Editors' Suggestion

Chiral Gravitons in Fractional Quantum Hall Liquids

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We elucidate the nature of neutral collective excitations of fractional quantum Hall liquids in the long-wavelength limit. We demonstrate that they are chiral gravitons carrying angular momentum −2, which are quanta of quantum motion of an internal metric, and show up as resonance peaks in the system's response to what is the fractional Hall analog of gravitational waves. The relation with existing and possible future experimental work that can detect these fractional quantum Hall gravitons and reveal their chirality is discussed.

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Introduction and motivation.—The study of neutral excitation spectra of fractional quantum Hall (FQH) liquids has a long history. It is now well understood that there is a sharp magnetoroton mode exhibiting a "roton" minimum at a finite wave vector [1], which has been observed experimentally [2]. The observability of this mode is related to the fact that it is a bound state of a Laughlin quasiparticle-quasihole pair (or a quasiexciton), which can be created by the (Landau-level projected) density operator, which is dipole active; it can thus be excited electromagnetically at the appropriate *finite* wave vector. The sharpness of the magnetoroton mode is tied to the fact that its energy is below the continuum formed by more complicated multi-quasiparticle or quasihole excitations (or multirotons).

On the other hand, the understanding of collective excitations at long-wavelengths is far from complete. The magnetoroton dispersion enters the continuum as the wave vector decreases, making it very hard to identify, and even casting doubt on its presence. More serious is the fact that Kohn's theorem dictates that dipole spectral weight is exhausted by the cyclotron mode, making the single mode approximation [1] completely ineffective at zero wave vector. This also renders any long-wavelength intra-Landau level collective excitation invisible to electromagnetic probe in the linear response regime [3].

In a parallel line of work, one of us [4] pointed out that there exists an internal geometrical degree of freedom (or internal metric) responsible for the intra-Landau level dynamics of the system, which is not properly captured by the standard description of FQH liquids in terms of topological quantum field theories. Physical implications of this geometrical degree of freedom have been discussed extensively [5–9], in particular its experimental observability [10–15]. Furthermore, it has also been argued [4, 16–19] that this internal metric has its own quantum dynamics, which gives rise to the long-wavelength collective excitations in FQH liquids that can be viewed as "gravitons." This provides a new insight into the invisibility of the long-wavelength intra-Landau level collective mode to electromagnetic probes: the graviton carries total angular momentum 2, mismatching that of the photon which carries angular momentum 1. In a recent paper, another of us [20] argued that these gravitons can instead be excited and probed using acoustic waves in the crystal, whose effects mimic those of gravitational waves.

In this Letter we present numerical results that demonstrate unequivocally the presence of the graviton mode, which shows up as a pronounced peak in the spectral function of the dynamical gravitational response [20]. We further reveal the chiral nature of the gravitons, namely, that they come with a specific polarization corresponding to angular momentum [21], −2. We will discuss possible experimental probes of these gravitons and, in particular, their polarization, as well as the relation between our results and closely related works.

Spectral functions.—Ref. [20] considered the coupling between an oscillating effective mass tensor and the intra-Landau level degrees of freedom of a two-dimensional electron gas (2DEG) confined to the lowest Landau level (LLL). For a two-body interaction of the form

$$V^{(2)} = \sum_{i<j} V(\mathbf{r}_i - \mathbf{r}_j) = \frac{1}{2}\sum_{\mathbf{q}} V_{\mathbf{q}}\rho_{\mathbf{q}}\rho_{-\mathbf{q}}, \tag{1}$$

where $V_{\mathbf{q}}$ is the Fourier transform of electron-electron interaction potential $V(\mathbf{r})$ and

$$\rho_{\mathbf{q}} = \sum_i e^{i\mathbf{q}\cdot\mathbf{r}_i} \tag{2}$$

is the density operator, it was found that the coupling is described by the operator

$$\hat{O}^{(2)} = \sum_{\mathbf{q}} (q_y^2 - q_x^2) V_{\mathbf{q}} e^{-(1/2)q^2\ell^2} \bar{\rho}_{\mathbf{q}}\bar{\rho}_{-\mathbf{q}}, \tag{3}$$

