Emergent Solution to the Strong *CP* Problem

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We construct a theory in which the solution to the strong *CP* problem is an emergent property of the background of the dark matter in the Universe. The role of the axion degree of freedom is played by multibody collective excitations similar to spin waves in the medium of the dark matter of the Galactic halo. The dark matter is a vector particle whose low energy interactions with the standard model take the form of its spin density coupled to $G\tilde{G}$, which induces a potential on the average spin density inducing it to compensate $\bar{\theta}$, effectively removing *CP* violation in the strong sector in regions of the Universe with sufficient dark matter density. We discuss the viable parameter space, finding that light dark matter masses within a few orders of magnitude of the fuzzy limit are preferred, and discuss the associated signals with this type of solution to the strong *CP* problem.

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Introduction.—The theory of the strong interactions is well established as quantum chromodynamics (QCD), based on an $\mathrm{SU}(3)_c$ gauge symmetry with vectorlike quarks in the fundamental representation. A wealth of observational data ranging from high energies where the theory is described as weakly coupled quarks and gluons down to low energies where they are confined into color-neutral hadrons has established QCD as an integral building block of the standard model (SM).

Despite this unquestionable success, the structure of QCD contains a deep mystery: the symmetries of the theory admit a dimension four interaction term for the gluons which violates *CP*:

$$\frac{\alpha_s}{8\pi}\bar{\theta}G_a^{\mu\nu}\tilde{G}^a_{\mu\nu} \tag{1}$$

where $\bar{\theta} \equiv \theta + \mathrm{ArgDet}M_q$ is the basis-independent quantity characterizing the physical combination of the strong phase θ and a phase in the quark Yukawa interactions. Null searches for an electric dipole moment of the neutron [1] require $\bar{\theta} \lesssim 10^{-10}$, in contrast to the naïve expectation that it be order 1. While it is possible that such a tiny value is simply one of the parameters that nature has handed us, the extraordinarily minute experimental limit is suggestive that we explore physical explanations.

The most popular explanation invokes a fundamental axion field [2–7], arising as the pseudo-Nambu Goldstone boson of a spontaneous broken $\mathrm{U}(1)_{PQ}$ symmetry [8,9], resulting in a coupling of the form

$$\frac{a(x)}{f_a}G_a^{\mu\nu}\tilde{G}^a_{\mu\nu}. \tag{2}$$

At low scales, nonperturbative QCD dynamics induce a potential which is schematically of the form $-\Lambda^4\cos(a/f_a - \bar{\theta})$, inducing a vacuum expectation value for a which effectively cancels the net coefficient of the *CP*-violating term. There is a vibrant experimental program underway to search for axions in various ranges of mass [10].

In this Letter, we propose a new class of solution to the strong *CP* problem. We consider a theory in which there is no fundamental axion field, but in which the dark matter, necessary to explain cosmological observations, is composed of light vector particles which couple to the gluons in such a way that the net local spin density acts in some ways like an emergent degree of freedom which cancels $\bar{\theta}$. The axion can be understood as an emergent phenomenon, similar in character to the spin-wave excitations observed in condensed matter systems.

Dark matter.—The dark matter is assumed to be a massive vector A_μ described by the free Lagrangian,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu, \tag{3}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the usual field strength tensor, and m can be understood as either a Stückelberg mass or as arising from a dark Higgs sector. We introduce an interaction between the dark matter and the SM gluons through operators of the form,

$$\frac{\alpha_s}{16\pi}\frac{1}{M_*^{(6+2n)}}S^{\mu\nu\rho}S_{\mu\nu\rho}(A^\alpha A_\alpha)^n G^a_{\sigma\lambda}\tilde{G}^{\sigma\lambda}_a, \quad (4)$$

where

$$S^{\mu\nu\rho}[A] \equiv F^{\mu\nu}A^\rho - F^{\mu\rho}A^\nu \quad (5)$$

is the functional of A_μ representing the position-independent portion of the Noether current corresponding to rotations, and thus corresponds in the nonrelativistic limit to the net spin density carried by the A_μ field, $\vec{S}_i \sim \epsilon_{ijk}S^{0jk}$. M_* characterizes the strength of the interaction and has units of energy, and n is an integer. Such interactions could be generated, for example, by integrating out heavy $\mathrm{SU}(3)_c$-charged degrees of freedom which couple to the dark matter (see Fig. 1). In that case, one would expect the low energy theory to contain the whole family of operators for all values of n.

