

## Rotating Binary Bose-Einstein Condensates and Vortex Clusters in Quantum Droplets

M. Nilsson Tengstrand,<sup>1</sup> P. Stürmer,<sup>1</sup> E. Ö. Karabulut,<sup>2</sup> and S. M. Reimann<sup>1</sup>

<sup>1</sup>*Mathematical Physics and NanoLund, LTH, Lund University, P.O. Box 118, 22100 Lund, Sweden*

<sup>2</sup>*Department of Physics, Faculty of Science, Selcuk University, TR-42075 Konya, Turkey*



(Received 7 June 2019; published 18 October 2019)

Quantum droplets may form out of a gaseous Bose-Einstein condensate, stabilized by quantum fluctuations beyond mean field. Determining the ground state of a rotating binary condensate, we show that multiple singly quantized vortices may form in these droplets at moderate angular momenta in two dimensions. Droplets carrying these precursors of an Abrikosov lattice remain self-bound for certain timescales after switching off an initial harmonic confinement. Intriguingly, we find evidence of a metastable persistent current in these new types of binary condensates. We discuss how this finding can be used to experimentally generate vortex-carrying quantum droplets.

DOI: [10.1103/PhysRevLett.123.160405](https://doi.org/10.1103/PhysRevLett.123.160405)

The formation of self-bound droplets is a well-known macroscopic phenomenon. For an exemplary droplet of water, stability and shape rely on the balance of effective forces between its constituent particles—attractive ones that keep it together, and repulsive ones that prevent it from collapse. Their interplay defines the droplets’ surface tension, stabilizing the system in a metastable state. Such droplets do not only occur at a macroscopic level, but are ubiquitous also in the quantum realm, where nuclei [1] and superfluid helium droplets [2–4] are prominent examples. While these are rather dense and strongly interacting many-body systems, recent experiments with ultracold quantum gases of bosonic atoms uncovered a novel type of quantum liquid: Self-bound droplets may form out of a gaseous Bose-Einstein condensate (BEC) of dysprosium [5–9] or erbium [10], atomic species that are known for their strong dipolar interactions [11–13]. Similar droplet states have more recently also been realized with binary Bose gases of potassium in different hyperfine states [14,15], where the inter- and intracomponent interactions are short ranged. These quantum droplets can be large, containing thousands of atoms. Importantly, they are very dilute—by more than 8 orders of magnitude when compared with liquid helium [14]. While the discovery with dysprosium [5–7] at first came as a surprise, the binary self-bound droplet states were theoretically predicted a year before [16] for a scenario similar to the experiments with potassium, and also in lower dimensions [17]. That higher-order corrections beyond mean field may lead to self-bound states was discussed earlier in a different setting in Refs. [18,19]. For the dipolar or binary self-bound bosonic systems of Refs. [5–7,14,15] the physical mechanism of droplet formation is based on tuning the interactions in the gas such that only a weak effective attraction remains. While in pure mean field this would lead to a collapse of the system, weak first-order corrections to the mean field

energy, often referred to as the Lee-Huang-Yang (LHY) correction [20], can become comparable in size and may thus stabilize the system.

Bound states that are merely a consequence of quantum corrections beyond mean field have been known since long from alkali-metallic clusters [21,22], where the jellium of the ionic charge background cancels the electronic Hartree term, resulting in a liquidlike electronic state bound mainly due to a balance between kinetic and exchange-correlation energy [23]. The fact that self-bound bosonic droplets can form with atoms as intrinsically different as lanthanides and alkali metals shows that this phenomenon for ultra-cold gases is a general one, giving evidence for a novel state of quantum matter with new and unexpected properties the exploration of which is yet “at its infancy” (see the recent focus article [24]).

In atomic quantum droplets the effective interactions between the constituent bosonic atoms are relatively weak. This eases their theoretical description, making it largely accessible to lowest-order corrections beyond mean field [16]. On the theoretical side, progress has been made in the framework of the extended Gross-Pitaevskii approach [10,25–31] where the LHY correction as well as atom losses are added to the nonlinear Schrödinger equation in an efficient *ad hoc* manner, by quantum Monte Carlo approaches [32–35], or by solving the Bogoliubov–de Gennes equations [28,29].