in which

$$\bar{\rho}_{\mathbf{q}} = \sum_i e^{i\mathbf{q}\cdot\mathbf{R}_i} \tag{4}$$

is the LLL projected density operator and **R** is the guiding center coordinate. It is straightforward to generalize the above to multiparticle interactions; of particular relevance to our later discussion is the case of a three-body interaction

$$V^{(3)} = \frac{1}{6}\sum_{\mathbf{q}_1\mathbf{q}_2} V_{\mathbf{q}_1\mathbf{q}_2}\rho_{\mathbf{q}_1}\rho_{\mathbf{q}_2}\rho_{-\mathbf{q}_1-\mathbf{q}_2}, \tag{5}$$

in which case the coupling is given by

$$\hat{O}^{(3)} = \sum_{\mathbf{q}_1\mathbf{q}_2} V_{\mathbf{q}_1\mathbf{q}_2}(q_{1y}^2 + q_{2y}^2 + q_{1y}q_{2y} - q_{1x}^2 - q_{2x}^2 - q_{1x}q_{2x}) \times e^{-\frac{\ell^2}{2}(q_1^2+q_2^2+|\mathbf{q}_1+\mathbf{q}_2|^2)}\bar{\rho}_{\mathbf{q}_1}\bar{\rho}_{\mathbf{q}_2}\bar{\rho}_{-\mathbf{q}_1-\mathbf{q}_2}. \tag{6}$$

Our numerical studies are based on the calculation of spectral functions of $\hat{O}$ and its close relatives $\hat{O}_\sigma$ (to be specified below) in finite-size systems:

$$I_\sigma(\omega) = \sum_n |\langle n|\hat{O}_\sigma|0\rangle|^2\delta(\omega - \omega_n), \tag{7}$$

where $|0\rangle$ is the ground state and $|n\rangle$ is an excited state with excitation energy $\hbar\omega_n$ (from here on we set $\hbar = 1$.) This is the system's transition rate due to an oscillating effective mass tensor metric, which is analogous to an oscillating metric induced by a gravitational wave.

To establish the chiral nature of the graviton, it is convenient to study operators that have "handedness" instead of the ones given in Eqs. (3) and (6). Therefore, we define

$$\hat{O}^{(2)}_{\mp} = \sum_{\mathbf{q}} (q_x \mp iq_y)^2 V_{\mathbf{q}} e^{-(1/2)q^2\ell^2}\bar{\rho}_{\mathbf{q}}\bar{\rho}_{-\mathbf{q}}, \tag{8}$$

where we have discarded an overall minus sign. A similar extension can be done for the three-body, or for that matter, to the n-body case of the Read-Rezayi [22] sequence. In this Letter we will focus on the Moore-Read (MR) state [23] with three-body interactions. As we will see, $\hat{O}_{\mp}$ are the creation and annihilation operators of the graviton, respectively, while $\hat{O} = (\hat{O}_+ + \hat{O}_-)/2$ is equivalent to the displacement operator in a harmonic oscillator, which couples to a linearly polarized "gravitational wave" [20].

Numerical results.—To compare $I_\sigma(\omega)$ for different sizes it is convenient to normalize it, by dividing out the factor $\langle 0|\hat{O}^\dagger_\sigma\hat{O}_\sigma|0\rangle$, so that

$$\int_{-\infty}^{\infty} I_\sigma(\omega)d\omega = 1. \tag{9}$$

We start with the Laughlin states. We consider both cases of fermions ($\nu = 1/3$) as well as bosons ($\nu = 1/2$) on toroidal geometries and evaluate Eq. (7). We have studied sizes up to 12 particles. The latter is generally believed to be larger than the correlation length (or size) of the system beyond which thermodynamic behavior becomes visible. However, quantitative effects would still persist. For small sizes almost the entire weight is exhausted by a single graviton peak. For larger sizes we observe broadening of the resonance and the appearance of smaller nearby peaks. However, the integrated weight of the resonance increases linearly with size, a trend that generally is not followed by the height of a single peak. For all the sizes that we considered the graviton resonance produces the largest response in the system. Figures 1 and 2 show these cases for fermions and bosons, respectively.

