Generalized Hydrodynamics with Space-Time Inhomogeneous Interactions

Alvise Bastianello, Vincenzo Alba, and Jean-Sébastien Caux
Institute for Theoretical Physics, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, Netherlands

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We provide a new hydrodynamic framework to describe out-of-equilibrium integrable systems with space-time inhomogeneous interactions. Our result builds up on the recently introduced generalized hydrodynamics (GHD). The method allows us to analytically describe the dynamics during generic space-time-dependent smooth modulations of the interactions. As a proof of concept, we study experimentally motivated interaction quenches in the trapped interacting Bose gas, which cannot be treated with current analytical or numerical methods. We also benchmark our results in the *XXZ* spin chain and in the classical sinh-Gordon model.

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Introduction.—Exploring the out-of-equilibrium behavior of quantum many-body systems is nowadays among the most active research areas in physics, due to a successful synergy between theoretical and experimental advances [1–4].

How, and is what sense, does a coarse-grained thermodynamic description emerge through dynamical evolution in isolated out-of-equilibrium many-body systems? One-dimensional systems represent an ideal playground to address this question: there, remarkably powerful tools exist, both theoretical (such as conformal field theory [5] and integrability [6,7]) and computational (such as matrix product states methods [8]). Integrability is ubiquitous in the low-dimensional world (and experimentally realized [9]), with applications ranging from spin chains [6] to continuum models, the latter having Lorentz [7] or Galilean [10,11] invariance, or neither [12]. Integrable models are characterized by the presence of infinitely many conserved charges $\hat{\mathcal{Q}}_j$, which can be used to exactly determine their thermodynamics [13]. In recent times, the importance of quasilocal charges has moreover been underlined [14].

The last decade has witnessed exact results reaching out-of-equilibrium protocols as well: great attention has been devoted to the homogeneous sudden quantum quench [15] (see also Ref. [16] and reference therein). Because of the conserved quantities, the system exhibits local relaxation to a state that is not thermal [17–25], but rather emerges from a quench action [26,27] or (where applicable) a generalized Gibbs ensemble (GGE) [28,29] which accounts for all the relevant charges.

More recently, the focus has been on quenches from spatially inhomogeneous systems. A new theoretical toolbox, dubbed generalized hydrodynamics (GHD) [30,31] allows us to address this problem. In Refs. [30,31] GHD dealt with inhomogeneous states evolving under a homogeneous Hamiltonian. Several applications have been explored [32–57], for instance, including diffusive corrections [58–61] or applying it to classical models [62–66]. Also, combined with the quasiparticle picture for integrable systems [67,68], GHD allows us to describe the entanglement spreading after inhomogeneous quenches [69–72]. Very recently, it has been shown that GHD provides the correct theoretical framework to describe atom-chip experiments [73].

When comparing with actual experiments, inhomogeneities, for instance due to external trapping potentials, should ideally be kept into account. Strictly speaking, inhomogeneities break integrability, but smooth variations can still be captured by invoking local relaxation to a (locally homogeneous) integrable model.

Inhomogeneities in the dynamics have already been studied with some limitations for either spatial [32,65] or temporal changes [54], opening the possibility, for example, of studying the famous quantum Newton cradle experiment [74] through GHD [47]. However, the current state of the art cannot capture changes in the interparticle interactions, leaving perturbative methods [75,76] or bosonization techniques [77–83] as the only methods to tackle this experimentally relevant situation.

In this Letter, we present a complete GHD approach that allows us to treat the dynamics under integrable

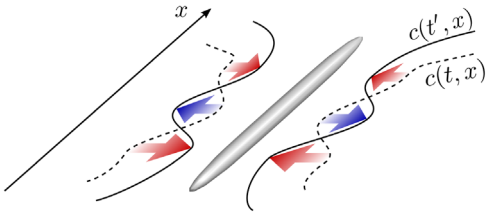

FIG. 1. Prototypical experimental setup that can be addressed with our method. A Bose gas is trapped in a one-dimensional tube. The space-time dependent interparticle interaction strength $c(t, x)$ is modified by modulating the transverse trapping potential (see also Fig. 2).

