Beyond Chu's Limit with Floquet Impedance Matching

Huanan Li,[1] Ahmed Mekawy,[1,2] and Andrea Alù[1,2,3,*]

[1]*Photonics Initiative, Advanced Science Research Center, City University of New York, New York, New York 10031, USA*

[2]*Department of Electrical Engineering, City College of The City University of New York, New York, New York 10031, USA*

[3]*Physics Program, Graduate Center, City University of New York, New York, New York 10016, USA*

(Received 13 March 2019; published 16 October 2019)

Chu's limit determines the minimum radiation quality factor Q of an electrically small resonator, and hence its maximum operational bandwidth, which is inversely proportional to its volume. This bound imposes severe restrictions in several areas of technology, from wireless communications to nanophotonics and metamaterials. We show that a suitably tailored temporal modulation of the matching network, combined with proper detuning of the feeding impedance, can overcome this limit and enable radiation over broader bandwidths, which scale as $1/\sqrt{Q}$, ensuring at the same time stability. Our findings open opportunities for communication systems, nanophotonics, and sensor technology.

DOI: 10.1103/PhysRevLett.123.164102

Antenna minimization is of central importance in modern wireless communications due to the ever-increasing demand for compact versatile devices [1]. Electrically small antennas and resonators are crucial for portable wireless devices, medical equipment, sensors, and communication systems, among other applications. Chu's limit, originally derived over 70 years ago [2], determines a minimum bound on the attainable radiation Q factor of small antennas, inversely proportional to their electrical volume, imposing severe constraints on their bandwidth of operation. Given the generality of this bound, which applies to any linear, passive, time-invariant open resonator, the efficiency-bandwidth product of small antennas is inherently limited and proportional to their volume [3]. This in turn implies a severe trade-off between the size of an antenna or a resonator and its bandwidth of operation.

To overcome Chu's limit, extensive research has been focused on active impedance matching techniques [4], using antenna loads that draw energy from amplifiers violating Foster's reactance theorem [5–9]; see also Ref. [10]. Non-Foster antennas typically take advantage of negative-impedance converters to impedance match the antenna load over large bandwidths. However, they typically suffer from inherent instabilities and detrimental noise stemming from a combination of the active nature of the involved circuitry and its complexity and parasitics [11]. Recently, time-varying networks based on switched systems triggered when a pulse signal impinges on the load have been explored to overcome the bandwidth-size trade-off [12–15]. However, this approach requires synchronization of the incoming signals with the time variations in the matching network, and it is limited to pulsed operation.

In recent years, Floquet systems have been gaining increasing attention in the physics and engineering communities, highlighting how time modulation can empower systems with highly unusual properties overcoming limitations of passive time-invariant systems. Time modulation has been employed to enable Floquet topological phases [16–18], modulation-induced nonreciprocity [19–22], Floquet PT-symmetric systems [23,24], Floquet engineering [25–28], and scattering [29,30]. Here, we apply the exotic physics of temporally modulated systems to impedance matching to overcome Chu's limit for signal transmission and gain in small antennas. We prove that Floquet impedance matching can overcome the gain bandwidth vs size trade-off of passive antenna systems, and it addresses the issue of instabilities and noise of non-Foster amplifier-based approaches. We validate our results within a realistic loop antenna loaded by a modulated varactor and discuss the possible impact of these findings on other relevant technologies.

Electrically small antennas and Chu's limit.—An antenna is electrically small if its electrical characteristic length $ka < 1/2$, where k is the wave number in free space and a is the radius of a sphere circumscribing it [4]. Small antennas are poor radiators due to their high radiation quality factor $Q = 2\omega_r \max\{W_E, W_M\}/P_{\mathrm{rad}}$, where W_E (W_M) is the time-averaged stored electric (magnetic) energy, P_{rad} is the radiated power, and ω_r is the frequency of operation [2,31]. In this definition, it is assumed that at ω_r the antenna is impedance matched, canceling its large reactance via a matching network and bringing it to resonance. Chu's limit determines a lower bound for the corresponding Q factor: an electric dipole, for instance, has an approximate input impedance $Z_{\mathrm{in}} \approx \eta[1/(jka) + (ka)^2]$ [2], characterized by a large reactance associated with the capacitance $C = \varepsilon_0 a$, and a small radiation resistance $R_r = \eta(ka)^2$, where ε_0 is the vacuum permittivity and η the impedance of free space (Fig. 1). The quality factor $Q_{\mathrm{Chu}} = 1/(\omega_r R_r C) = 1/(k_r a)^3$, where $k_r = \omega_r/c_0$ (with c_0

