FFA model [72,75]. This model is the first representative of a class of interacting integrable models specific to the Floquet setting, featuring stable chiral quasiparticles.

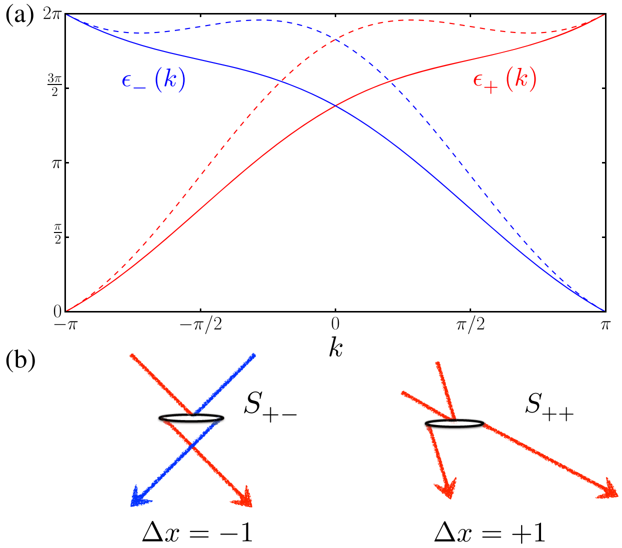

FIG. 1. Chiral quasiparticles in the DFFA model. (a) Dispersion relations showing the (bare) single-particle Floquet quasienergies $\varepsilon(k)$ for both $+$ and $-$ quasiparticles, for $\lambda = 0.3$ (solid lines) and $\lambda = 0.65$ (dashed lines). Both bands are topological (chiral) as they wrap around the periodic quasienergy direction and can only exist in a periodically driven system. Note that, for $\lambda = 0.65$, the $\pm$ particles can be left (right) moving for some range of momenta. (b) Soliton gas picture. The scattering events in the DFFA model factorize onto simple two-body processes, which semiclassically correspond to a displacement $\Delta x = \pm 1$ after a collision, independent of the momenta of the quasiparticles.

Model.—We consider a chain of $2L$ qubits (spins-$\frac{1}{2}$) with dynamics generated by the repeated application of the unitary evolution (Floquet) operator

$$\hat{F}(\lambda) = e^{-i\lambda\hat{H}} \prod_{j\text{even}} \hat{U}_{j-1,j,j+1} \prod_{j\text{odd}} \hat{U}_{j-1,j,j+1}, \tag{1}$$

with gates $\hat{U}_{j-1,j,j+1} \equiv \mathrm{CNOT}(1 \to 2)\mathrm{CNOT}(3 \to 2)\,\mathrm{Toff}(1,3 \to 2)$, in terms of controlled NOT (CNOT) and Toffoli gates [76], and H is a Hamiltonian that we will specify below. In simpler terms, $\hat{U}_{j-1,j,j+1}$ is the instruction "flip spin j if one or both of its nearest neighbors is up." For $\lambda = 0$, this model reduces to FFA, $\hat{F}(0) = \hat{F}_0$.

FFA limit.—On its own, $\hat{F}_0$ hosts two species of chiral quasiparticle excitations above the vacuum state $|0\rangle = |\downarrow\downarrow\ldots\downarrow\rangle$, indexed $\nu = +1$ for "right movers" and $\nu = -1$ for "left movers." We regard the $2L$ physical sites as L unit cells: the nth unit cell contains the A site $2n-1$ and B site $2n$. If both of these sites are $\uparrow$, then there is a $\nu = +1$ right-moving doublon in cell n; if the B site of cell $n-1$ and A site of n are both $\uparrow$, there is a $\nu = -1$ left-moving doublon in cell n. Additionally, we refer to isolated $\uparrow$'s as "molecules", which contain one of each mover: a molecule on the A site of cell n corresponds to both $\nu = \pm 1$ movers in cell n; a B molecule in cell n corresponds to a $+$ at n and $-$ at $n+1$. The molecule states $\downarrow\uparrow\downarrow$ arise during collisions between the two species. Apart from these collisions, $\hat{F}_0$ acts by changing the positions of the $\pm$ particles by ± 1 unit cell and conserves independently the number of each, $N_\pm$.