As these droplets form out of a BEC there is good reason to assume that they have superfluid properties. One of the signatures of superfluidity are vortices—topologically nontrivial states well known from harmonically trapped BECs (see, e.g., Refs. [36–44] or the reviews [45,46]), characterized by a depletion of the density accompanied by a phase shift. So far, however, experimental evidence for vortices in these droplets appears elusive. Only a few theoretical works yet considered the LHY-stabilized

quantum droplets' rotational properties. Very recent work found metastable necklacelike clustered droplets carrying angular momentum [47]. Stationary states with a singly quantized vortex were discussed for a prolate dipolar droplet that under time evolution can split into two smaller droplet fragments [48,49]. A similar fragment formation was also reported for binary droplets in both two [50] and three dimensions, in the latter case carrying up to two units of angular momentum in a region of experimentally accessible parameters [51]. A doubly quantized vortex was found to decay into two singly quantized vortices upon a quadrupolar deformation [52]. In the above studies, the vorticity was imprinted on the droplet by a phase factor of the initial state.

In this Letter we demonstrate in a novel way the ubiquity of vortices in quantum droplets carrying angular momentum. Instead of relying on the method of imprinting vorticity, here the *ground state* of a rotating trapped binary BEC is studied. We find that metastable vortex clusters may form after a sufficiently slow release from a weakly confining trap. We find that these droplets carry angular momentum by forming multiple singly quantized vortices along with rigid-body rotation, in contrast to the imprinted single multiply quantized vortex solutions previously found in the literature. We show that intriguingly, these LHY-amended condensates may support a metastable persistent current, and suggest how this phenomenon can be used in order to experimentally realize self-bound vortex droplets.

Let us now consider a species-symmetric binary BEC confined to two dimensions [17] that is interacting weakly via short-range interactions, where the inter- and intra-species interactions are assumed to be attractive and repulsive, respectively. For such a binary BEC with equal masses of the atoms in the two components, the coupled Gross-Pitaevskii equations reduce to that of a one-component BEC with an accordingly modified interaction term [16,17]. We initially confine the gas in a harmonic trap with an added Gaussian at the trap center. The extended Gross-Pitaevskii equation for such a system in a frame rotating with angular frequency  $\Omega$  can then be written as

$$i \frac{\partial \psi}{\partial t} = \left( -\frac{1}{2} \nabla^2 + \frac{1}{2} \omega^2 r^2 + V_0 e^{-(r/a_\perp)^2} + |\psi|^2 \ln |\psi|^2 - i K_3 |\psi|^4 - \Omega L_z \right) \psi, \quad (1)$$

where the scaling invariances of the system have been used to bring the equation into this dimensionless form. Here  $\omega$  is the harmonic trapping frequency,  $V_0$  the amplitude of the Gaussian,  $a_\perp = 1/\sqrt{\omega}$  the oscillator length,  $K_3$  the rate of three-body losses, and  $L_z = x p_y - y p_x$ . The order parameter is normalized according to  $\int d^2 \mathbf{r} |\psi|^2 = N$ . The energy in the nonrotating frame without losses is

