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Constraining the Mass of the Graviton with the Planetary Ephemeris INPOP

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We use the planetary ephemeris INPOP17b to constrain the existence of a Yukawa suppression to the Newtonian potential, generically associated with the graviton's mass. We also give an interpretation of this result for a specific case of fifth force framework. We find that the residuals for the Cassini spacecraft significantly (90% C.L.) degrade for Compton wavelengths of the graviton smaller than 1.83×10^{13} km, which correspond to a graviton mass bigger than 6.76×10^{-23} eV$/c^2$. This limit is comparable in magnitude to the one obtained by the LIGO-Virgo Collaboration in the radiative regime. We also use this specific example to defend that constraints on alternative theories of gravity obtained from postfit residuals may be generically overestimated.

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Introduction.—From a particle physics point of view, general relativity can be thought of as a theory of a massless spin-2 particle—hereafter named a *graviton*. From this perspective, it is legitimate to investigate whether or not the graviton could actually possess a mass—even if it is minute. Such an eventuality has been scrutinized from a theoretical point of view since the late 1930s, with the pioneering work of Fierz and Pauli [1]. There is a wide set of massive gravity theories that lead to various phenomenologies [2]. One of the generic predictions from several models—although not all of them (in particular, usually not for models prone to the Vainshtein mechanism [2])—is that the usual $1/r$ falloff of the Newtonian potential acquires a Yukawa suppression [2]. In the present Letter, we aim to test this particular phenomenology, regardless of the specificity of the theoretical model that produced it. For more information on the status of current theoretical models, we refer the reader to [2,3]. As a consequence, we assume that the line element in space-time curved by a spherical massive object at rest, at leading order in the Newtonian regime, reads

$$ds^2 = \left(-1 + \frac{2GM}{c^2 R} e^{-R/\lambda_g}\right) c^2 dT^2 + \left(1 + \frac{2GM}{c^2 R} e^{-R/\lambda_g}\right) dL^2, \quad (1)$$

with $dL^2 \equiv dX^2 + dY^2 + dZ^2$, $R \equiv \sqrt{X^2 + Y^2 + Z^2}$, and λ_g is the Compton wavelength of the graviton—although we will see that our constraints can be applied on a wider range of massive and nonmassive gravity metrics. Obviously, as long as λ_g is big enough, the gravitational phenomenology in the Newtonian regime can be reduced to one of general relativity in any given level of accuracy.

Another generic feature of many massive gravity theories is that, if the graviton is massive, its dispersion relation may be modified according to $E^2 = p^2c^2 + m_g^2 c^4$ [2], such that the speed of gravitational waves depends on its energy (or frequency) $v_g^2/c^2 = c^2p^2/E^2 \simeq 1 - h^2c^2/(\lambda_g^2 E^2)$. Therefore, the waveform of gravitational waves would be modified during their propagation, while, at the same time, sources of gravitational waves have been seen up to more than 1420 Mpc (at the 90% C.L.) [4]. As a consequence, waveform match filtering can be used to constrain the graviton mass from gravitational waves detections [5,6].

Combining bounds from several events in the catalog GWTC-1 [4] leads to $\lambda_g \geq 2.6 \times 10^{13}$ km [respectively, $m_g \leq 5.0 \times 10^{-23}$ eV$/c^2$ [4,7]—with the definition $m_g = h/(c\lambda_g)$] at the 90% C.L. (assuming that the graviton mass affects the propagation only, and not the binaries dynamics). It is important to keep in mind that this limit is obtained in the radiative regime, while we focus here on the Newtonian regime. Although, one could expect λ_g to have the same value in both regimes for most massive gravity theories, it may not be true for all massive gravity theories. Therefore, both constraints should be considered independently from an agnostic point of view. See, e.g., [2] for a review on the graviton mass constraints.

Direct constraints from the Solar System.—Twenty years ago [5], and more recently [8], Will argued that Solar System observations could be used to improve—or at least be comparable with—the constraints on λ_g obtained from

TABLE I. Examples of correlations between various INPOP17b parameters and the Compton wavelength λ_g. a, EMB and $M_\odot$ state for semimajor axes, the Earth-Moon barycenter and the mass of the Sun, respectively.