The Hamiltonian for both cases consists of a single pseudopotential for relative angular momentum 1 (fermions) or 0 (bosons). Our energies are given in units of the strength of these pseudopotentials. We note that the energies for which we see graviton responses are consistent with previous numerical calculations [24], where the graviton is the $k = 0$ energy of the Girvin-MacDonald-Platzman magnetoroton collective mode inside the excited states continuum.

To establish that gravitons are chiral on the torus we employ the chiral operators $\hat{O}_{\mp}$. Interestingly, $\hat{O}_-$ has the

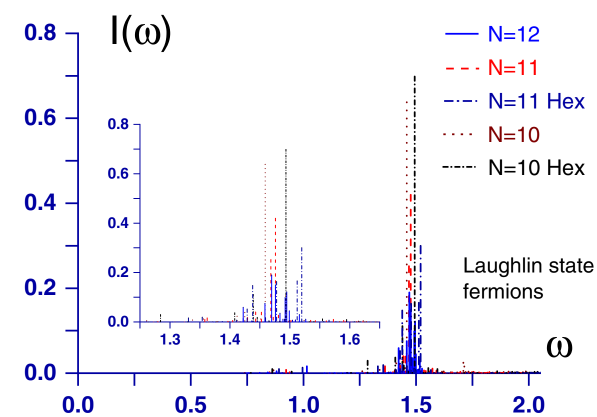

FIG. 1. A bird's-eye view of the $I_-(\omega)$ from the fermionic Laughlin ground state at $\nu = 1/3$ for various sizes and geometries (Hex stands for hexagon geometry; square geometry otherwise). The graviton response can clearly be seen. The inset shows more details near energies where the response is strong. For $N = 10$ and 11 we recovered over 97% of the total weights. Unlike the case of bosons (shown below) the "background noise" (very small amplitude scatter of the data not near the graviton signal) seems to be stronger for fermions, which are also computationally more costly than bosons. Accordingly, for $N = 12$ we recovered 90% of the weight over 87% of which is in the window of the inset. The rest, at higher energies, appears to be background noise. For example, we recover over 99% of the weight for 10 electrons. However, no significant peaks other than those shown in the inset were seen. The total weight in the inset is only 87% of the total weight. In other words about 12%, at higher energies, is just background noise.

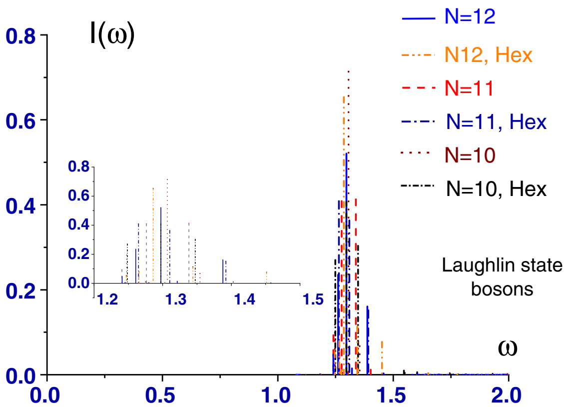

FIG. 2. Same as Fig. 1 except for bosons at $\nu = 1/2$. In this case we have collected 99% of the total weight. The graviton response again stands out against the background noise. For all sizes and geometries the weights shown in the inset constitute over 98% of the total.

same effect as the operator of Eq. (3), while $\hat{O}_+$ annihilates the model state, and so $I_+(\omega)$ is identically zero. The chiral operators, as noted, can be generalized to n-body pseudopotential Hamiltonians for the Read-Rezayi sequence. We have explicitly verified this for the three-body Hamiltonian of the MR state. In fact, in all our plots (except the Coulomb interaction case below) we show $I_-(\omega)$.