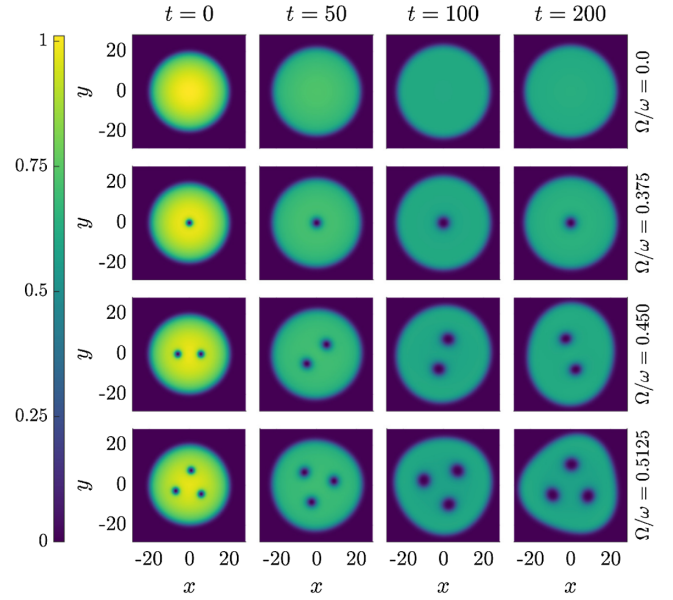


FIG. 1. Time evolution of various rotational ground state densities in a harmonic trap ( $V_0 = 0$ ) at different rotation frequencies for  $N = 1000$ ,  $\omega = 0.04$ , and  $K_3 = 0$  (i.e., no three-body losses). The harmonic trap frequency is linearly decreased in time such that it is zero at  $t = 100$ . See the Supplemental Material [54] for movies of the full time evolution of the densities.

$$E = \int d^2 \mathbf{r} \left[ \frac{1}{2} |\nabla \psi|^2 + \frac{1}{2} \omega^2 r^2 |\psi|^2 + V_0 e^{-(r/a_\perp)^2} |\psi|^2 + \frac{1}{2} |\psi|^4 \ln \left( \frac{|\psi|^2}{\sqrt{e}} \right) \right] \quad (2)$$

and the angular momentum  $L = \int d^2 \mathbf{r} \psi^* L_z \psi$ . Equation (1) is solved with the usual split-step Fourier method [53] in real and imaginary time. For the imaginary time propagation we use a set of different randomly perturbed initial conditions in order to avoid local minima in the energy landscape.

We first look for the ground state of free droplets in a rotating frame, but before convergence can be reached, the droplets are found to decay to fragments similar to those in the three-dimensional case [51]. As a first remedy to these inherently unstable solutions, we add a weak stabilizing harmonic confinement to the system. The dimensionless parameters considered here to illustrate our findings are  $N = 1000$  and  $\omega = 0.04$ . For these values the corresponding free droplet has the characteristic flat-top shape [16,17,30,31,50], and the trapping frequency is sufficiently weak to keep the droplet at a density close to its (free) equilibrium value. We first of all consider a purely harmonic trap ( $V_0 = 0$ ) and identify the rotational ground states at distinct rotation frequencies  $\Omega$ . The density distributions for some ground states at different  $\Omega$  are shown in the leftmost column of Fig. 1. Clearly, with

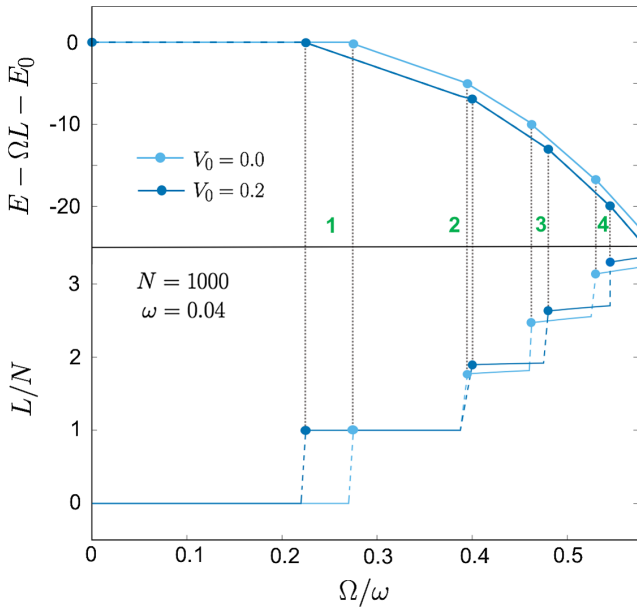


FIG. 2. Upper panel: Ground state energy in the rotating frame as a function of rotation frequency for  $N = 1000$  and  $\omega = 0.04$ , in a harmonic trap ( $V_0 = 0$ , light blue) and with a Gaussian added at the center ( $V_0 = 0.2$ , dark blue). Both curves are plotted against the energy of their respective nonrotating ground state  $E_0$ . Lower panel: Angular momentum as a function of rotation frequency for the same parameters. Distinct kinks in the energy correspond to an increase in the number of singly quantized vortices (green numbers) displayed as steps in the angular momentum shown in the lower panel.

increased rotation, vortices are induced in a way similar to that of a one-component condensate, proceeding from the formation of a unit vortex at the trap center, to two- and three-vortex states with the usual two- and threefold symmetries. Figure 2 shows the ground state energy in the rotating frame  $E - \Omega L$  and the corresponding angular momentum as a function of the trap rotation. The first three steps in  $L$ , corresponding to kinks in the rotational energy, are seen for  $L/N \approx 1.0$ , 1.8, and 2.5 for the first three vortex states shown in Fig. 1 similarly to scalar BECs [36,40].