	λ_g	a Mercury	a Mars	a Saturn	a Venus	a EMB	$GM_\odot$
λ_g	1	0.50	0.49	0.04	0.39	0.05	0.66
a Mercury	⋯	1	0.21	0.001	0.97	0.82	0.96
a Mars	⋯	⋯	1	0.03	0.29	0.53	0.06
a Saturn	⋯	⋯	⋯	1	0.003	0.02	0.01
a Venus	⋯	⋯	⋯	⋯	1	0.86	0.94
a EMB	⋯	⋯	⋯	⋯	⋯	1	0.73
$GM_\odot$	⋯	⋯	⋯	⋯	⋯	⋯	1

the LIGO-Virgo Collaboration—assuming that the parameters λ_g appearing in both the radiative and Newtonian limits are the same. A graviton mass would indeed lead to a modification of the perihelion advance of Solar System bodies. Hence, based on current constraints on the perihelion advance of Mars—or on the post-Newtonian parameters γ and β—derived from Mars Reconnaissance Orbiter (MRO) data, Will estimates that the graviton's Compton wavelength should be bigger than $(1.2 - 2.2) \times 10^{14}$ km (respectively, $m_g < (5.6 - 10) \times 10^{-24}$ eV$/c^2$), depending on the specific analysis. However, as an input for his analysis, Will uses results based on the statistics of residuals of the Solar System ephemerides that are performed without including the effect of a massive graviton. But various parameters of the ephemeris (e.g., masses, semimajor axes, Compton parameter, etc.) are all more or less correlated to λ_g (see Table I). Therefore, any signal introduced by $\lambda_g < +\infty$—for instance, a modification of a perihelion advance—can in part be reabsorbed during the fit of other parameters that are correlated with the mass of the graviton. (See Supplemental Material [9], which includes Refs. [10–15]). This, we believe, necessarily leads to a decrease of the constraining power of the ephemeris on the graviton mass with respect to postfit estimates. As a corollary, we believe that all analyses based solely on postfit residuals tend to overestimate the constraints on alternative theories of gravity due to the lack of information on the correlations between the various parameters. Eventually, we think that one cannot produce conservative estimates of any parameter without going through the whole procedure of integrating the equations of motion and fitting the parameters with respect to actual observations—which is the very *raison d'être* of the ephemeris INPOP.

INPOP (Intégrateur Numérique Planétaire de l'Observatoire de Paris) [16] is a planetary ephemeris that is built by integrating numerically the equations of motion of the Solar System following the formulation of [17], and by adjusting to Solar System observations such as lunar laser ranging or space missions observations. In addition to adjusting the astronomical intrinsic parameters, it can be used to adjust parameters that encode deviations from general relativity [18–21], such as λ_g. The latest released version of INPOP, INPOP17a [22], benefits of an improved modeling of the Earth-Moon system, as well as an update of the observational sample used for the fit [21]—especially including the latest Mars orbiter data. For this work we use an extension of INPOP17a, called INPOP17b, fitted over an extended sample of Messenger data up to the end of the mission, provided by [23].

In the present Letter, our goal is to use the latest planetary ephemeris INPOP17b in order to constrain a hypothetical graviton mass directly at the level of the numerical integration of the equations of motion and the resulting adjusting procedure. By doing so, the various correlations between the parameters are intrinsically taken into account, such that we can deliver a conservative constraint on the graviton mass from Solar System observations—details about the global adjusting procedure are given in Supplemental Material [9].

Modelization for Solar System phenomenology.—Following Will [8], we develop perturbatively the potential in terms of r/λ_g, such that the line element (1) now reads

$$ds^2 = \left(-1 + \frac{2GM}{c^2 r}\left[1 + \frac{1}{2}\frac{r^2}{\lambda_g^2}\right]\right)c^2 dt^2 + \left(1 + \frac{2GM}{c^2 r}\left[1 + \frac{1}{2}\frac{r^2}{\lambda_g^2}\right]\right)dl^2 + \mathcal{O}(c^{-3}\lambda_g^{-2}), \qquad (2)$$

albeit with a change of coordinate system (we assume that the underlying theory of gravity is covariant, such that this change of coordinates has no impact on the derivation of the actual *observables*):

$$T = \frac{t}{\sqrt{1 + \frac{GM}{c^2 \lambda_g}}}, \qquad X^i = \frac{x^i}{\sqrt{1 - \frac{GM}{c^2 \lambda_g}}}. \qquad (3)$$