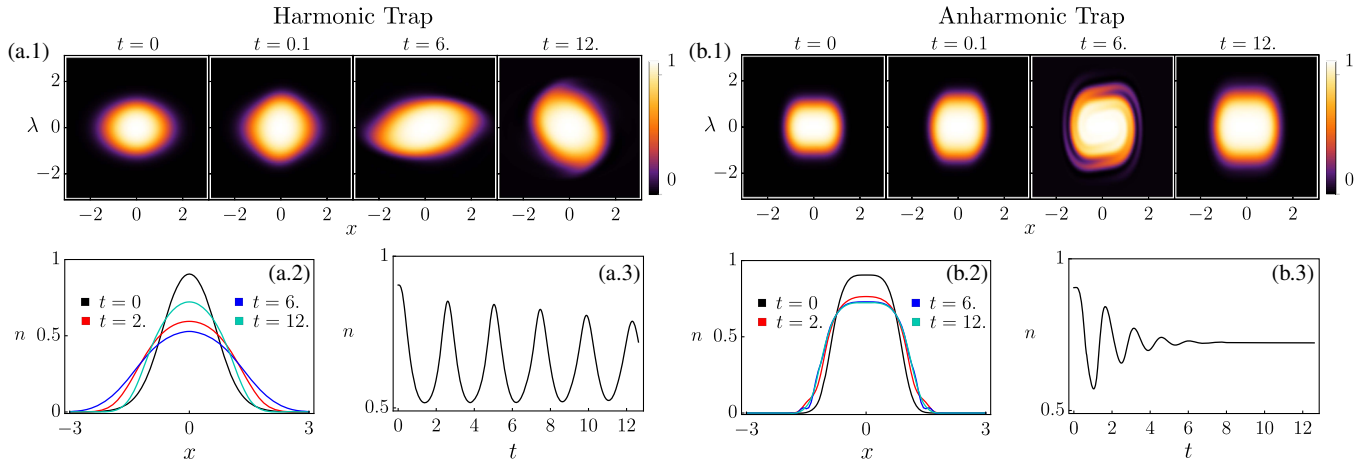

FIG. 2. Evolution of the trapped one-dimensional Lieb-Liniger gas. The interaction strength is changed as $c(t) = 0.3 + \tanh(3t)$ during the evolution. The left and right panels correspond to the harmonic and anharmonic trapping potentials $V(x) = x^2/2 - 0.5$ and $V(x) = x^4/2 - 0.5$, respectively. In both cases the initial state is a thermal state at inverse temperature $\beta = 2$. In (a.1) and (b.1) we show the space dependence of the quasiparticle filling functions at different times. In each subfigure, the y axis λ is the quasiparticle rapidity. The x axis shows the position inside the trap. (a.2) Particle densities $n(t,x) \equiv \langle \psi^\dagger(x)\psi(x)\rangle$ as a function of x, for several times. Subfigure (a.3): density at the center of the trap as a function of time. Subfigures (b.2) and (b.3): the same as in (a.2) and (a.3) for the quench in the anharmonic trap.

Hamiltonians with space-time inhomogeneous interactions. Our results significantly extend the current GHD framework exhausting all the possible inhomogeneities which can be considered on a pure hydrodynamic level, disclosing the full power of GHD in describing experimentally relevant protocols. We discuss the potential applications of our result to interaction changes in the Lieb-Liniger model [10,11] (Figs. 1 and 2), which is of primary experimental interest. So far, the primary analytical tool used in dealing with time-dependent interactions has been the Luttinger liquid approach [84,85], recently generalized to include spatial inhomogeneities [86–92]. In contrast with GHD, this method is nevertheless confined to the low-energy excitations. We numerically benchmark the GHD predictions both in the quantum and classical realms, considering the XXZ spin chain and the classical sinh-Gordon field theory, showing once again the wide applicability of our results. Furthermore, we improve the numerical method proposed in Ref. [34] to solve GHD equations, promoting it from a first order to a second order algorithm in the time step, providing a great stability enhancement.