Instead of minimizing the energy by solving Eq. (1) for fixed  $\Omega$ , one may instead study the energy at a fixed angular momentum by minimizing the dimensionless quantity  $\tilde{E} = E + C(L - L_0)^2$  [55], where  $C$  and  $L_0$  are dimensionless constants.  $\tilde{E}$  has a minimum at  $L = L_0 - E'/(2C)$  provided the curvature  $\tilde{E}'' = E'' + 2C$  is positive, which is the case when  $C$  is a sufficiently large number compared to  $E''$ . Furthermore, when  $C$  is much larger than  $E'$ , the minimum occurs at  $L \approx L_0$  with  $\tilde{E} \approx E$ , making it possible to obtain solutions for states that are not rotational ground states, i.e., for arbitrary  $L$ .

The dispersion relation obtained in this way is displayed in Fig. 3 for angular momenta up to and beyond the unit vortex that nucleates at the trap center at  $L/N = 1.0$ . At this value, for  $V_0 = 0$ , the energy has a kink, which can turn

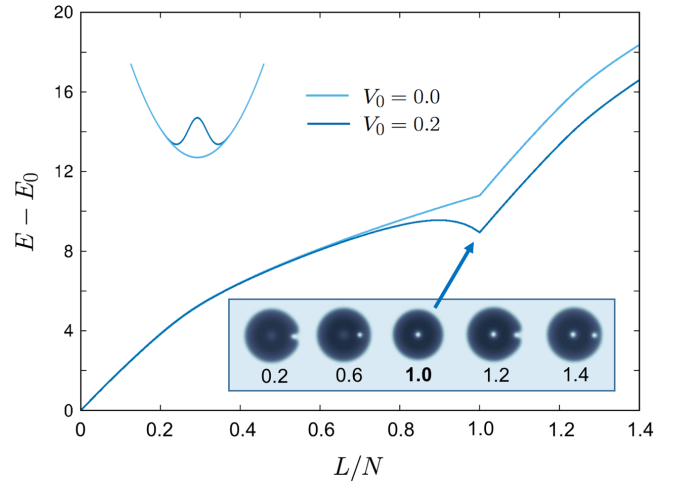


FIG. 3. Energy as a function of angular momentum for  $N = 1000$  and  $\omega = 0.04$ , in a harmonic trap ( $V_0 = 0$ , light blue) and with a Gaussian added at the trap center ( $V_0 = 0.2$ , dark blue), as in Fig. 2. Both curves are plotted against the energy of their respective nonrotating ground state  $E_0$ .

into an energetic minimum in the rotating frame when the energy  $E - \Omega L$  is tilted downwards by a constant slope, resulting in the first step in  $L$  when solving Eq. (1) for a corresponding value of  $\Omega$ ; see Fig. 2. For higher vortex numbers the mechanism is similar, with kinks in  $E$  leading to the plateaus in  $L$  for certain values of  $\Omega$ . In the absence of three-body losses,  $K_3 = 0$ , Eq. (1) conserves angular momentum. This implies that a condensate in a unit vortex ground state in the rotating frame will remain so even after the rotation ceases by virtue of this conservation law. However, since there is no local minimum in the dispersion relation for the purely harmonic case, this state is susceptible to small perturbations. It will thus slide down in energy to the nonrotating ground state. Let us next consider a nonmonotonic trapping potential by adding a Gaussian to the center of the harmonic trap (as has been realized experimentally; see, e.g., Ref. [56]). Trapping potentials of this kind have been shown to cause local minima in the dispersion relation for scalar BECs [57]. The energy as a function of angular momentum for  $V_0 = 0.2$  shown in Fig. 3 confirms that this is also the case for this binary system. Such a mexican-hat type of confinement can thus support a metastable persistent current even in the presence of weak perturbations. The energy in the rotating frame and the corresponding angular momentum as a function of rotation frequency with this central Gaussian is shown in Fig. 2.