In the FFA model, all quasiparticle excitations have two possible velocities and no dispersion. The structure of conservation laws in this model differs from that of generic interacting integrable models, in which a generalized Gibbs ensemble (GGE) [77] can be fully specified through the distribution of quasiparticle velocities. In the FFA model, there are only two velocities, which do not fully specify a state. The remaining conservation laws correspond to asymptotic "spacings" between adjacent quasiparticles of the same species [69]. In the zero-density limit, the bare spacings between same-species quasiparticles are conserved, since all such quasiparticles have the same velocity. At nonzero densities, one can define an asymptotic spacing by accounting for interaction effects: e.g., suppose we have two $+$ quasiparticles that are n steps apart; the quasiparticle on the right collides first with a $-$ quasiparticle and is time delayed by one step. Therefore, while there is a $-$ quasiparticle between them, the two $+$ quasiparticles will be exactly $n-1$ steps apart if their asymptotic spacing is n. Given a spin configuration, its asymptotic spacings can be found numerically by simulating its free expansion into vacuum [69].

Adding dispersion.—We now construct $\hat{H}$, the Hamiltonian part of (1), to generate dispersion while maintaining integrability. Conservation of particle number automatically precludes many simple terms, i.e., most single spin processes. Even a pair-hopping term like $\hat{\sigma}_i^+\hat{\sigma}_{i+1}^+\hat{\sigma}_{i+2}^-\hat{\sigma}_{i+3}^-$ will not conserve $N_\pm$: it can bring two $+$ doublons to neighboring unit cells, producing a $-$ doublon on the intervening bond. The simplest $N_\pm$-conserving operator that disperses quasiparticles is $\hat{h}_j \equiv \hat{d}_{j-1}\hat{\sigma}_j^+\hat{\sigma}_{j+1}^+\hat{\sigma}_{j+2}^-\hat{\sigma}_{j+3}^-\hat{d}_{j+4}$, where $\hat{d}_j \equiv \frac{1}{2}(1-\hat{\sigma}_j^z)$ [analogously, $\hat{u}_j \equiv \frac{1}{2}(1+\hat{\sigma}_j^z)$]. This term "checks" that it would not create any new quasiparticles before moving one. Setting $\hat{H} = \sum_j(\hat{h}_j + \hat{h}_j^\dagger)$ would give a simple dispersive extension of the FFA model; however, we cannot confirm that this preserves integrability, so we add other terms,

$$\begin{aligned}\hat{H} = \sum_i (&\hat{d}_i\hat{\sigma}_{i+1}^+\hat{\sigma}_{i+2}^+\hat{\sigma}_{i+3}^-\hat{\sigma}_{i+4}^-\hat{d}_{i+5} + \hat{d}_i\hat{\sigma}_{i+1}^+\hat{\sigma}_{i+2}^-\hat{d}_{i+3}\\ &+ \hat{d}_i\hat{\sigma}_{i+1}^+\hat{\sigma}_{i+2}^+\hat{u}_{i+3}\hat{d}_{i+4} + \mathrm{refl}\\ &+ \hat{d}_i\hat{\sigma}_{i+1}^+\hat{\sigma}_{i+2}^+\hat{\sigma}_{i+3}^-\hat{u}_{i+4}\hat{u}_{i+5} + \mathrm{refl}\\ &+ \hat{d}_i\hat{u}_{i+1}\hat{\sigma}_{i+2}^+\hat{u}_{i+3}\hat{u}_{i+4} + \mathrm{refl}\\ &+ \hat{u}_i\hat{u}_{i+1}\hat{\sigma}_{i+2}^+\hat{\sigma}_{i+3}^-\hat{u}_{i+4}\hat{u}_{i+5}) + \mathrm{H.c.},\end{aligned} \tag{2}$$

where "refl." indicates that one should reverse the sequence of indices in the previous term. In the quasiparticle language, $\hat{H}$ (2) maps a configuration σ to a uniform superposition of all configurations σ' with a single quasiparticle moved by one unit cell, provided $N_\pm$ are preserved.

The Hamiltonian (2) commutes with $\hat{F}_0$, but nevertheless acts nontrivially because $\hat{F}_0$ has exponentially degenerate eigenstates: for a given $N_\pm$ in a system of size L, there are only $O(L^2)$ eigenvalues, but exponentially many basis states, corresponding to different quasiparticle positions. Equation (2) lifts the degeneracy in this subspace and thus makes the dynamics fully quantum. This perturbation cures many pathological features of the FFA model that are due to these degeneracies, such as its failure to equilibrate to the diagonal ensemble [69].