The operator $\hat{O}_-$ creates excitations with angular momentum -2, which is also the angular momentum of the gravitons they create. To reveal this chirality more explicitly we also investigate the two Laughlin states on disk geometry where angular momentum is a good quantum number. Instead of using Eq. (8), we express $\hat{O}^{(2)}_{\mp}$ in terms of anisotropic complex pseudopotentials [25]: $O^{(2)}_+ \propto \sum_M |m+2, M\rangle\langle m, M|$ and $O^{(2)}_- \propto \sum_M |m, M\rangle\langle m+2, M|$, where $|m, M\rangle$ is a two-body state with the relative angular momentum m and center-of-mass angular momentum M, with $m = 1$ for fermions and $m = 0$ for bosons. We now see why $\hat{O}_+$ annihilates the Laughlin state: it tries to turn a pair with relative angular momentum m into $m+2$, which does not exist in the Laughlin state. As a result, $I_+(\omega)$ is zero everywhere. On the other hand, Figs. 3(a) and 3(b) show strong graviton peaks in $I_-(\omega)$ for the fermionic and bosonic cases, respectively. Comparing to the cases on the torus, we find good agreement in peak positions, and noticeably less broadening and background noise. The results are not sensitive to the number of orbitals we keep as long as we have enough orbitals to accommodate the Laughlin states; see Figs. 3(c) and 3(d).

We now return to toroidal geometry and investigate the graviton contribution to the spectral functions for the Coulomb potential at $\nu = 1/3$, which is the experimentally most relevant case. Figure 4 shows $I_-(\omega)$ for electrons at $\nu = 1/3$ on a square torus, where we use the Coulomb interaction including finite quantum-well thickness

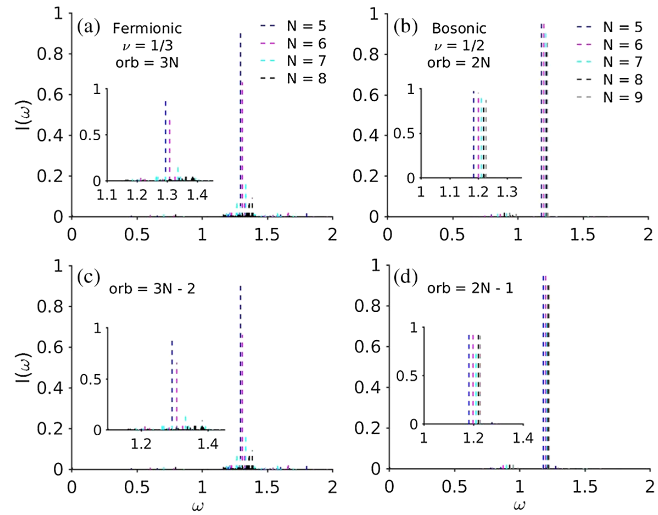

FIG. 3. A bird's-eye view of (a) and (c) fermionic $I_-(\omega)$ with $3N$ and $3N-2$ orbitals, and of (b) and (d) bosonic $I(\omega)$ with $2N$ and $2N-1$ orbitals for N particles on disk geometry. For all sizes, we recovered up to 97% of the total weight for the fermionic case and up to 99% for the bosonic case. In all cases, the graviton absorptions stand out against the background noise.

appropriate for the samples of Ref. [26], whose relation with our work is discussed below. While the weights are smaller than those of the model states, a clear signature of the graviton is discernible in comparison to other peaks further up in the continuum. In this case, $\hat{O}_+$ does not annihilate the Coulomb ground state, because there do exist pairs with relative angular momentum $m = 1$ in the ground