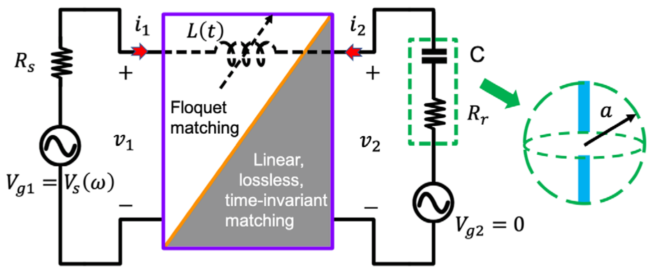

FIG. 1. Schematic of the Floquet matching network for an electric dipole, which is modeled as a series capacitance-resistance circuit. Floquet matching is achieved with a time-varying inductor $L(t) = L_0[1 + m\cos(\omega_M t)]$ (see the dashed line).

being the speed of light in vacuum), increases inversely with the electrical volume $(k_r a)^3$, and thus becomes very large for small antennas. This value represents the minimum bound of the radiation Q factor of a small antenna, known as Chu's limit [2,3,32–34]. Practical antennas can yield a larger Q factor only by inducing additional stored energy within their physical structure. This in turn fundamentally limits the fractional bandwidth of operation, which is inversely proportional to Q.

Chu's limit is fundamentally rooted within the more general Bode-Fano bound on gain bandwidth [35–37]. For passive, linear, time-invariant systems consisting of an antenna (e.g., a series resistance and capacitance) and the associated matching network (Fig. 1), we have [38]

$$\int_0^{\infty} \frac{1}{\tilde{\omega}^2} \ln \frac{1}{\sqrt{1 - T_0}} d\tilde{\omega} < \frac{\pi}{Q}, \tag{1}$$

where the normalized (angular) frequency $\tilde{\omega} = \omega/\omega_r$, and the transducer gain T_0 is defined as the average power delivered to R_r over the maximum available average power at the source V_s. Equation (1) is particularly restricting when matching an electrically small antenna due to its high-Q nature. In this case, we necessarily expect a narrow gain peak around the operation frequency ω_r [4].

Floquet impedance matching.—We establish a Floquet matching framework to go beyond the Bode-Fano bound for electrically small open resonators. For simplicity, we analyze the case of a small electric dipole antenna, impedance matched to the source through a time-modulated inductor $L(t) = L_0[1 + m\cos(\omega_M t)]$ (see Fig. 1). The static part L_0 is employed to compensate for the antenna reactance and bring it to resonance at $\omega_r = 1/\sqrt{L_0 C}$. The modulation is introduced to impart parametric gain and overcome Chu's limit for passive networks; see also Refs. [39,40].

In order to evaluate the transducer gain $T_0(\omega)$ in this Floquet matching circuit, and meanwhile study its stability, we construct the (normalized) Floquet scattering matrix $\overleftrightarrow{S}(s)$ in the complex excitation-frequency plane $s = \sigma + j\omega$, $\omega \in (0, \omega_M)$. The Floquet $\overleftrightarrow{S}$ matrix, defined as $\vec{b} = \overleftrightarrow{S}\,\vec{a}$, connects the outgoing power wave amplitudes at different harmonics with the incoming ones [38], written collectively as output-input vectors $\vec{b}$ and $\vec{a}$:

$$\vec{a} \equiv \frac{1}{2\overleftrightarrow{k}}[\overleftrightarrow{y} \cdot \vec{V} + \vec{I}], \qquad \vec{b} \equiv \frac{1}{2\overleftrightarrow{k}_*}[\overleftrightarrow{y}_* \cdot \vec{V} - \vec{I}], \tag{2}$$