The interaction, Eq. (4), is not manifestly gauge invariant, and can be understood to be written in the unitary gauge. As dictated by dark gauge-invariance, the dependence of M_* on the underlying UV parameters depends on the form of the UV theory. For example, if the $\mathrm{SU}(3)_c$-charged fermions in the loop are chiral, and get their mass from the same dark Higgs vacuum expectation value v_D which breaks the dark gauge symmetry, one would expect the coefficient of the interaction to get a contribution at 1-loop of the form $\int d^4k\, k^4 m_\Psi^{(2+2n)}/(k^2-m^2)^{(6+2n)} \sim \partial^4/M_*^{6+2n}$, where $m_\Psi = y v_D$ is the mass generated by the Yukawa interaction, and the vector fields A_μ would be longitudinal modes arising from the would-be Goldstone bosons. Note that the loop integral goes to zero when v_D goes to zero, as expected from gauge invariance.

This operator allows collisions at high energy colliders to produce (multiparticle) dark matter states, and is bounded by searches for monojets recoiling against missing momentum [11,12]. While detailed analyses for this specific interaction do not exist, existing monojet searches are expected to require $M_* \gtrsim$ a few hundred GeV [13].

Effective local theta.—As we will see below, the necessary masses for the dark matter are very small, and we assume that the local dark matter in the galactic halo can be described as a coherent state characterized by its expectation values of energy and the quantity $\langle S^{0ij}S_{0ij}A^{2n}\rangle$ contained in the interaction Eq. (4). These two quantities are simultaneously measurable, as can be demonstrated by observing that the Hamiltonian density $\mathcal{H} \equiv T^{00}$ is the 00 component of the energy momentum tensor, which in the noninteracting limit takes the form $T^{\mu\nu} = F^{\mu\alpha}F^\nu_\alpha + \frac{1}{4}\eta^{\mu\nu}F^{\rho\sigma}F_{\rho\sigma} + m^2(A^\mu A^\nu - \frac{1}{2}\eta^{\mu\nu}A^\rho A_\rho)$, and satisfies $[S^{0ij}, \mathcal{H}] = 0$. In the nonrelativistic limit, $\mathcal{H}$ reduces to m^2A^2, such that $S^{0ij}S_{0ij}\mathcal{H}^n/m^{2n} \to S^{0ij}S_{0ij}A^{2n}$.

The dynamics of the dark matter in a region of space close to the solar location is described by a partition function with the UV dynamics of QCD encoded (schematically) by a short distance potential and the long distance influence of the gravitational dynamics of the galaxy represented by an external potential:

$$-\Lambda^4\cos\left(\frac{S^{\mu\nu\rho}S_{\mu\nu\rho}(A^2)^n}{M_*^{(6+2n)}} - \bar{\theta}\right) - \mu T^{00}, \quad (6)$$

with μ adjusted such that it enforces the local energy density consistent with the Galactic gravitational dynamics,

$$\langle T^{00}\rangle = \rho_\odot \sim 0.3\ \mathrm{GeV/cm^3} \sim 3\times 10^{-7}\ \mathrm{eV}^4. \quad (7)$$

In a particular region of space, the contribution from the dark matter to the effective θ term is bounded by the maximum spin density consistent with the local number density of the dark matter. In terms of the amplitude of the coherent state $\mathcal{A}$, the derivatives scale as $\langle\partial_0 A\rangle \sim m\mathcal{A}$, $\langle\partial_i A\rangle \sim mv\mathcal{A}$ (where $v \sim 10^{-3}$ is the typical velocity dispersion), and $\langle S^{0ij}\rangle \sim sm\mathcal{A}^2$, where $0 \le s \le 1$ characterizes the degree to which the field is polarized. In this language, the long distance contribution to the effective potential determines $\mathcal{A}$, and the QCD contribution acts to prefer a local value of s which minimizes the effective θ term in that region of space.