Single-quasiparticle sectors.—We first find eigenstates of (1) for a single $\pm$ quasiparticle, $|j,\pm\rangle = \sigma^x_{2j}\sigma^x_{2j\pm1-1}|0\rangle$. The Fourier transform is an eigenstate of $\hat{F}(\lambda)$,

$$\hat{F}(\lambda)|k,\nu\rangle = e^{-i\nu k - 2i\lambda\cos k}|k,\nu\rangle, \tag{3}$$

where $\nu = \pm 1$. Here, λ controls the strength of the dispersing term, and $k = 2\pi m/L$ for integer m, with L the system size in unit cells. This model thus has two chiral bands (Fig. 1). For $\lambda < 1/2$, all $+$ $(-)$ quasiparticles have right-(left-)moving group velocities, but for $\lambda > 1/2$, both species have left- and right-moving quasiparticles. The group velocities of $\pm$ quasiparticles are given by $v^0_{\pm,k} = \pm 1 - 2\lambda \sin k$. These chiral bands are characterized by a quantized winding number $\nu = \int_{-\pi}^{\pi}(dk/2\pi)v^0_{\pm,k} = \pm 1$, which is the invariant characterizing Thouless pumping [6,32,33].

Bethe ansatz solution.—We now move on to multiparticle sectors. We note, first, that in the absence of left movers, the FFA evolution is just a trivial global translation. In this purely right-moving sector, the dynamics of $+$ quasiparticles consists of hopping and hardcore nearest-neighbor repulsion. The scattering phase shift between particles of the same species is thus $S_{++}(k_2,k_1) = S_{-}(k_2,k_1) = \mathcal{S}(k_2,k_1) = -e^{i(k_2-k_1)}$. Meanwhile, the scattering between left and right movers is engineered to retain the FFA form such that the phase shift after a collision is $S_{-+}(k_+,k_-) = \tilde{\mathcal{S}}(k_+,k_-) = +e^{i(k_+-k_-)}$, and no meaning is ascribed to the order of the arguments. Higher-body collisions factorize onto the two-body scattering processes, ensuring integrability. For a many-body state with fixed (N_+,N_-), where $\{k_j^\pm\}$ refer to the momenta of the $\pm$ quasiparticles, we find the following quantization condition (see Supplemental Material [78]):

$$e^{ik_j^+L} = \prod_{\substack{n=1\\ n\neq j}}^{N_+}\mathcal{S}(k_j^+,k_n^+)\prod_{m=1}^{N_-}\tilde{\mathcal{S}}^*(k_j^+,k_m^-),$$
$$e^{ik_j^-L} = \prod_{\substack{n=1\\ n\neq j}}^{N_-}\mathcal{S}(k_j^-,k_n^-)\prod_{m=1}^{N_+}\tilde{\mathcal{S}}(k_j^-,k_m^+). \tag{4}$$

These quantization conditions have the same form as Bethe equations in Hamiltonian systems. Translational invariance and the recurrence properties of the FFA model [with which the Hamiltonian (2) commutes] impose two further constraints. We require, first, that $\sum_j k_j^+ + \sum_j k_j^- = K$, where $K = 2\pi m/L$ with m is one of the allowed global momenta, and second, that the relative momentum $\sum_j k_j^+ - \sum_j k_j^- = \Theta$, where

$$\Theta = \frac{2\pi N_\theta + (N_+ - N_- - L)K}{L + N_- + N_+}, \tag{5}$$

with $1 \le N_\theta \le (L + N_- + N_+)$ an integer, unless $L + N_- + N_+$ is even, in which case N_θ must be as well (see Supplemental Material [78]). Finally, no two quasiparticles of the same species may occupy the same momentum state. With these constraints, the solutions (4) fully characterize the eigenstates in a finite system, and the corresponding quasienergy $e^{-i\varepsilon}$ of the Floquet unitary (1) reads $\varepsilon = \sum_{\nu=\pm}\sum_{n=1}^{N_\nu}(\nu k_n^\nu + 2\lambda\cos k_n^\nu)$.

Remarkably, these equations are simple enough that they can be solved exactly for any finite system. The set of allowed momenta for either species ν is

$$k_j^\nu = \frac{\pi(2m_j^\nu + N_\nu - 1) - \nu\Theta}{L - N_\nu + N_{\bar{\nu}}}, \tag{6}$$

with $\bar{\nu} = -\nu$ and $1 \le m_j^\nu \le L - N_\nu + N_{\bar{\nu}}$. The number of available $m_j^\pm$ decreases with the total number of $\pm$ movers because neighboring unit cells cannot both host $\pm$'s without a $\mp$ between them. We also note that the quantization condition depends on the total number and momentum of the $\pm$ quasiparticles, not on the details of their distribution. Relatedly, (4) and (6) do not depend on λ and thus also apply to $\hat{F}_0$, though in that model, the phase shift between quasiparticles of the same species is ill-defined, as they move in unison and never collide.