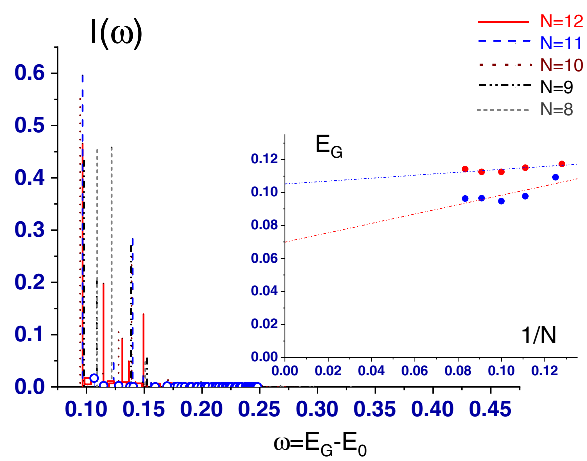

FIG. 4. The graviton response of the Coulomb potential $\nu = 1/3$ ground state with hexagonal unit cell. The x axis represents the excitation energy measured from the ground state energy. The large symbols at the bottom represent the relative weight [to quantify the relative strengths, $I_+(\omega)$ is normalized by the total weight of the $I_-(\omega)$] of $I_+(\omega)$ ($N = 12$ square and $N = 11$ circular symbols). The inset shows scaling of the graviton energy vs inverse of the system size. The lower points are energies of the main peak, whereas the upper points are the average energies weighted by the size of the corresponding peaks.

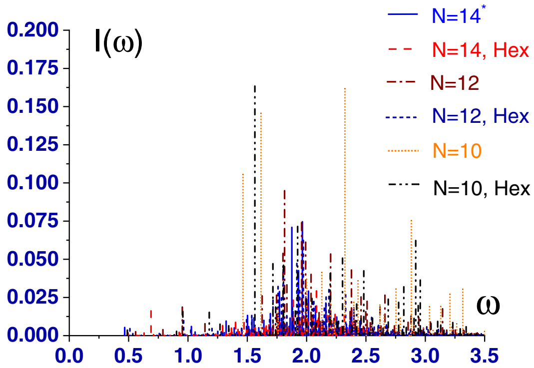

FIG. 5. The spectral function $I_-(\omega)$ for the MR state of fermions with even number of particles at $\nu = 1/2$.

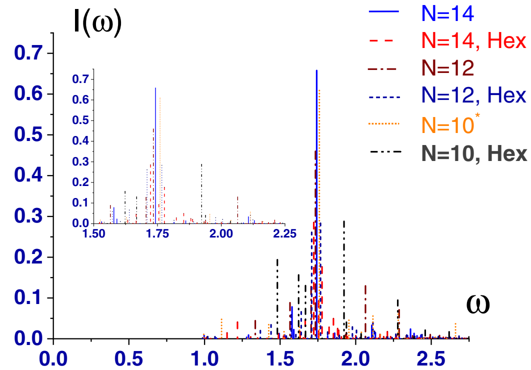

FIG. 6. The spectral function $I(\omega)$ for the MR state of bosons with an even number of particles at $\nu = 1$.

state. Nonetheless, the chiral nature of the graviton is evident by the strong suppression of the $I_+(\omega)$ compared to $I_-(\omega)$. This is because such pairs are rare, reflecting the Laughlin correlation.

Our bounds on the energy of the graviton (0.07–0.105 in units of $e^2/4\pi\varepsilon_0\ell$, ℓ is the magnetic length) shown in the inset of Fig. 4, are consistent with the resonance energy (0.084) found in the inelastic light scattering measurement of Ref. [26]. In such a two-photon process (one photon absorbed and one photon emitted, also known as Raman scattering), the total angular momentum of the two photons matches that of the graviton if they have the appropriate polarization; as a result gravitons can be excited [18]. We therefore identify the resonance of Ref. [26] as due to the graviton excitation.

To obtain a more definitive prediction one may need a more systematic sampling of the resonance energies, which appear to be broadened and hence introduce the scatter we observe in the data.

We next consider the MR states. As in the case of the Laughlin states, for even numbers of particles the graviton has (orbital) spin of 2 and obeys Bose statistics. However, for an odd number of particles, in addition to the graviton, in Ref. [16] it was shown that there exists a fermionic "gravitino" resonance, which has spin 3/2. In this Letter we study the former, but defer the investigation of gravitino to future studies. Figure 5 shows $I(\omega)$ (which is equivalent to $I_-(\omega)$ for the same reason as the Laughlin case) for a three-body interaction that makes the MR state the exact ground state for fermions at $\nu = 1/2$. For this part, we have again studied the square and hexagonal torus, which are the two highest symmetry geometries.