The dark matter contribution to the effective θ is parametrically,

$$\frac{s^2m^2\mathcal{A}^{(4+2n)}}{M_*^{(6+2n)}} \sim s^2\frac{\rho^{(2+n)}}{M_*^{(6+2n)}m^{(2+2n)}}. \quad (8)$$

In order to cancel a $\bar{\theta}$ of order one near the Sun, the mass of the dark matter must satisfy,

$$m \lesssim \left(\frac{\rho_\odot^{(2+n)}}{M_*^{(6+2n)}}\right)^{[1/(2+2n)]}. \quad (9)$$

The maximum m as a function of the operator dimension n is plotted for $M_* = 1$ TeV in Fig. 2. For $n \ge 3$, masses large enough to be consistent with the bound on the fuzziness of dark matter on small scales [14–17] are

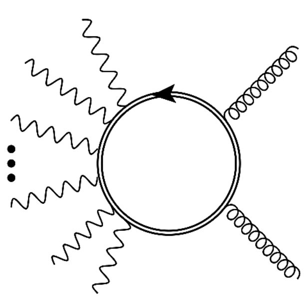

FIG. 1. Representative Feynman diagram indicating how integrating out $\mathrm{SU}(3)_c$-charged fermions can generate an interaction between the dark matter and gluons.

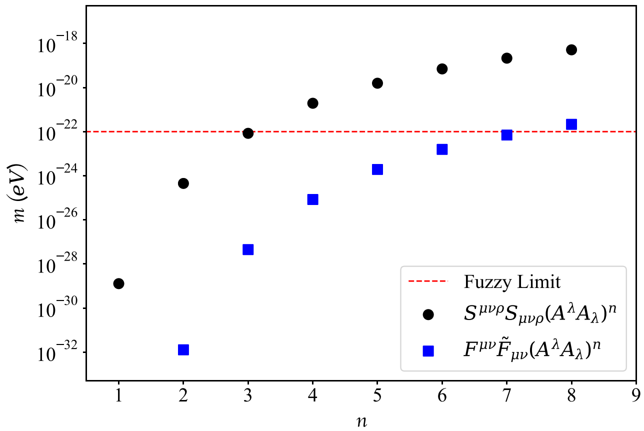

FIG. 2. Maximum dark matter mass consistent with solving the strong *CP* problem near Earth, as a function of the operator dimension n (black circles). The red dashed line indicates the bound on the dark matter mass from a small scale structure [14–16]. The blue squares indicate the maximum masses from the alternative interaction, Eq. (21).

consistent with the emergent solution to the strong *CP* problem.

While operators containing larger values of n are necessary to consistently cancel $\bar{\theta}$ near Earth, it is clear that the (unavoidable) presence of operators with lower n are not problematic. Given the local density of dark matter, the lower n operators make a negligible contribution to the local effective θ term. Operators with higher n occur at the same order in the loop expansion, though they are suppressed by additional powers of M_*.

Additional contributions to $\theta_{\rm eff}$: Our analysis so far has assumed that the QCD potential represents the only important dynamics influencing the dark matter spin density. It is crucial that any other contributions be sufficiently subdominant that they deflect s from the minimum of Eq. (6) such that the effective θ term remains $\lesssim 10^{-10}$.