The DFFA model corrects several pathological features of the FFA model. We show this numerically using exact diagonalization (ED) to analyze the quasienergy level statistics, which does not show level repulsion (Fig. 2), consistent with integrability. We also checked using ED that the value of the "r ratio" [37] is consistent with a Poisson distribution for all $\lambda > 0$.

Thermodynamics.—In the thermodynamic limit, one defines densities of quasiparticles at a given species and rapidity, $\rho_\pm(k)$, as well as total densities of states $\rho_\pm^{\rm tot}(k)$, related via the Bethe equations

$$2\pi\rho_\pm^{\rm tot}(q) = 1 + n_\mp - n_\pm, \tag{7}$$

where $n_\pm \equiv \int_{-\pi}^{\pi} dq\rho_\pm(q) = N_\pm/L$. These equations follow from the continuum limit of (4), with the scattering

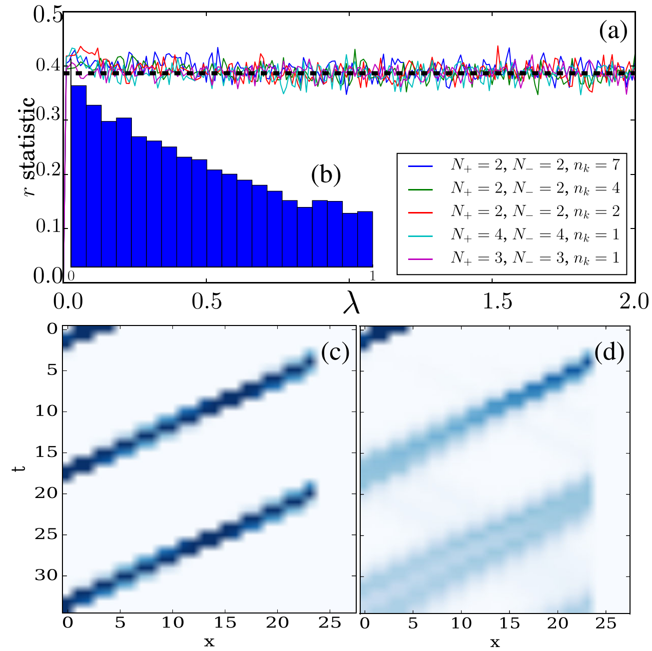

FIG. 2. Numerical results. Quasienergy level statistics as a function of λ, for several values of $N_\pm$ and K at $L = 9$. (a) The r ratio shows good agreement with a Poisson distribution (dashed line) for all $\lambda > 0$. (b) (Inset) The distribution of r for $\lambda = 1.0$ does not show level repulsion, consistent with integrability. Plot of the OTOC $C(t)$ for $L = 14$ unit cells with $N_+ = 1$ and $N_- = 2$, for (c) $\lambda = 0$, corresponding to the FFA model, and (d) $\lambda = 0.05$, where we see that the OTOC does not "fill in" behind the front except through the dispersion of the perturbed quasiparticle. All data obtained from exact diagonalization.

kernels $\mathcal{K}_{\nu\nu'} = (1/2\pi i)(d/dk)\ln S_{\nu\nu'}$ with $\nu, \nu' \in \{+,-\}$ given by $\mathcal{K}_{++} = \mathcal{K}_{--} = 1/(2\pi)$, $\mathcal{K}_{+-} = \mathcal{K}_{-+} = -1/(2\pi)$.

Starting with these equations, one can straightforwardly construct generalized equilibrium states of this Floquet system. We emphasize that, since the DFFA model is integrable, its dynamics lead to nontrivial steady states that are distinct from featureless infinite temperature states that would be expected for generic interacting Floquet systems. For concreteness, we focus on generalized equilibrium states characterized by a given density of $\pm$ quasiparticles via the partition function $\mathcal{Z} = \sum_{\{\sigma\}} e^{-\mu_- N_- - \mu_+ N_+}$, but our discussion extends naturally to arbitrary GGEs for this model. In terms of quasiparticle densities, the partition function reads $\mathcal{Z} \sim \int \mathcal{D}\rho e^{L\int dk S_{\mathrm{YY}}} e^{-L\mu_+ \int dk\rho_{k,+} - L\mu_- \int dk \rho_{k,-}}$, where S_{YY} is the so-called Yang-Yang entropy [79,80] associated with the occupation of quasiparticle states. In the thermodynamic limit, these integrals are dominated by their saddle point, giving rise to thermodynamic Bethe ansatz (TBA) equations analogous to Hamiltonian integrable systems [80]. This leads to the following equations for the occupation numbers (Fermi factors) $\theta_\nu(k) \equiv \rho_\nu(k)/\rho_\nu^{\mathrm{tot}}(k) \equiv (1 + e^{\epsilon_\nu(q)})^{-1}$, which turn out to be independent of k,

$$\epsilon_\pm = \mu_\pm + \log\left(\frac{1 + e^{-\epsilon_\pm}}{1 + e^{-\epsilon_\mp}}\right). \tag{8}$$