Since we are interested in rotational properties of self-bound condensates, we now imagine a scenario where the ground state at a particular rotation frequency is maintained even after the trap rotation has stopped (as could be realistic in an experimental setting when there exists a local minimum in the dispersion relation). The condensate is then released from the trap by decreasing  $\omega$  linearly in time



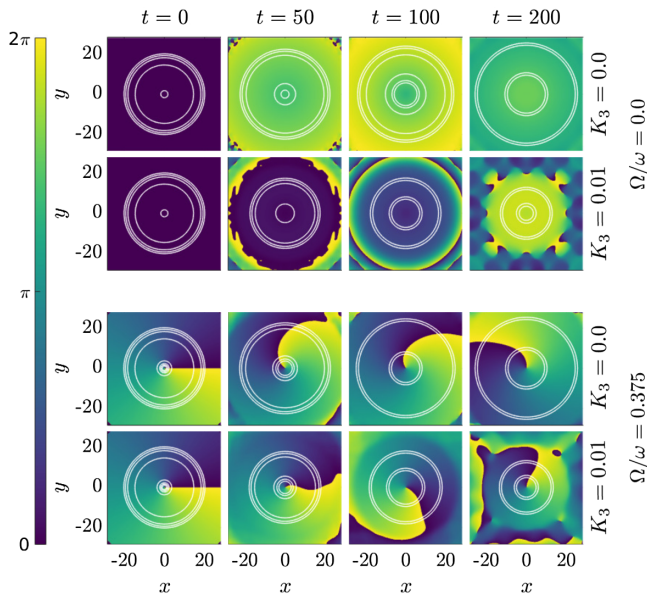


FIG. 4. Phases and superimposed density contours of the time evolution of ground states with zero or one vortex in a harmonic trap with a Gaussian ( $V_0 = 0.2$ ) for  $N = 1000$  and  $\omega = 0.04$ . Rows one and three show evolutions without three-body losses  $K_3 = 0$ , and rows two and four with  $K_3 = 0.01$ . The harmonic trap frequency is linearly decreased in time such that it is zero at  $t = 100$  while leaving the Gaussian unchanged. See the Supplemental Material [54] for movies of the full time evolution of the densities.

until  $\omega = 0$  in order to reduce the radial velocity that results from the expansion. The real time propagation for the purely harmonic case is shown in Fig. 1. Intriguingly, in this ideal case where conservation laws are intact, the droplets stay stable even after the trap is fully turned off while still carrying angular momentum in the form of vortices and rigid-body rotation. Additionally, the shape of the droplets is deformed according to the number of vortices. For a condensate in a trap with the Gaussian discussed previously, we consider a similar release, but now leaving the Gaussian even after the harmonic trap is turned off. Density contours and phases for the release of zero- and unit-vortex states in such a setup are displayed in Fig. 4, where we have also included a comparison with the corresponding systems including three-body losses with  $K_3 = 0.01$ . For both the cases with and without a vortex, the droplet is pinned to the remaining Gaussian, and they stay metastable and self-bound even when the symmetry-breaking three-body losses term is present.

In order to relate our results to experimental values, let us consider  $^{39}\text{K}$  atoms tightly confined by a harmonic trap in the transversal direction with an oscillator length  $\ell_z = 0.1 \mu\text{m}$ , and three dimensional  $s$ -wave scattering lengths equal to  $a_{\uparrow\downarrow} = -50.0a_0$  and  $a = 50.5a_0$  for the inter- and intraspecies interactions, respectively ( $a_0$  is the Bohr radius). Note that these values for the scattering lengths correspond to the stable gas phase in three

dimensions [14]; the liquefaction is due to the transition to two dimensions [17]. This choice, when transformed from the dimensionless parameters used above, corresponds roughly to  $\omega \sim 10$  Hz,  $N \sim 10^5$ , and  $n \sim 10^{14} \text{ cm}^{-3}$ , with units of time and space in  $\sim \text{ms}$  and  $\sim \mu\text{m}$ , respectively [58]. The value used for the three-body losses in Fig. 4 corresponds approximately to  $10^{-27} \text{ cm}^6/\text{s}$ .