The change of coordinate system is meant to get rid of the nonobservable constant terms that appear in the line element of Eq. (1) after expanding in terms of λ_g^{-1}. Considering a N-body system, the resulting additional acceleration to incorporate in INPOP's code is

$$\delta a^i = \frac{1}{2}\sum_P \frac{GM_P}{\lambda_g^2}\frac{x^i - x_P^i}{r} + \mathcal{O}(\lambda_g^{-3}), \qquad (4)$$

where M_P and x_P^i are, respectively, the mass and the position of the gravitational source P. In what follows, we make the standard assumption that the underlying theory is such that light propagates along null geodesics [3]. From the null condition $ds^2 = 0$ and Eq. (2), the resulting additional Shapiro delay at the perturbative level reads

$$\delta T_{\rm ER} = \frac{1}{2}\sum_P \frac{GM_P}{c^3\lambda_g^2}\left[\vec{N}_{\rm ER}(\vec{R}_{\rm PR}R_{\rm PR} - \vec{R}_{\rm PE}R_{\rm PE}) + b_P^2 \ln\left(\frac{R_{\rm PR} + \vec{R}_{\rm PR}\vec{N}_{\rm ER}}{R_{\rm PE} + \vec{R}_{\rm PE}\vec{N}_{\rm ER}}\right)\right] + \mathcal{O}(c^{-3}\lambda_g^{-3}), \quad (5)$$

where $\vec{R}_{XY} = \vec{x}_Y - \vec{x}_X$, $R_{XY} = |\vec{R}_{XY}|$, $\vec{N}_{XY} = \vec{R}_{XY}/R_{XY}$, and $b_P = \sqrt{R_{\rm PE}^2 - (\vec{R}_{\rm PE}\vec{N}_{\rm ER})^2}$. One can notice the correction to the Shapiro delay scales as $(L_c/\lambda_g)^2$ with respect to the usual delay, where L_c is a characteristic distance of a given geometrical configuration. Given the old acknowledged constraint from Solar System observations on the graviton mass ($\lambda_g > 2.8 \times 10^{12}$ km [5,8,24]), one deduces that the correction from the Yukawa potential on the Shapiro delay is negligible for past, current, and forthcoming radioscience observations in the Solar System (however, note that the scaling of the correction to the Shapiro delay illustrates the breakdown of the λ_g^{-1} development in cases where the characteristic distances involved are large with respect to the Compton wavelength—as should be expected).

On the other hand, the fifth force formalism predicts an additional Yukawa term to the Newtonian potential [25]

$$V = \frac{Gm}{r}(1 + \alpha e^{-r/\lambda}) \quad (6)$$

If we assume that $\lambda \gg r$ and $\alpha > 0$, we can also expand the Yukawa term, such that our result on λ_g can be transposed to $\lambda/\sqrt{\alpha}$—although, one first has to rescale the gravitational constant to $\tilde{G} = G(1+\alpha)$, and then to make the same coordinate change as in Eq. (3), but while substituting λ_g with λ/α. Note that a fifth force is also one of the generic features of several massive gravity theories [2].

Evaluation of the significance of the residuals deterioration.—To give a confidence interval for λ_g, we proceed as follows. For each value of λ_g, we perform a global fit of all other parameters to observations using the same data that for the reference solution INPOP17b—therefore, for the same number of observations. After the global fit procedure, we compute the residuals at the same dates for the reference solutions and observe how they are degraded or improved with respect to λ_g. The result is that Cassini residuals are the first to degrade significantly while λ_g decreases (see Supplemental Material [9] for details).

To quantify the statistical meaning of this degradation, we perform a Pearson [26] χ^2 test between both residuals in order to look at the probability that they were both built from the same distribution. To compute the χ^2, we build an optimal histogram with the Cassini residuals of INPOP17b using the method described in [27], assuming the Gaussianity of the distribution of the residuals. We determine the optimal bins in which are counted the residuals to build the histogram. Then, using the same bins, we build an histogram for the Cassini residuals obtained by the solution