Thermodynamics of integrable models.—The thermodynamic Bethe ansatz (TBA) technique is nowadays a textbook topic [13]: here we present the basic concepts for the sake of a self-contained exposition. The Hilbert space of integrable models can be understood in terms of multiparticle states $|\{\lambda\}_{i=1}^N\rangle$, labeled by suitable parameters λ called rapidities [6,7]. Quasiparticles undergo pairwise elastic scatterings, which are described by an interaction-dependent scattering matrix $S(\lambda)$. These states are common eigenstates of the full set of (quasi-)local charges. In the thermodynamic limit (TDL), we switch to a coarse-grained description through a rapidity (root) density $\rho(\lambda)$ [13], which gives the density of rapidities within the interval $(\lambda, \lambda + d\lambda)$. The root densities are in a one-to-one correspondence with the possible thermodynamic states of the system, such as GGEs [93] or thermal states and fix the (extensive part of) the expectation value of the local charges

$$\lim_{\mathrm{TL}} \frac{1}{L} \langle \{\lambda\}_{i=1}^N | \hat{\mathcal{Q}}_j | \{\lambda\}_{i=1}^N \rangle = \int d\lambda q_j(\lambda)\rho(\lambda), \qquad (1)$$

together with any other local (in real space) property of the system, according to the quench action approach [26,27,94]. The function $q_j(\lambda)$ in Eq. (1) is called the charge eigenvalue.

Nontrivial interactions induce collective effects. For example, the group velocity of the quasiparticles, which is defined as $v(\lambda) = \partial_\lambda \epsilon / \partial_\lambda p$, with $\epsilon(\lambda)$, $p(\lambda)$ the energy and momentum eigenvalues respectively, is "dressed" as $v^{\mathrm{eff}}(\lambda) = (\partial_\lambda \epsilon)^{\mathrm{dr}} / (\partial_\lambda p)^{\mathrm{dr}}$, with ϵ^{dr} and p^{dr} the dressed quasiparticle energy and momentum. These are obtained by using that an arbitrary dressed quantity $\tau^{\mathrm{dr}}(\lambda)$ is defined through the integral equation

$$\tau^{\mathrm{dr}}(\lambda) = \tau(\lambda) - \int \frac{d\mu}{2\pi} \partial_\lambda \Theta(\lambda - \mu)\vartheta(\mu)\tau^{\mathrm{dr}}(\mu), \qquad (2)$$

with $\tau(\lambda)$ the "bare" quantity. Here $\Theta(\lambda) = -i \log S(\lambda)$, with $S(\lambda)$ the two-body scattering matrix encoding the interaction, while $\vartheta(\lambda) = 2\pi\rho(\lambda)/(\partial_\lambda p)^{\mathrm{dr}}$ is the so-called filling function. We summarized the TBA considering a single

particle species, but the construction is easily generalized to several species of excitations and bound states.

Emergent hydrodynamics with space-time inhomogeneous interactions.—TBA describes homogeneous stationary states. Instead, we now consider smooth space-time inhomogeneities, both in the initial state and in the Hamiltonian. We imagine a family of integrable models parametrized by a coupling α, with Hamiltonians

$$\hat{H}(\alpha) = \int dx \hat{\mathbf{h}}(x, \alpha(t,x)). \tag{3}$$

Crucially, in Eq. (3) α is a function of both space and time. We consider models in the continuum for simplicity, but the same construction can be repeated on the lattice.

Spatial inhomogeneities of the initial state on the same typical length scale of the variation of α are allowed. We are then interested in describing the system at the Eulerian scales $(\Delta t, \Delta x) \sim \big((\partial_t \alpha)^{-1}, (\partial_x \alpha)^{-1}\big)$, considering at the same time the limit of infinitely smooth variations $\partial_t \alpha \sim \partial_x \alpha \to 0$.