where the Floquet voltage $\vec{V} = \binom{\vec{V}_1}{\vec{V}_2}$ and current vectors $\vec{I} = \binom{\vec{I}_1}{\vec{I}_2}$ consist of the frequency component of the voltages $\{v_1(t), v_2(t)\}$ and currents $\{i_1(t), i_2(t)\}$, as shown in Fig. 1, e.g., $v_1(t) = \sum_n (\vec{V}_1)_n e^{s_n t} + \text{c.c.}$ with $s_n \equiv s + nj\omega_M$, etc.; the diagonal Floquet source admittance matrix $\overleftrightarrow{y} = \begin{pmatrix} \overleftrightarrow{y}_1 & 0 \\ 0 & \overleftrightarrow{y}_2 \end{pmatrix}$ consists of two diagonal admittance submatrices $\overleftrightarrow{y}_1$ and $\overleftrightarrow{y}_2$, with diagonal entries $(\overleftrightarrow{y}_1)_{n,n} = 1/R_s$ and $(\overleftrightarrow{y}_2)_{n,n} = 1/[R_r + 1/(Cs_n)]$, respectively; the diagonal matrix $\overleftrightarrow{k} = \begin{pmatrix} \overleftrightarrow{k}_1 & 0 \\ 0 & \overleftrightarrow{k}_2 \end{pmatrix}$ consists of two diagonal submatrices $\overleftrightarrow{k}_1$ and $\overleftrightarrow{k}_2$ with entries $(\overleftrightarrow{k}_1)_{n,n} = 1/\sqrt{R_s}$ and $(\overleftrightarrow{k}_2)_{n,n} = \sqrt{R_r} C s_n/(R_r C s_n + 1)$, and finally $\overleftrightarrow{k}_*(\{s_n\}) = \overleftrightarrow{k}^T(\{-s_n\})$ and $\overleftrightarrow{y}_*(\{s_n\}) = \overleftrightarrow{y}^T(\{-s_n\})$. The amplitude squared of the matrix components $\overleftrightarrow{S}_{\alpha_r n_r, \alpha_c n_c}(j\omega)$ measures the average power delivered to port $\alpha_r = 1, 2$ at frequency $\omega + n_r\omega_M$, $n_r \in \mathbb{Z}$, over the maximum available average power at the monochromatic source V_s of frequency $\omega + n_c\omega_M$, $n_c \in \mathbb{Z}$ and located at port $\alpha_c = 1, 2$. Specifically, we have [41]

$$\overleftrightarrow{S} = -\overleftrightarrow{k}\,\overleftrightarrow{k}_*^{-1} + 2\overleftrightarrow{k}(\overleftrightarrow{y} + \overleftrightarrow{Y})^{-1}\overleftrightarrow{k}, \tag{3}$$

where the Floquet admittance matrix $\overleftrightarrow{Y}$ of the time-periodic matching inductor is $\overleftrightarrow{Y} = \begin{pmatrix} \overleftrightarrow{L}^{-1} & -\overleftrightarrow{L}^{-1} \\ -\overleftrightarrow{L}^{-1} & \overleftrightarrow{L}^{-1} \end{pmatrix}$, with submatrix $\overleftrightarrow{L}$ with components $(\overleftrightarrow{L})_{n_r,n_c} = s_{n_r} L_{n_r - n_c}$, $L_n \equiv \frac{\omega_M}{2\pi}\int_0^{2\pi/\omega_M} L(t) e^{-jn\omega_M t} dt$.

We point out the structure similarity of Eq. (3) for the Floquet $\overleftrightarrow{S}$ matrix with a conventional coupled-mode formulation [22]. Hence, Eq. (3) enables an effective static-circuit description to be given for the time-dependent scattering [29] under the high modulation-frequency expansion. This effective static circuit, resembling the concept of an effective Hamiltonian for periodically driven quantum systems [25,28], describes the response of the time-modulated circuit and allows one to define the transducer gain $T_0(\omega) = |\overleftrightarrow{S}_{20,10}(j\omega)|^2$ in Eq. (1). In contrast to static circuits, $T_0(\omega)$ can go beyond bound (1) because the system is time varying and energy exchanges with the modulation network are allowed. Here, to calculate $T_0(\omega)$, we can directly work with the Floquet $\overleftrightarrow{S}$ matrix, or more conveniently, the Floquet transmission matrix $\overleftrightarrow{S}_{21}$ with input (output) at port 1 (port 2), a subblock of $\overleftrightarrow{S}$, which reads

$$\overleftrightarrow{S}_{21} = 2\overleftrightarrow{k}_2 \overleftrightarrow{y}_2^{-1}(\overleftrightarrow{L} + \overleftrightarrow{y}_2^{-1} + \overleftrightarrow{y}_1^{-1})^{-1}\overleftrightarrow{y}_1^{-1}\overleftrightarrow{k}_1. \tag{4}$$