Moreover, we see that  $1/|\ln n_{2\text{D}} a_{2\text{D}}^2| \ll 1$  [where  $n_{2\text{D}}$  is the two-dimensional density and  $a_{2\text{D}} = \ell_z \sqrt{(\pi/B)}$   $\exp(-\sqrt{\pi/2} \ell_z/a)$ , with  $B \approx 0.905$ , for a transversal harmonic trapping [58]], which puts us in a regime where beyond-mean-field predictions are valid for two dimensions [59]. The requirement for a quasi-2D system  $4na\ell_z^2/\pi \lesssim 0.03$  established in Ref. [60] is also met. It should be noted, however, that both of these results were derived for a system confined transversally by a box with periodic boundary conditions and not for a harmonic oscillator.

In conclusion, we have shown that binary self-bound bosonic droplets as realized in recent experiments with potassium [14,15] exhibit the formation of singly quantized vortices in a way similar to scalar BECs with weak short-range interactions, but with the addition of a deformation to the droplets' shape. In order to stabilize these droplets and prevent their decay into fragments (such as they were found both in two and three dimensions; see Refs. [50,51]), we found that it is crucial to first stabilize the droplets by a weak harmonic confinement, chosen such that it barely confines the self-bound droplet. When switching off the trap rotation and slowly releasing the droplet, cusps in the dispersion relation  $E(L)$  lead to rotational ground states that can generate rotating droplets containing multiple singly quantized vortices. To study a more perturbation-resistant system, we considered a binary BEC trapped in a mexican-hat potential, where we found the existence of a metastable persistent current that potentially could be utilized in order to produce droplets carrying angular momentum in the form of vortices. The findings presented here should be in the range of present experiments for binary condensates. We expect that similar metastable persistent currents may occur in dipolar quantum droplets. While the present analysis made use of the two-dimensional extended Gross-Pitaevskii approach [17], it will also be interesting to study the crossover between two and three dimensions [59,60] from the perspective of vortex formation.

Persistent currents, as here newly found in these binary condensates, have earlier been experimentally realized for ring-trapped BECs in Ref. [61], pointing out their importance for future atomtronics applications. Ring-shaped BECs have recently even been used to mimic the redshift effect [62]. The latter experiments were concerned with rapid expansion; however, an intriguing opportunity is similar investigations with a self-bound LHY ring carrying a persistent current, as here analyzed in the adiabatic limit. Likewise, ring-shaped waveguides recently showed a new

pathway towards guided matter-wave interferometry [63], for which persistent currents in LHY gases open up many new perspectives.

We thank in particular G. Kavoulakis for his help and useful comments at the initial stage of the project. We also thank J. Bengtsson, J. Bjerlin, D. Boholm, G. Eriksson, B. Mottelson, and R. Sachdeva for discussions. This work is financially supported by The Swedish Research Council and the Knut and Alice Wallenberg Foundation.