Closely following the same argument presented in Refs. [30,31], in this limit we can invoke local relaxation to an inhomogeneous GGE, associated with a weakly inhomogeneous root density $\rho(t,x,\lambda)$. We report the details of the derivation of the GHD equations in the Supplemental Material (SM) [95]. Here, we rather present the result, discussing its physical interpretation and validity regime, together with possible applications.

Our main result is that $\rho(t,x,\lambda)$ satisfies the following hydrodynamic equations as

$$\partial_t \rho + \partial_x (v^{\mathrm{eff}} \rho) + \partial_\lambda \left(\frac{\partial_t \alpha f^{\mathrm{dr}} + \partial_x \alpha \Lambda^{\mathrm{dr}}}{(\partial_\lambda p)^{\mathrm{dr}}} \rho \right) = 0 \tag{4}$$

where we dropped the space-time dependence to lighten the notation. In Eq. (4) v^{eff} is the dressed velocity of the quasiparticles. Only first-order derivatives appear, implying that the equation is invariant under the rescaling $(t,x) \to (At, Ax)$, with $A \in \mathbb{R}^+$. For a space-time homogeneous dynamics ($\partial_x \alpha = \partial_t \alpha = 0$), the standard GHD equations are obtained [30,31]. The forces f and Λ are obtained by solving

$$f(\lambda) = -\partial_\alpha p(\lambda) + \int \frac{d\mu}{2\pi} \partial_\alpha \Theta(\lambda - \mu) (\partial_\mu p)^{\mathrm{dr}} \vartheta(\mu), \tag{5}$$

$$\Lambda(\lambda) = -\partial_\alpha \epsilon(\lambda) + \int \frac{d\mu}{2\pi} \partial_\alpha \Theta(\lambda - \mu) (\partial_\mu \epsilon)^{\mathrm{dr}} \vartheta(\mu). \tag{6}$$

Here $\vartheta = 2\pi\rho/(\partial_\lambda p)^{\mathrm{dr}}$ is the filling function. As usual in GHD, Eq. (4) has a clear semiclassical interpretation: $\rho(t,x,\lambda)$ locally describes the phase-space density of a collection of quasiparticles, moving with velocity v^{eff} and subjected to force terms induced by the inhomogeneities, which can change the quasiparticles rapidity. The force terms account for both single particle as well as collective effects. The former are contained in the terms $\partial_\alpha p$ and $\partial_\alpha \epsilon$ in Eqs. (5) and (6). These are the energy-momentum changes of a single excitation of rapidity λ induced by the inhomogeneities: the change in the dispersion relation causes the excitation to accelerate. Force terms due to inhomogeneities have been previously derived in Ref. [32], for spatially inhomogeneous potentials linearly coupled to the charge densities, which nevertheless cannot induce any inhomogeneity in the scattering data of the model. In Ref. [54], slow magnetic flux changes in the XXZ spin chain have been studied. In both cases, only single-particle effects arise in the GHD equation and can now be regarded as a particular case of our more general findings.

The integrals in Eqs. (5) and (6) are entirely due to collective behaviors and have never been derived in previous studies. Because of the modifications in the interparticle interactions, encoded in the scattering phase Θ, the excitations experience force fields caused by the surrounding particles.

For spatial-homogeneous interactions, i.e., $\partial_x \alpha = 0$, we are able to derive Eq. (4) for rather generic integrable models [95]. In the presence of spatial inhomogeneities, thus $\partial_x \alpha \neq 0$, Eq. (4) is derived in the presence of Lorentz invariance [95] and in Galilean invariant models through a nonrelativistic limit [96–99] (see SM [95]). Outside of the mentioned cases, we present Eq. (6) as a conjecture, although well supported by numerical evidence (see Fig. 3). As a further nontrivial check, thermal states are shown to be steady states of the GHD equation [Eq. (4)] with $\partial_x \alpha \neq 0$ [95].