We can fully characterize the system using four dimensionless control parameters: two are driving independent, the quality factor Q and the resistance ratio $r = R_s/R_r$, the other two are driving related, the relative modulation amplitude m and the normalized modulation frequency $\tilde{\omega}_M = \omega_M/\omega_r$. Once the quality factor Q and the operation frequency ω_r are specified, the parameters of the Chu-limited passive antenna, i.e., radiation resistance $R_r = \eta/Q^{2/3}$, capacitance $C = 1/(Q^{1/3}\omega_r\eta)$, size $a = c_0/(Q^{1/3}\omega_r)$, and static matching inductance $L_0 = \eta Q^{1/3}/\omega_r$, are determined. For example, if we choose $Q = 100$ and $\omega_r = 2\pi \times 30$ MHz in the radio-wave spectrum, we have $R_r \approx 17.5\ \Omega$, $C \approx 3$ pF, $a \approx 34$ cm, and $L_0 \approx 9.3\ \mu$H.

The temporal modulation of the matching network makes the system nonconservative at the operational frequency ω_r, and parametric gain can be used to reduce the numerator of Q (the so-called Q energy [31]), hence improving the bandwidth for a fixed antenna size. We realize parametric gain by choosing $\tilde{\omega}_M = 2$, which couples ω_r with $-\omega_r$ [42]. Around ω_r, the response of the system is well approximated by the two-mode truncation of $\overleftrightarrow{S}_{21}$ in Eq. (4), yielding

$$\overleftrightarrow{S}_{21}^{(2)} \approx 2\frac{\sqrt{r}}{Q}\begin{pmatrix} \tilde{s}_{-1} + \frac{1}{\tilde{s}_{-1}} + \frac{1+r}{Q} & \tilde{s}_{-1}\frac{1}{2}m \\ \tilde{s}_0\frac{1}{2}m & \tilde{s}_0 + \frac{1}{\tilde{s}_0} + \frac{1+r}{Q} \end{pmatrix}^{-1}, \tag{5}$$

where the normalized complex frequencies $\tilde{s}_n = s_n/\omega_r$, and the transducer gain, $T_0(\omega) \approx |\overleftrightarrow{S}_{21}^{(2)}(2,2)|^2$, with the argument indicating the entry of the matrix.

Parametric phenomena, however, are inherently frequency selective; hence, the achieved parametric gain through modulation is expectedly narrow in bandwidth, rather than enhancing it, and it commonly induces instabilities [43]. This is seen in Fig. 2(a), where we plot the transducer gain versus frequency for different values of m for the same resistance ratio, $r = 1$, which ensures maximum gain in the static scenario. Starting from $m = 0$ (red line), we increase the modulation amplitude and achieve enhanced gain beyond the limit of passive systems, but over a reduced bandwidth. Here, we chose $m = 0.02,\ 0.05$ as two relevant examples, with the inset showing the evolution of the peak of T_0 with respect to m, which can be derived analytically from Eq. (5), $\max(T_0) = 1/(1 - \frac{1}{16}m^2Q^2)^2$. The realizable gain diverges for $m = 4/Q$, indicating the boundary after which the system becomes unstable for