- 
- [1] A. Bohr and B. Mottelson, *Nuclear Structure*, Nuclear Structure No. Vol. 1 (World Scientific, Singapore, 1998).
  - [2] R. Donnelly, *Quantized Vortices in Helium II* (Cambridge University Press, Cambridge, England, 1991), Vol. 3.
  - [3] J. P. Toennies and A. F. Vilesov, *Angew. Chem. Int. Ed.* **43**, 2622 (2004).
  - [4] F. Ancilotto, M. Barranco, and M. Pi, *Phys. Rev. B* **97**, 184515 (2018).
  - [5] H. Kadau, M. Schmitt, M. Wenzel, C. Wink, T. Maier, I. Ferrier-Barbut, and T. Pfau, *Nature (London)* **530**, 194 (2016).
  - [6] M. Schmitt, M. Wenzel, F. Böttcher, I. Ferrier-Barbut, and T. Pfau, *Nature (London)* **539**, 259 (2016).
  - [7] I. Ferrier-Barbut, H. Kadau, M. Schmitt, M. Wenzel, and T. Pfau, *Phys. Rev. Lett.* **116**, 215301 (2016).
  - [8] I. Ferrier-Barbut, M. Schmitt, M. Wenzel, H. Kadau, and T. Pfau, *J. Phys. B* **49**, 214004 (2016).
  - [9] I. Ferrier-Barbut, M. Wenzel, F. Böttcher, T. Langen, M. Isoard, S. Stringari, and T. Pfau, *Phys. Rev. Lett.* **120**, 160402 (2018).
  - [10] L. Chomaz, S. Baier, D. Petter, M. J. Mark, F. Wächtler, L. Santos, and F. Ferlaino, *Phys. Rev. X* **6**, 041039 (2016).
  - [11] M. Lu, S. H. Youn, and B. L. Lev, *Phys. Rev. Lett.* **104**, 063001 (2010).
  - [12] M. Lu, N. Q. Burdick, S. H. Youn, and B. L. Lev, *Phys. Rev. Lett.* **107**, 190401 (2011).
  - [13] K. Aikawa, A. Frisch, M. Mark, S. Baier, A. Rietzler, R. Grimm, and F. Ferlaino, *Phys. Rev. Lett.* **108**, 210401 (2012).
  - [14] C. Cabrera, L. Tanzi, J. Sanz, B. Naylor, P. Thomas, P. Cheiney, and L. Tarruell, *Science* **359**, 301 (2018).
  - [15] G. Semeghini, G. Ferioli, L. Masi, C. Mazzinghi, L. Wolswijk, F. Minardi, M. Modugno, G. Modugno, M. Inguscio, and M. Fattori, *Phys. Rev. Lett.* **120**, 235301 (2018).
  - [16] D. S. Petrov, *Phys. Rev. Lett.* **115**, 155302 (2015).
  - [17] D. S. Petrov and G. E. Astrakharchik, *Phys. Rev. Lett.* **117**, 100401 (2016).
  - [18] A. Bulgac, *Phys. Rev. Lett.* **89**, 050402 (2002).
  - [19] H.-W. Hammer and D. T. Son, *Phys. Rev. Lett.* **93**, 250408 (2004).
  - [20] T. D. Lee, K. Huang, and C. N. Yang, *Phys. Rev.* **106**, 1135 (1957).
  - [21] W. D. Knight, K. Clemenger, W. A. de Heer, W. A. Saunders, M. Y. Chou, and M. L. Cohen, *Phys. Rev. Lett.* **52**, 2141 (1984).
  - [22] H. Nishioka, K. Hansen, and B. R. Mottelson, *Phys. Rev. B* **42**, 9377 (1990).
  - [23] M. Koskinen, P. O. Lipas, and M. Manninen, *Z. Phys. D* **35**, 285 (1995).
  - [24] I. Ferrier-Barbut, *Phys. Today* **72**, No. 4, 46 (2019).
  - [25] D. Baillie, R. M. Wilson, R. N. Bisset, and P. B. Blakie, *Phys. Rev. A* **94**, 021602(R) (2016).
  - [26] F. Wächtler and L. Santos, *Phys. Rev. A* **93**, 061603(R) (2016).
  - [27] F. Wächtler and L. Santos, *Phys. Rev. A* **94**, 043618 (2016).
  - [28] R. N. Bisset, R. M. Wilson, D. Baillie, and P. B. Blakie, *Phys. Rev. A* **94**, 033619 (2016).
  - [29] D. Baillie, R. M. Wilson, and P. B. Blakie, *Phys. Rev. Lett.* **119**, 255302 (2017).
  - [30] A. Boudjemâa, *Phys. Rev. A* **97**, 033627 (2018).
  - [31] A. Boudjemâa, *Phys. Rev. A* **98**, 033612 (2018).
  - [32] H. Saito, *J. Phys. Soc. Jpn.* **85**, 053001 (2016).
  - [33] A. Macia, J. Sánchez-Baena, J. Boronat, and F. Mazzanti, *Phys. Rev. Lett.* **117**, 205301 (2016).
  - [34] F. Cinti, A. Cappellaro, L. Salasnich, and T. Macrì, *Phys. Rev. Lett.* **119**, 215302 (2017).
  - [35] V. Cikojević, K. Dželalija, P. Stipanović, L. Vranješ Markić, and J. Boronat, *Phys. Rev. B* **97**, 140502(R) (2018).
  - [36] D. A. Butts and D. S. Rokhsar, *Nature (London)* **397**, 327 (1999).
  - [37] M. R. Matthews, B. P. Anderson, P. C. Haljan, D. S. Hall, C. E. Wieman, and E. A. Cornell, *Phys. Rev. Lett.* **83**, 2498 (1999).
  - [38] K. W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, *Phys. Rev. Lett.* **84**, 806 (2000).
  - [39] F. Chevy, K. W. Madison, and J. Dalibard, *Phys. Rev. Lett.* **85**, 2223 (2000).
  - [40] G. M. Kavoulakis, B. Mottelson, and C. J. Pethick, *Phys. Rev. A* **62**, 063605 (2000).
  - [41] J. R. Abo-Shaeer, C. Raman, J. M. Vogels, and W. Ketterle, *Science* **292**, 476 (2001).
  - [42] C. Raman, J. R. Abo-Shaeer, J. M. Vogels, K. Xu, and W. Ketterle, *Phys. Rev. Lett.* **87**, 210402 (2001).
  - [43] K. W. Madison, F. Chevy, V. Bretin, and J. Dalibard, *Phys. Rev. Lett.* **86**, 4443 (2001).
  - [44] P. C. Haljan, I. Coddington, P. Engels, and E. A. Cornell, *Phys. Rev. Lett.* **87**, 210403 (2001).
  - [45] A. L. Fetter, *Rev. Mod. Phys.* **81**, 647 (2009).
  - [46] H. Saarikoski, S. M. Reimann, A. Harju, and M. Manninen, *Rev. Mod. Phys.* **82**, 2785 (2010).
  - [47] Y. V. Kartashov, B. A. Malomed, and L. Torner, *Phys. Rev. Lett.* **122**, 193902 (2019).
  - [48] A. Cidrim, F. E. A. dos Santos, E. A. L. Henn, and T. Macrì, *Phys. Rev. A* **98**, 023618 (2018).
  - [49] A.-C. Lee, D. Baillie, R. N. Bisset, and P. B. Blakie, *Phys. Rev. A* **98**, 063620 (2018).
  - [50] Y. Li, Z. Chen, Z. Luo, C. Huang, H. Tan, W. Pang, and B. A. Malomed, *Phys. Rev. A* **98**, 063602 (2018).
  - [51] Y. V. Kartashov, B. A. Malomed, L. Tarruell, and L. Torner, *Phys. Rev. A* **98**, 013612 (2018).
  - [52] F. Ancilotto, M. Barranco, M. Guilleumas, and M. Pi, *Phys. Rev. A* **98**, 053623 (2018).
  - [53] S. A. Chin and E. Krotscheck, *Phys. Rev. E* **72**, 036705 (2005).