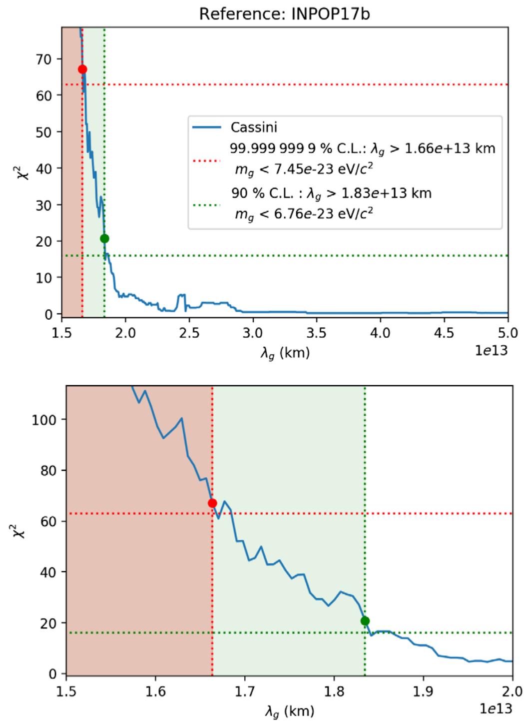

FIG. 1. Plot of $\chi^2(\lambda_g)$ and the constraints deduced for λ_g. The probabilities $p = 90\%$ and $p = 99.9999999\%$ correspond to critical values of χ^2 equal to, respectively, 15.99 and 62.94.

to be tested with a given value of λ_g. Note that the first bin left bound is $-\infty$ and the last bin right bound is $+\infty$. Let $(C_i)_i$ be the bins in which are counted the values of the residuals, and let N_i^I, N_i^G be the number of residuals of INPOP17b and the solution to be tested, respectively, counted in bin number i. One can then compute

$$\chi^2(\lambda_g) = \sum_{i=1}^{n} \frac{(N_i^G - N_i^I)^2}{N_i^I}. \quad (7)$$

For Cassini data, it occurs that the optimal binning gives 10 bins. As a result, this χ^2 follows a χ^2 law with 10 degrees of freedom. If the computed χ^2 is then greater than its quantile for a given confidence probability p, we can say that the distribution of the residuals obtained for λ_g is different from the residuals obtained by the reference solution with a probability p. This test can be done for both a positive detection of a physical effect and a rejection of the existence of a physical effect. If the computed $\chi^2(\lambda_g)$ becomes then greater than its critical value for a probability p, one has to check if residuals are smaller or bigger than those obtained by the reference solution. In the first case (smaller—or better—residuals), it means that the added effect increases significantly the quality of the residuals and is probably (with a probability p) a true physical effect. On the contrary, in the second case (bigger—or degraded—residuals), it means that the added effect is probably physically false. In our work, the critical increasing of $\chi^2(\lambda_g)$ corresponds to a degradation of the residuals (see the Supplemental Material [9] for a detailed analysis). The massive graviton can then be rejected for high enough values of the mass (or low enough values of λ_g).

Results.—In Fig. 1 we plot the χ^2 as a function of λ_g. In this plot, we give two values of quantiles associated to two probabilities of significance, $p = 90\%$ and $p = 99.9999999\%$, which correspond to critical values of χ^2 equal to 15.99 and 62.94, respectively, for a 10 degrees of freedom χ^2 distribution. We obtain respectively $\lambda_g > 1.83 \times 10^{13}$ km (respectively, $m_g < 6.76 \times 10^{-23}$ eV$/c^2$) and $\lambda_g > 1.66 \times 10^{13}$ km (respectively, $m_g < 7.45 \times 10^{-23}$ eV$/c^2$). These results are shown in Fig. 1. We also provide an enlargement of the main figure in order to show that the χ^2 is not monotonic for small differences of λ_g. However, if a given limit is crossed several times, our algorithm automatically takes the most conservative value in the discrete set of λ_g, as can be seen in Fig. 1.

Conclusion.—In the present Letter, we deliver the first conservative estimate of the graviton mass from an actual fit of a combination of Solar System data using a criterion based on a state of the art Solar System ephemerides: INPOP17b. The bound reads $\lambda_g > 1.83 \times 10^{13}$ km (respectively $m_g < 6.76 \times 10^{-23}$ eV$/c^2$) with a confidence of 90% and $\lambda_g > 1.66 \times 10^{13}$ km (respectively, $m_g < 7.45 \times 10^{-23}$ eV$/c^2$) with a confidence of 99.999 999 9%. As previously explained, in terms of a fifth force, the constraint on λ_g can be translated into a constraint on $\lambda/\sqrt{\alpha}$, simply by substituting λ_g by $\lambda/\sqrt{\alpha}$, if $\alpha > 0$.

The fact that our 90% C.L. bound is comparable in magnitude to the one obtained by the LIGO-Virgo Collaboration in the radiative regime [7,28] is a pure coincidence: the two bounds rely on totally different types of observation—gravitational waves versus radioscience in the Solar System—and probe different aspects of the massive graviton phenomenology—radiative versus orbital.

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