We stress that, in order to have a weakly varying (locally) integrable model, a smooth dependence of $\hat{\mathbf{h}}(x,\alpha)$ [Eq. (3)] on the coupling does not suffice: the whole set of (quasi-) local charges must be smooth as a function of α. For example, our method cannot be applied to interaction changes in the XXZ spin chain with $|\Delta| < 1$, which has a fractal dependence on the coupling [13].

Applications and numerical checks.—We now show the wide applicability of our results. GHD equations are numerically solved according with the method described in the SM [95], where we also present a short summary of the TBA of the models here investigated. In Fig. 2 we show a possible application to an experimentally relevant setup, namely, a (slow) interaction quench in the interacting Bose gas [10,11]. We mention that there are no alternative analytical and numerical methods to address this type of protocols. Closely related setups have already been experimentally addressed [100].

The Hamiltonian of the Lieb-Liniger model reads $\hat{H} = \int dx \{\partial_x \hat{\psi}^\dagger \partial_x \hat{\psi} + c(t)(\hat{\psi}^\dagger)^2 (\hat{\psi})^2 + V(x) \hat{\psi}^\dagger \hat{\psi}\}$, with $[\hat{\psi}(x), \psi^\dagger(y)] = \delta(x-y)$. The gas is loaded in a harmonic trap in a low-temperature state, the interaction $c(t) > 0$ is then slowly increased. This induces a nontrivial evolution

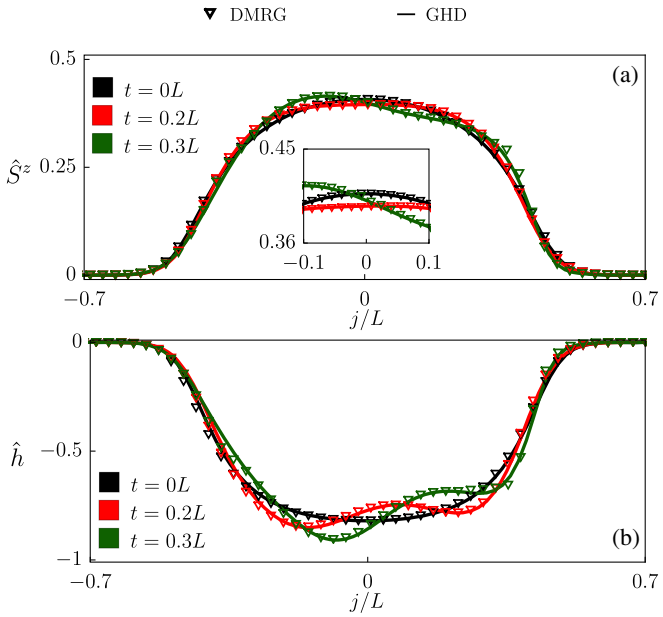

FIG. 3. Slow quench in the trapped XXZ spin chain. We apply the external static magnetic field $B_j = -1 - 8(j/L)^2$, with j the distance from the chain center and L its length. The initial state is a thermal one with $\beta = 4$. We evolve the system with the XXZ chain with $\Delta_j(t) = 1.5 + 0.3\tanh(3t/L)\sin[4\pi(j-t)/L]$. (a) Profile of the local magnetization $\hat{S}^z_j$ as a function of j/L and several times. The curves are GHD results. The symbols are tDMRG simulations for a chain with $L = 128$, and are in good agreement with the GHD. The inset shows a zoom around the center of the system. (b) Profile of the local energy density $\hat{h}_j = \hat{S}^x_j\hat{S}^x_{j+1} + \hat{S}^y_j\hat{S}^y_{j+1} + \Delta_j(t)\hat{S}^z_j\hat{S}^z_{j+1} - \Delta_j(t)/4$.