In the case of the square geometry, the threefold degeneracy of the MR state for an even number of electrons is split into two wave vectors that are not related by symmetry: one in the Brillouin zone (BZ) corner and a pair of states on the BZ boundary (shown by asterisks in the size labels in our plots). In both cases of the fermions and bosons we have included some of these ground states. In some cases for the same size and geometry the two distinct ground states either show a single strong peak or a few smaller neighboring peaks, but with comparable total weights, which appear to indicate broadened resonance with a shorter lifetime. For the hexagonal geometry all 3 ground states have wave vectors related by symmetry, and depending on size, they exhibit both sharp and broadened peaks.

In Fig. 6 we present the results for boson MR states at $\nu = 1$. For this case we have also calculated $I_-(\omega)$ on a disc; see Fig. 7. The results are consistent with those obtained on the torus, although the graviton peak appears sharper. Note that on a disc subtleties associated with the threefold degeneracy and differences between even and odd particle number cases do not exist, which may be contributing factors to better quality data.

As seen by the size of the weights in these figures the noise level increases considerably in the three-body case and is worse for fermions. A similar trend was observed for the Laughlin states as noted earlier. Nevertheless, the graviton peaks do stand out against the background noise.

Experimental observability and future work.—In Ref. [20] it was suggested that the graviton will show

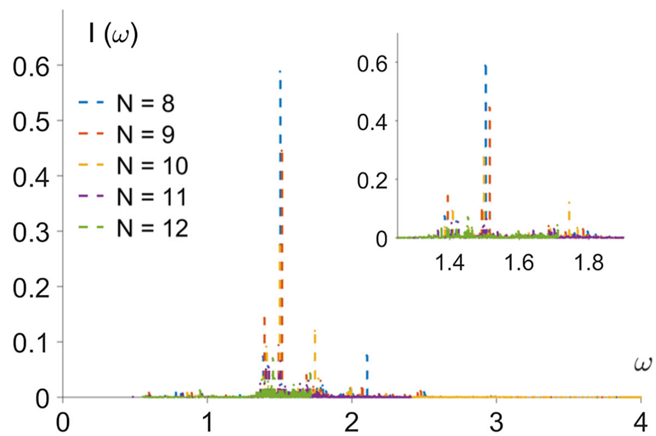

FIG. 7. The spectral function $I(\omega)$ for the MR state of bosons at $\nu = 1$ on a disc, where the orbital number used is the same as the particle number N.

up as a sharp resonance peak in the absorption spectrum of the acoustic wave propagating perpendicular to the 2DEG, which is an analog of the gravitational wave. Our numerical results support this, as the spectral functions calculated here are those describing the coupling between acoustic waves and 2DEG. Such experiments have yet to be performed. More recently it was argued that the graviton will dominate quench dynamics in a FQH liquid [24]. As discussed above, we identify the resonance peak in the inelastic light scattering at $\nu = 1/3$ [26] as due to the graviton. To gain a more detailed understanding we need to calculate the appropriate spectral function for such processes. As pointed out previously [18] the chirality of the graviton can be revealed through the polarization of the light in a Raman process. Here we are able to make a much more specific prediction: In order to excite the chiral graviton with angular momentum -2, the incoming light needs to be circularly polarized to have angular momentum -1, while the (Raman or inelastically) scattered light will have the opposite polarization and have angular momentum $+1$, thus transferring a net angular momentum -2 to the 2DEG. In a very recent experimental work [27], inelastic light scattering was performed on the second Landau level states. We tentatively attribute the sharp resonance at $\nu = 7/3$ (which is termed a new plasmon) to the graviton, similar to that at $\nu = 1/3$ [28]; the much broadened peak at $\nu = 5/2$ is consistent with the broadening we found in our calculations for the MR state. But much more detailed studies using interactions and operators appropriate to the second Landau level, which involve a more complicated form factor, are needed. We leave these and other details to future work.

In summary, we have found a clear signature of a chiral graviton mode for both Laughlin and MR states, and particularly for the ground state of the Coulomb interaction at $\nu = 1/3$. In all cases of torus studies the total weights, the bulk of which constitute the graviton resonance, scale linearly with system size. Our results are consistent with the inelastic light scattering experiment of Pinczuk *et al.* [26] that sees a resonance with zero momentum.

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