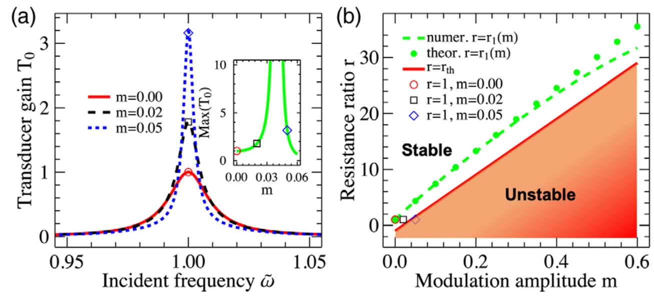

FIG. 2. (a) Transducer gain T_0 as a function of the (normalized) excitation frequency $\tilde{\omega}$ of the source for (relative) modulation amplitude $m = 0,\ 0.02,\ 0.05$. (Inset) Maximum value of T_0 versus m. (b) Stability phase diagram of the Floquet matching system in Fig. 1 in the parameter space (m, r). Numerical (green dashed line) and theoretical (green circle) evolution of $r = r_1(m)$ for $\max[T_0(\omega_r)] = 1$. Here, $Q = 100$, $\tilde{\omega}_M = 2$, and three typical scenarios for different m are marked with different symbols.

$r = 1$. A clear trade-off arises between maximum achievable gain and stability of the system.

The phase diagram reported in Fig. 2(b) and the associated stability region is a critical consideration when analyzing the Floquet matching scenario, as we need to avoid instabilities that often jeopardize the design of non-Foster-matching networks in other approaches [6]. The key advantage here is that the dispersion of parametric gain is carefully controlled through the modulation parameters. For each driving amplitude m, the instability threshold value r_{th} for the resistance ratio is determined by solving $\sigma_{\mathrm{pole}} = \mathrm{Re}(s_{\mathrm{pole}}) = 0$ for the first pole of the Floquet scattering matrix $\overleftrightarrow{S}$ in Eq. (3). Driving-affected poles can be obtained from $\det(\overleftrightarrow{y} + \overleftrightarrow{Y}) = 0$ in Eq. (3), leading to $\det(\overleftrightarrow{L} + \overleftrightarrow{y}_2^{-1} + \overleftrightarrow{y}_1^{-1}) = 0$. Under the two-mode approximation used to derive Eq. (5), we get $\tilde{\omega}_{\mathrm{th}} = \mathrm{Im}(s_{\mathrm{pole}}) = 1$ and $r_{\mathrm{th}} = \frac{1}{2}mQ - 1$. Moreover, $(\partial\sigma/\partial r)|_{\sigma=0} = -1/(2Q) < 0$, which ensures that the poles move away from the unstable half s plane $\mathrm{Re}(s) > 0$ when the resistance ratio further increases from $r = r_{\mathrm{th}}$. Therefore, $r > r_{\mathrm{th}} = \frac{1}{2}mQ - 1$ ensures stability for a given m and Q. The inset of Figs. 2(a) and 2(b) explicitly show with different markers the location of the three gain curves in Fig. 2(a) for the considered values of m.

An available degree of freedom to ensure stability of our Floquet matching network is to control the source impedance in order to purposely introduce an impedance mismatch, and hence to reduce the peak of the transducer gain. Impedance detuning acts as a form of negative feedback in the system, reducing the peak gain and proportionally enlarging the bandwidth of operation. We proceed to verify this prediction by choosing the resistance ratio $r_1 = R_s/R_r$ for each modulation amplitude m so that $\max[T_0(\omega_r)] = 1$, i.e., a peak gain equal to the maximum in the nonmodulated scenario. Figure 2(b) shows the evolution of $r_1(m)$, which consistently lies in the stable region of the phase diagram. Controlling the source resistance can be obtained with a multisection transformer [41], and it is not bandwidth limited because there is no reactance involved, consistent with Eq. (1). Using Eq. (5), we find an explicit equation for $r_1(m)$:

$$(1+r_1)(1+r_1-2\sqrt{r_1}) = \frac{1}{4}m^2Q^2, \tag{6}$$

which determines a unique real-valued function $r_1(m) > r_{\mathrm{th}}$ lying in the stable portion of Fig. 2(b). As expected, from Eq. (6), $r_1 = 1$ when $m = 0$ produces the (static) impedance matching condition. In addition, $r_1(m) \propto m$ when mQ becomes large.