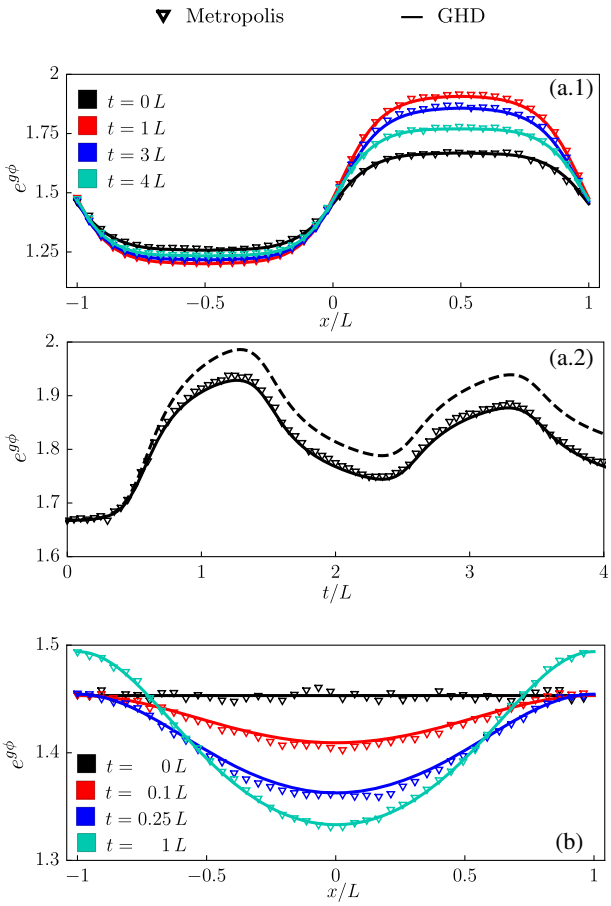

FIG. 4. Classical sinh-Gordon model: GHD results compared with classical Monte Carlo simulations. Panels (a.1) and (a.2): the system is prepared in an inhomogeneous thermal state with inverse temperature $\beta(x) = 1.25 + 0.25\tanh[2\sin(2\pi x/L)]$, with L the system length. The system is evolved with the sinh-Gordon Hamiltonian with inhomogeneous coupling $g(x) = 1.5 + 0.5\tanh[2\sin(2\pi x/L)]$ and $m = 1$. (a.1) Profile of the vertex operator $\langle e^{g\phi}\rangle$ as a function of x/L at different times. The curves are GHD predictions. Symbols are classical Monte Carlo simulations for $L = 30$. (a.2) Vertex operator $\langle e^{g\phi}\rangle$ at $x = L/2$ as a function of time. Now the continuous line is the GHD prediction. The dashed line is the result obtained by ignoring the force fields, i.e. $\Lambda = 0$ in Eq. (4), which is inaccurate, as expected. Panel (b): the system is prepared in an homogeneous thermal ensemble with $m = 1$, $g = 1.5$ and $\beta = 1.25$. At $t > 0$ we vary the mass $m \to m(t,x) = m + \Delta m 4t/L[\cos(2\pi x/L) - 1]$ for $t \le L/4$ and $m(t,x) = m + \Delta m[\cos(2\pi x/L) - 1]$ for $t > L/4$, we choose $\Delta m = 0.25$ and the interaction g is kept constant. The profiles of the vertex operator at different times are displayed.

of the quasiparticle densities, which are reported in Fig. 2(a.1). As the interparticle repulsion is increased quasiparticles increase their rapidity λ (reflected in the stretching of the initial blob along the vertical direction) and escape from the center of the trap. The local total density $n(t,x) = \int d\lambda \rho(\lambda)$ of the quasiparticles is shown in Fig. 2(a.2). Interestingly, the quench induces a breathing mode, which is long-lived in harmonic potentials [100]. This is clear from Fig. 2(a.3), where we show the density n in the center of the trap as a function of time. In Figs. 2(b.1) and 2(b.3) we focus on the slow quench in an anharmonic trap. As it is clear from Fig. 2(b.1) the anharmonicity causes a spiral motion in the filling which develops a fractal structure as times passes [47,66], which is smoothed due to the discretization used to solve the GHD equation [Eq. (4)]. Now a much faster relaxation is observed as compared with the harmonic case, due to dephasing [47,101].