- [54] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.123.160405> for movies showing the full time evolution for the results shown in Fig. 1.
- [55] S. Komineas, N.R. Cooper, and N. Papanicolaou, *Phys. Rev. A* **72**, 053624 (2005).
- [56] V. Bretin, S. Stock, Y. Seurin, and J. Dalibard, *Phys. Rev. Lett.* **92**, 050403 (2004).
- [57] K. Kärkkäinen, J. Christensson, G. Reinisch, G.M. Kavoulakis, and S.M. Reimann, *Phys. Rev. A* **76**, 043627 (2007).
- [58] D.S. Petrov and G.V. Shlyapnikov, *Phys. Rev. A* **64**, 012706 (2001).
- [59] T. Ilg, J. Kumlin, L. Santos, D. S. Petrov, and H. P. Büchler, *Phys. Rev. A* **98**, 051604(R) (2018).
- [60] P. Zin, M. Pylak, T. Wasak, M. Gajda, and Z. Idziaszek, *Phys. Rev. A* **98**, 051603(R) (2018).
- [61] S. Eckel, J. G. Lee, F. Jendrzejewski, N. Murray, C. W. Clark, C. J. Lobb, W. D. Phillips, M. Edwards, and G. K. Campbell, *Nature (London)* **506**, 200 (2014).
- [62] S. Eckel, A. Kumar, T. Jacobson, I. B. Spielman, and G. K. Campbell, *Phys. Rev. X* **8**, 021021 (2018).
- [63] S. Pandey, H. Mas, G. Drougakis, P. Thekkepatt, V. Bolpasi, G. Vasilakis, K. Poullos, and W. von Klitzing, *Nature (London)* **570**, 205 (2019).