The same dynamics which gives rise to the operator connecting the dark matter to $G\tilde{G}$ will also lead to operators containing dependence on s which is unaligned with $\bar{\theta}$. These operators take the form

$$\frac{a_p}{16\pi^2}\frac{1}{M_*^{(8+2p)}}(S^{\mu\nu\rho}S_{\mu\nu\rho})^2(A_\lambda A^\lambda)^p, \tag{10}$$

where p is an integer which characterizes the operator making the dominant contribution, and a_p is a dimensionless coefficient which could be computed given a more concretely realized UV theory. This operator will shift s from the minimum cancelling $\bar{\theta}$, inducing an effective θ term of order:

$$\delta\theta \sim \frac{\rho_\odot^2}{\Lambda^4 m^2 M_*^2}\times\left(\frac{\rho_\odot}{m^2 M_*^2}\right)^{p-n}. \tag{11}$$

For $m \sim 10^{-18}$ eV and $M_* \sim 1$ TeV, the effective local θ term is acceptably small provided $p \lesssim n+5$.

The local environment may also impose a preference on the net dark matter spin density. For example, the dark matter may possess a magnetic dipole moment, described by, e.g.,

$$\frac{e\lambda_m}{16\pi^2 M_*^4}F_{\rm EM}^{\mu\nu}\partial^2(F_{\nu\rho})F_\mu^\rho, \tag{12}$$

where $F_{\rm EM}$ is the electromagnetic field strength, e is the electric coupling, and λ_m is a dimensionless quantity. If the mediator fermions carry electroweak charge, one would expect the magnetic dipole is induced at one loop, and $\lambda_m \sim 1$, whereas if not it will nonetheless be induced at three loops, $\lambda_m \sim [\alpha_S(M_*)/4\pi]^2$. At the surface of Earth, this induces a shift in the effective theta term of order,

$$\delta\theta \sim \frac{e\lambda_m}{32\pi^2}\frac{B_\oplus m^{(3+n)}M_*^{(-1+n)}}{\Lambda^4\rho_\odot^{n/2}}, \tag{13}$$

where $B_\oplus \sim 3\times 10^{-3}$ eV^2 is the strength of Earth's magnetic field at its surface. Even for $\lambda_m \sim 1$, this is far too small to be important for the masses of interest.

If the dark matter interacts directly with electrons with coupling g_D (e.g., through a small amount of kinetic mixing with the hypercharge interaction), it will typically induce a magnetic moment that is larger by $\lambda_m \sim g_D^2 M_*^2/m_e^2$, where m_e is the mass of the electron. Even for order one coupling strengths $g_D \sim 1$, this is small enough as to not significantly destabilize the local effective value of θ.

Even in the absence of a magnetic moment, there is a gravitational interaction between the dark matter spin and the spin of Earth. These corrections are encapsulated by the potential on the net dark matter spin density induced by the Earth's gravitational field, described as a background Kerr metric characterized by its Schwarzschild radius $r_s = 2GM_\oplus \sim 10^5$ eV^{-1} and angular momentum per unit mass $\vec{a} = \vec{J}_\oplus/M_\oplus$; $|\vec{a}| \sim 10^5$ eV^{-1}. To linear order in r_s and $\vec{a}$, the term in the effective Lagrangian at a position $\vec{r}$ from the center of the Earth reads,

$$\frac{r_s m}{2r^3}(\vec{r}\times\vec{a})\cdot\{(\vec{A}\times\vec{\partial})\times\vec{A}\} + \frac{r_s m^2}{r^3}A_0\vec{a}\cdot(\vec{r}\times\vec{A}). \tag{14}$$

The correction to the local value of the effective θ is,

$$\frac{r_s|\vec{a}|v}{R_\oplus^2}\frac{M_*^{3+n}m^{1+n}}{\Lambda^4\rho_\odot^{n/2}}, \tag{15}$$

where $R_{\oplus}$ is the radius of Earth. For the parameters of interest, this is negligibly small.

Phenomenology.—Cosmological production: As with any ultralight boson playing the role of dark matter, it is necessary to invoke a nonthermal production mechanism which results in a nonrelativistic momentum distribution. For the low masses of interest here, production through inflationary fluctuations is thought to be inefficient given the current upper bound on the inflationary scale [18–20]. Production through a generic tachyonic instability is possible, though it requires some fine-tuning [21–23]. Masses as low as $\sim 10^{-18}$ eV can be accommodated if the vector mass results from a dark Higgs whose mass is close to the dark matter mass [24].