In Fig. 3 we focus on the XXZ spin chain with Hamiltonian $\hat{H} = \sum_{j=-L}^{L}\{\hat{S}^x_j\hat{S}^x_{j+1} + \hat{S}^y_j\hat{S}^y_{j+1} + \Delta_j(t)\hat{S}^z_j\hat{S}^z_{j+1} + B_j\hat{S}^z_j\}$, where $\hat{S}^\alpha_j$ are standard spin-1/2 operators. The system is initialized in a confining magnetic field and in a low temperature thermal state, with a uniform interaction $\Delta_j > 1$. Then, Δ_j is slowly changed with time in the form of a traveling wave (see Fig. 3). In Fig. 3 we compare the GHD predictions for the local magnetization and the local energy density with time-dependent density matrix renormalization group (tDMRG) simulations [102–104], finding excellent agreement. tDMRG methods suffer strong limitations in the accessible time scale; therefore we revert to the classical world to explore longer time scales. In Fig. 4 we benchmark the GHD in the classical sinh-Gordon model (see Ref. [105] and [62] for the TBA). The model describes a scalar field ϕ with

Lagrangian $\mathcal{L}=\int dx\{\partial_\mu\phi\partial^\mu\phi/2-(m/g)^2[\cosh(g\phi)-1]\}$, with m the mass and g the interaction parameter. We consider two protocols. In the first case [panels (a.1) and (a.2)] the system is initially prepared in a thermal state with an inhomogeneous temperature profile. The system is then evolved with the sinh-Gordon Hamiltonian with inhomogeneous coupling $g\to g(x)$. Such an inhomogeneity does not affect the single-particle dispersion law (see SM [95]), providing an ideal benchmark to test the collective effects in Eqs. (5) and (6). Symbols are Monte Carlo data [62], whereas the lines are the GHD results. In (a.1) we show the expectation value of the vertex operator $e^{g\phi}$ as a function of position x/L for different times, in (a.2) we plot $e^{g\phi}$ at $x=L/2$ as a function of time. The agreement with the GHD is spectacular. The dashed line is the GHD result neglecting the collective effects, i.e., the integrals in the right-hand side in Eq. (5), which clearly have a crucial role. In the second protocol [panel (b)], we start from a homogeneous thermal ensemble, then a mass inhomogeneity is slowly activated with a linear ramp in time up to $t=L/4$ and then kept constant. Mass changes do not affect the scattering data [95], implying that only single-particle effects in Eqs. (5) and (6) play a role. Despite the form of the force terms, mass inhomogeneities do not belong to the class of inhomogeneities described in Ref. [32] and thus provide a nontrivial benchmark of our findings.

Conclusions and outlook.—The success of hydrodynamic approaches is hard to overestimate. GHD merges the hydrodynamic framework with integrability, providing unprecedented levels of accuracy in describing out-of-equilibrium systems. In this Letter we extended the reach of this program, providing hydrodynamic equations which account for arbitrary (smooth) inhomogeneities in the couplings and state. Several interesting questions are left out for the future. Our analysis holds true when the model has a smooth dependence on the inhomogeneous coupling, but there could be special points (or regions) where this hypothesis breaks down. Understanding the behavior of protocols overcoming such special points is surely a compelling quest, which can unveil a rich phenomenology (see Ref. [54] for a closely related problem). Including higher order corrections in the derivative expansion at the root of Eq. (4) is another important direction. Finally, it is important to devise numerical schemes based on molecular dynamics, such as the flea gas [33], to simulate the GHD equations [Eq. (4)].

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