By employing Eqs. (5) and (6) and considering $\tilde{\omega} \equiv \omega/\omega_r \approx 1$, we obtain the transducer gain dispersion $T_0(\omega)$ explicitly,

$$T_0(\omega) \approx \frac{1}{1+4(\tilde{\omega}-1)^2\frac{Q^2}{r_1}(1-\frac{\sqrt{r_1}}{1+r_1})^2}, \tag{7}$$

indicating that the bandwidth B_T (in units of ω_r), i.e., the FWHM, of the transducer gain $T_0(\omega)$ in this optimal scenario is $B_T = \sqrt{r_1}/\{Q[1-\sqrt{r_1}/(1+r_1)]\} \to \sqrt{m/(2Q)}$ in the large mQ limit, much larger than $B_T \propto 1/Q$ in the static scenario [44]. Interestingly, the bandwidth of our stable Floquet matching network scales with $Q^{-1/2}$, ensuring significant broadening for electrically small antennas.

In Fig. 3, we numerically illustrate the response of the active Floquet matching network and the realistic opportunity of breaking Chu's limit in electrically small antennas based on this principle. Also here $Q = 100$, corresponding to $k_r a \approx 0.215$. As m grows, the bandwidth of $T_0(\omega)$ increases considerably compared to the passive matching scenario when $m = 0$ [Fig. 3(a)]. The theoretical curve is derived from Eq. (7) with $r = r_1(m)$, given in Eq. (6).

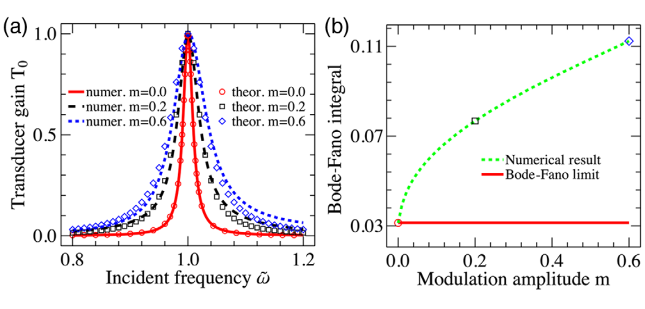

FIG. 3. Breaking Chu's limit using Floquet impedance matching with parametric gain. (a) Transducer gain T_0 versus (normalized) excitation frequency $\tilde{\omega}$ of the source when the (relative) modulation amplitude $m = 0.0$, 0.2, 0.6. Numerical and theoretical results for $r_1(m)$ are adopted from Fig. 2(b). (b) Bode-Fano integral in Eq. (1) versus m, showing operation beyond the Bode-Fano bound (the red solid line). As m increases, the resistance ratio $r = R_s/R_r$ is correspondingly tuned as $r = r_1(m)$. Other common parameters are the same as in Fig. 2.

In reality, the transmission line supporting the incoming waves from the source has a fixed impedance R_t. Therefore, as a further confirmation, we demonstrate bandwidth enhancement as seen in Fig. 3(a) by incorporating an additional multisection transformer as part of the matching network, which is used to tune the Thevenin equivalent source resistance R_s as required for our purpose [41]. To see clearly the breaking of the Chu's limit, we show a direct comparison with the Bode-Fano bound in Eq. (1); see Fig. 3(b). When $m = 0$, the passive matching is close to the Bode-Fano bound, as expected for highly reactive loads, and the system is above the Chu limit on the Q factor [1]. As the modulation amplitude m increases, we go well beyond the Bode-Fano bound, without sacrificing transducer gain or stability. Finally, we explore possible intermodulation distortions, which may be caused by the normalized transmittance $T_{-1}(\omega)$ at sideband $n = -1$ with frequency $\omega_{-1} = \omega - j\omega_M$. In this case, $T_{-1}(\omega) \approx |\overleftrightarrow{S}_{21}^{(2)}(1,2)|^2$, and we have

$$T_{-1}(\omega) \approx \left(1 - \frac{2\sqrt{r_1}}{1+r_1}\right)\frac{1}{1+4\frac{Q^2}{r_1}(\tilde{\omega}-1)^2}. \tag{8}$$

Under the positive frequency convention, this major sideband $|\omega_{-1}|$ sits symmetrically on the other side of the design frequency ω_r since $\omega_M = 2\omega_r$. Its peak value, i.e., $\max\{T_{-1}(\omega)\} = 1 - 2\sqrt{r_1}/(1+r_1)$, is smaller than the one for the transducer gain $T_0(\omega)$ in Eq. (7). In other words, the power transfer efficiency to the load at the same frequency is larger than any other harmonic.