Structure of galaxies: For masses close to the fuzzy limit, small scale structures are prevented from forming, and the cusps of large galaxies are typically smoothed into cores [25,26]. For masses on the larger end of the range we consider, these effects are unlikely to be observable.

A potentially important feature stems from the fact that dense areas of dark matter have a smaller effective θ, and thus a lower vacuum energy. If one treats the background of dark energy as a cosmological constant, and tunes its value such that in regions with very little dark matter, the net vacuum energy reproduces the observed acceleration of the cosmological expansion, this implies that regions containing overdensities of dark matter experience a net negative contribution to their vacuum energy from QCD. This feature could lead to interesting modifications to the usual cosmology and history of structure formation (e.g., [27]). However, at face value, this picture implies a dramatic modification to the dynamics of galaxies, and may pose a serious challenge unless there is some mechanism which operates locally to cancel contributions to dark energy (perhaps as a solution to the cosmological constant problem).

A less dramatic solution would be to invoke $n \gtrsim 6$ and dark matter masses closer to the fuzzy limit, for which the cosmological density of dark matter is sufficient to solve the strong CP problem across the entire Universe. In that case, one adjusts the cosmological constant such that it leads to the observed cosmological acceleration, without any particular impact on galactic dynamics.

Signals at gravitational wave detectors: The mechanism by which the vector dark matter environmentally solves the strong CP problem is somewhat agnostic as to its interactions with the standard model fermions. There could be a small direct coupling, or one could be induced through kinetic mixing with the ordinary photon. In that case, the motion of Earth through the dark matter halo induces an additional time-dependent contribution to the force between objects at a tiny level which is nonetheless accessible to interferometers designed to detect gravitational waves [28]. In the mass range of interest, the current best constraints from the Eöt-Wash experiment [29,30] require the coupling to ordinary matter be less than about $e \times 10^{-23}$, depending on the details of which SM fermions interact with the light boson, and the LISA experiment is expected to eventually improve on these limits for masses $\gtrsim 10^{-18}$ eV [28].

Distant CP violation: Any environmental solution to the strong CP problem based on the background of dark matter can have an important consequence: regions without dark matter may be unable to completely cancel the effective θ, and thus have different microscopic physics compared with the solar system, characterized by the protons and neutrons in those regions of space possessing large electric dipole moments whose magnitude corresponds to the local value of $\theta_{\rm eff}$ and can be estimated from chiral perturbation theory [31,32],

$$d_p \simeq \frac{e g_A c_+ \tilde{m} \theta_{\rm eff}}{8\pi^2 f_\pi^2} \log\left(\frac{\Lambda^2}{m_\pi^2}\right), \qquad (16)$$

where the axial coupling $g_A \sim 1.27$ and $c_+ \sim 1.7$ are terms in the chiral Lagrangian, and $\tilde{m} \equiv m_u m_d/(m_u + m_d) \sim 1.2$ MeV is the reduced quark mass. In regions with $\theta_{\rm eff}$ of order one, d_p is of order 10^{-16} $e\,$cm. This large CP violation is unlikely to lead to large changes in stellar dynamics and evolution [33], but could potentially lead to observable deviations in the atomic physics of stars in regions with lower dark matter density, such as in the outskirts of the Milky Way, or in nearby globular clusters.

Since the bulk composition of stars is hydrogen, we examine the impact of a proton electric dipole moment on its atomic transitions. Treating the electric dipole as a perturbation, the first order correction to the nlm electronic wave function of a hydrogen atom, $|\delta\Psi_{nlm}\rangle$, is given by,

$$|\delta\Psi_{nlm}\rangle = \sum_{(n'l'm')} \frac{\langle \Psi_{n'l'm'}|\hat{H}'|\Psi_{nlm}\rangle}{E_{nlm} - E_{n'l'm'}} |\Psi_{n'l'm'}\rangle, \qquad (17)$$

where $\hat{H}'$ is the additional electric dipole field induced by the proton at the origin, and E_{nlm} and $|\Psi_{nlm}\rangle$ are the unperturbed energy level and unperturbed state vector of the nlm state.