Together with (7) this forms a complete characterization of the generalized Gibbs ensemble. The simple algebraic form of (8) resembles the high-energy limit of the TBA equations of other integrable systems [81,82]. For $\lambda = 0$ (FFA model), the properties of this ensemble can also be derived by a transfer-matrix calculation [72]; these approaches give equivalent results (see Supplemental Material [78]). These TBA solutions allow one to probe the physics of Thouless pumps in an interacting model even in the limit $L \to \infty$.

Hydrodynamics.—Coarse-grained dynamics in the DFFA model can be described using the recently developed theory of generalized hydrodynamics (GHD) [83–103]. GHD treats the system semiclassically as a soliton gas [89]. There are two species of solitons $\pm$ whose bare group velocities v_k^0 follow from the dispersion relation (3). When solitons collide, each pick up a semiclassical displacement $\Delta x = 2\pi\mathcal{K}$ in the direction of motion (Fig. 1). Like-species (unlike-species) collisions speed up (slow down) solitons by one step. Collisions lead to a dressing of the velocities [83,84,104], with the effective velocities in a state with quasiparticle densities $\rho_{\pm,k}$ given by

$$v_{\pm,k} = v_{\pm,k}^0 + \int dq(v_{\pm,k} - v_{\pm,q})\rho_{\pm,q} - \int dq(v_{\pm,k} - v_{\mp,q})\rho_{\mp,q}. \tag{9}$$

Higher-order corrections are derived in the Supplemental Material [78].

Operator dynamics.—The rapidity-independent scattering kernels in the DFFA model have important consequences for operator spreading, which is simpler here than in generic integrable models [72]. In the generic case, any operator creates a "light cone" that fills in at late times: a spatially local operator has a spread of momenta and thus of group velocities, and the velocity dependence of the scattering kernel implies that perturbing the velocity of one quasiparticle will affect the trajectories of all the others. This does not happen either in the hard rod gas or in the DFFA model, since the scattering kernel in these models is velocity independent and, consequently, any perturbation that preserves $N_\pm$ will only affect the state of one quasiparticle. Thus the butterfly cone, measured via the out-of-time-order correlator [105–108] (OTOC) $C(x,t) \equiv \frac{1}{2}|\mathrm{Tr}\{[\hat{h}_{j=2}, \hat{\sigma}_x^z(t)]^2\}|$ does not fill in except through the dispersion of the perturbed quasiparticle (Fig. 2). This property can also be seen from the coordinate Bethe ansatz: an operator that changes the velocity of a $+$ quasiparticle does not alter the quantization condition for any other quasiparticles and thus does not force a global rearrangement, in contrast to the generic case.

Conclusion.—In summary, we present and solve exactly a Floquet model that is the first of its kind in a number of respects. It is the first example of an interacting integrable Floquet model that is neither smoothly deformable to Hamiltonian dynamics [66] nor classically simulable (FFA). Our solution of the dispersing model has provided insight into the physics of the FFA model, which prior to this work was not confirmed to be integrable in the Yang-Baxter sense. The dispersing model regularizes several pathological features of the FFA model, but nonetheless preserves the chiral quasiparticle excitations of the FFA model, which realize topological Thouless pumping. Despite the complicated nature of the Hamiltonian terms, the resulting Bethe (4) and TBA equations (8) are the simplest of any interacting integrable model as far as we are aware. This model shows the existence of interacting Floquet models with stable chiral quasiparticles and suggests a route to finding others, building on integrable cellular automata [67,71,109,110].

Finally, we briefly discuss the experimental implications of this Letter. The FFA model comprises standard CNOT and Toffoli gates and is therefore simple to implement on existing "noisy intermediate-scale quantum computers" [111] based on ion traps, cold atoms, or superconducting qubits. The Hamiltonian (2) is more challenging, although a Trotterized version that preserves integrability may be implemented on small gate-based quantum simulators; transport, operator growth [112], and even level statistics [113] have been measured in this setting. There might also be simpler-to-realize deformations of the FFA model that retain integrability (e.g., models that only contain the doublon-hopping term). Exploring such deformations is an interesting topic for future work.

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