Realistic implementation in a loop antenna configuration.—To validate our results, we explore a realistic implementation in a loop antenna geometry [Fig. 4(a)]. We simulated this geometry with CST microwave studio

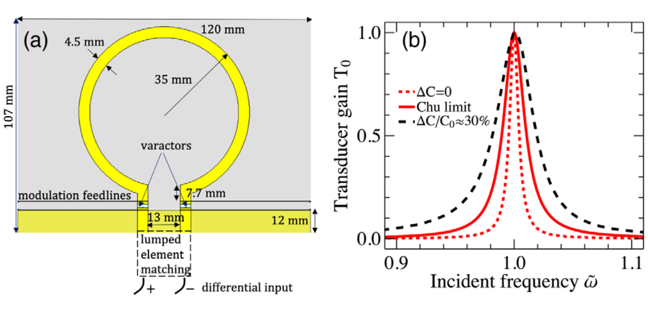

FIG. 4. Realistic simulation of Floquet matching of a loop antenna. (a) Geometry of a small loop antenna with electrical length $k_r a \approx 0.22$ at the design frequency $\omega_r/2\pi \approx 300$ MHz, matched via a variable capacitor. The quality factor $Q \approx 224$ of the passive antenna is larger than Chu's limit $Q_{\mathrm{Chu}} = 1/(k_r a)^3 \approx 94$. (b) Simulated transducer gain T_0 as a function (the black dashed line) of the normalized excitation frequency $\tilde{\omega} = \omega/\omega_r$ of the source, significantly surpassing passive matching (the red dashed line). As a reference, the result for an ideal Chu-limited antenna is also shown (red solid line).

[45]. In the Supplemental Material [41], we characterize this realistic setup and demonstrate its inherent stability via time-domain simulations. In Fig. 4(b), the dispersion of the transducer gain is compared to the one for a passive matching load, realized by a lumped series capacitance C_0 and a shunt inductance following an L-matching network. The antenna bandwidth B_T is 11.3 MHz, in contrast to 2.7 MHz for passive matching. For loop antennas characterized by series inductance and resistance, the Bode-Fano integral in Eq. (1) is modified as $\int_0^\infty \ln(1/\sqrt{1-T_0})d\tilde{\omega}$ [38], which in our result yields a value of 0.06, which is beyond $\pi/Q_{\mathrm{Chu}} \approx 0.03$ and thus confirms operation well beyond Chu's fundamental limit; see also Fig. 4(b). Our Floquet matching network is therefore able to dramatically enhance the bandwidth performance compared to the passive scenario.

Conclusion.—In this Letter, we discussed the concept of Floquet matching to broaden the gain bandwidth in electrically small open resonators. A modulated matching network providing parametric gain, combined with suitable detuning of the source impedance, ensures bandwidth broadening for the transmitted signal, combined with inherent stability, without sacrificing the transducer peak gain. Our theory shows that the performance of the small antenna (i.e., the gain-bandwidth product) increases proportionally to $1/\sqrt{Q}$. We validated our results in a loop antenna setup, including realistic parasitics. Similar concepts may be extended to time-varying nanophotonic circuit elements, with opportunities to broaden the resonant bandwidth in a variety of contexts, from sensing and nonlinearity enhancement to metamaterial functionalities [46] and quantum photonics. In this scheme, the overall efficiency of the system is still limited since a portion of the additional energy fed by the modulation network is routed back toward the source instead of being transmitted. Unwanted back reflections may be filtered out with nonreciprocal elements in the matching network, which may again be implemented using time modulation [20,47]. More generally, we envision using multiple modulated matching elements with suitably tailored phases to combine parametric gain with gauge-field-like bias, which may route the modulation network energy preferentially toward the antenna load, further enhancing the performance of Floquet matching networks.

This work was supported by the Air Force Office of Scientific Research with MURI Grant No. FA9550-18-1-0379, and the Defense Advanced Research Projects Agency.

*Corresponding author.
aalu@gc.cuny.edu

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