The dipole interaction induces mixing between the unperturbed $l = 0$ and $l = 1$ states, which allows for $E1$ single photon $2s \to 1s$ transitions through the correction to $|\delta\Psi_{200}\rangle$ proportional to $|\Psi_{n'10}\rangle$:

$$\begin{aligned} \langle \Psi_{n'10}|\delta\Psi_{200}\rangle = {} & \frac{d_p e}{4\pi\sqrt{3}\epsilon_0} \frac{C_{n'1} C_{20}}{E_{n'10} - E_{200}} \\ & \times \int_0^\infty dr e^{-(r/a_0)[(1/n') + \frac{1}{2}]} \\ & \times \frac{2r}{n' a_0} L^3_{n'-2}\left(\frac{2r}{n' a_0}\right) L^1_1\left(\frac{r}{a_0}\right), \end{aligned} \qquad (18)$$

where C_{nl} are the hydrogen wave function normalization coefficients, a_0 is the Bohr radius, $L_n^l(x)$ are the associated Laguerre polynomials, and the z axis has been chosen along the direction of the electric dipole.

The rate for $E1$ emission of a single photon via the transition from the $2s$ to the $1s$ state is [34],

$$\Gamma(2s \to 1s + \gamma) = \frac{e^2\omega^3}{3\pi}|\langle\Psi_{100}|\hat{r}|\delta\Psi_{200}\rangle|^2 \tag{19}$$

$$\simeq 10^{-24}\ \text{eV} \times \theta_{\text{eff}}^2, \tag{20}$$

where $\hat{r}$ is the position operator and $\omega \equiv E_{200} - E_{100}$. In regions where θ_{eff} is of order unity, this represents an enhancement of the rate for this transition by a factor of about 10^4 compared with the CP-conserving $M1$ transition [35]. In principle, a powerful telescope collecting spectroscopic information could potentially discern this transition line and infer its rate. Resolving this transition from the nearby CP-conserving $2p \to 1s$ line would require a wavelength resolution of order $\delta\lambda/\lambda \sim 10^6$, which is about an order of magnitude beyond the current capabilities of an instrument such as the Keck telescope [36].

Conclusions and outlook.—We have explored a novel solution to the strong CP problem based on the dark matter environment. The dark matter is an ultralight light vector particle with mass $\lesssim 10^{-18}$ eV, whose spin density is coupled to the gluon field in such a way as to allow it to cancel an order one $\bar{\theta}$ at the position of Earth. Regions with sufficiently small densities of dark matter cannot locally cancel an order one $\bar{\theta}$, perhaps leading to areas of the Universe in which CP is not locally conserved, and potentially a novel history for structure formation.

We have explored a particular operator, Eq. (4), in which the dark matter spin is coupled to the gluon $G\tilde{G}$. There are a wider array of possible operators, as any operator involving the dark matter spin (and enhanced by its number density) could potentially work. For example, the operator,

$$\frac{\alpha_s}{16\pi}\frac{1}{M_*^{(4+2n)}}F^{\mu\nu}\tilde{F}_{\mu\nu}(A^\lambda A_\lambda)^n G^a_{\sigma\lambda}\tilde{G}^{\sigma\lambda}_a, \tag{21}$$

is less suppressed by the interaction scale M_*, though additionally suppressed from the spatial derivatives of the dark matter field. From Fig. 2, we see that slightly lower masses for the dark matter, though nonetheless consistent with the fuzzy limits for $n \gtrsim 7$, are required to cancel an order one $\bar{\theta}$ at the position of Earth. This operator has the additional complication that $F\tilde{F}A^{2n}$ does not commute with the Hamiltonian, implying an intrinsically quantum mechanical dynamic for the evolution of the Galaxy. We leave more detailed thought concerning this interesting possibility for future work.

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